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Optimal Financial-Market Integration and Security Design*

I. Introduction

Innovations in financial markets have taken different forms, from the introduction of new assets to the integration of segmented markets. The new financial assets introduced in the final decades of the twentieth century include options, swaps, inflation-indexed, bonds, and various other derivatives, such as catastrophe bonds and credit protections. But many recent financial innovations represent instances of integration of segmented markets.[1] For instance, the introduction of electronic trading reduced the transaction costs for small brokers and retail investors, thereby integrating their trades in many national financial exchanges (Naik and Yadav 1999). Similarly, the

We study 2-period pure-exchange Capital Asset Pricing Model (CAPM) economies with incomplete financial markets and restricted participation. We characterize the optimal financial-market structure and efficient innovations consisting of both the introduction of new assets and the integration of segmented markets. Welfare gains from innovations are maximal when the endowments of affected agents are negatively correlated. Uncoordinated innovations lead to efficient market structures if all assets have identical participation structure or the markets being integrated trade identical assets. However, coordination failure in the introduction of new assets may result when assets have participation structures that overlap only partially.

1. The empirical evidence for the segmentation of financial markets is ample. Blume and Friend (1975), Mankiw and Zeldes (1991) and Blume and Zeldes (1993) document that investors within an economy do not seem to hold the entire market portfolio and, in fact, hold a very small number of individual securities. French and Poterba (1991), Kang and Stulz (1997), and Lewis (1999) document the "home bias puzzle," which is a form of limited participation by investors in foreign securities.

practice of securitization has allowed liabilities such as mortgages, credit card debt, and local bank debt, originally held by specific classes of creditors, to be traded by national financial markets (Kendall and Fishman 1998). Finally, the integration of international commerce among the OECD (Organization for Economic Cooperation and Development) countries has led to the integration of previously segmented financial markets, as evidenced by a decline in the "home bias" (Wei 1996 and Mann and Meade 2002).

The theoretical literature on financial innovation has analyzed the case where new financial assets are introduced to span the uncertainty of agents' endowments and production plans in economies with incomplete financial markets (see the surveys in Allen and Gale 1994 and Duffie and Rahi 1995). However, this literature has paid much less attention to the limited participation in financial markets and the integration of segmented markets as a form of financial innovation. This is true also for the study of innovations in international financial markets[2] and the theoretical and empirical analysis of optimal currency areas.[3] Such limited attention to issues of international financial integration is at odds with the reality of integration processes. For instance, the recent convergence of the European economies toward the Economic and Monetary Union (EMU) and the launch of the single currency have significantly affected the stock market integration of the member countries. Danthine, Giavazzi, and von Thadden (2000) report that foreign equity holdings of German investment funds as a share of total assets increased from less than 4% in 1990 to more than 20% in 1998; and Hardouvelis, Malliaropulos, and Priestley (2000) report that foreign equity holdings of pension funds (insurance companies) in EMU countries increased on average from 29% (11%) of total equity holdings in 1992 to more than 50% (30%) in 1999, in contrast to the non-EMU countries for which these numbers remained unchanged at roughly 20% (25%).[4]

In this paper, we study financial innovations consisting of both the introduction of new assets and the integration of segmented markets. Our analysis proposes factors that determine the welfare gains from financial integration, in terms of the properties of the income processes of member agents. Moreover, we identify conditions under which a

2. For instance, Wincoop (1994), Athanasoulis and Shiller (2001), and Davis, Nalewaik, and Willen (2001) deal with the introduction of new assets rather than with the integration of segmented financial markets.

3. The literature on currency areas, including Helpman and Razin (1982), Neumeyer (1998), Ching and Devereux (2000), and Alesina, Barro, and Tenreyro (2002) focus attention principally on trade liberalizations and currency unions for economies that already exhibit financial-market integration.

4. See also Bris, Koskinen, and Nilsson (2002), who report that the increasing capital-market integration in Europe also lowered the cost of debt capital for firms in member countries and these firms have also issued greater equity than before.

coordination of financial innovations (e.g., through a consolidation of local exchanges) is likely to be socially desirable.

We discuss our analysis and results through a leading example economy that is composed of large investors, such as firms and banks, and small investors, such as retail households. Large investors have unrestricted access to financial markets whereas small investors have access only to local markets. Local markets could be specialized financial markets or geographically restricted markets; but they could also represent national markets in an international context. Trade in local markets is organized through exchanges. Suppose diseconomies of scale prevent these local markets from fully integrating the trades of all investors in the economy.[5] Suppose also that complete financial markets are not feasible because of transaction costs. Thus, our example economy consists of financial markets that are incomplete and segmented. Exchanges in the economy have incentives to innovate in the form of a partial integration of their trades as well as the introduction of new financial assets.

We first characterize the optimal structure of financial assets and the optimal integration level of markets. Specifically, we address the following questions: What is the optimal collection of financial assets that provides the best risk-sharing opportunities in the economy? With what should the payoffs of these assets be correlated? On what factors do the welfare gains from integration of local financial markets depend? Which local markets are the best to integrate?

If lump-sum transfers can compensate agents in the economy for negative relative price effects, then we show that the optimality of financial structures is determined only by the associated "betas," that is, by the covariances of each agent's endowment with each asset's payoff, normalized by the variance of the asset payoff. The aggregate welfare associated with a financial-market structure increases with (an appropriate measure of) the dispersion of the betas. This characterization provides simple answers to the preceding questions.

Innovation in the form of a newly introduced asset proves optimal if it maximizes the sum of the distances between the betas of each pair of agents allowed to trade in the new asset. Further, the structure of financial assets is optimal if all assets are designed so as to maximize risk sharing. As a result, an optimal financial-market structure is achieved when asset payoffs are correlated with the most crucial factors (the principal components) that drive the dispersion of the endowments of agents.

Similarly, it is optimal to integrate two groups of agents in the same market if the distance between the mean betas of the agents in the two

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5. Diseconomies of scale can be due to monitoring costs or the costs of setting up an integrated clearing system. In international financial markets, for example, diseconomies of scale might be related to other aspects of integration, such as policy coordination in a currency union.

groups is maximal. For instance, two countries, whose financial markets are segmented because of the investors' home bias or the lack of harmonization of market regulations, gain maximally from financial integration if their aggregate endowment processes are negatively correlated. While this result naturally arises from the risk-sharing motive for financial integration, it is strikingly at odds with the prescriptions for integration deriving from other considerations, such as the integration of monetary policy in currency unions.[6] Our results suggest a need to reconsider the estimated benefits of the formation of unions that are also associated with the integration of financial markets. Since many countries in continental Europe are fairly similar, the literature on the European Union has often focused entirely on the monetary issues. In contrast, and in the spirit of our proposal, the approach of the U.K. Treasury has been to carefully weigh the trade-off between the loss of flexibility in monetary policy upon entering the euro area and the gains to be made through international capital-markets integration.[7]

These results concern the characterization of optimal innovations and financial-market structures. Financial-market structures, however, result from the innovations introduced by decentralized intermediaries. Hence, we also examine the efficiency properties of a class of decentralized innovation processes. We show that optimal financial structures do result from a decentralized innovation process whenever financial innovation consists of either the introduction of new assets into an economy without restricted participation constraints or the relaxation of restricted participation constraints for an existing asset. In contrast, optimal financial market structures may not necessarily result from a decentralized innovation process when the innovation consists of the introduction of new assets into economies with restricted participation.

Consider again the setting where large financial institutions participate in financial markets of all exchanges, but each exchange has a clientele of retail investors who participate only in the local exchange. In an international context, each exchange belongs to a different economy and has a clientele of domestic retail investors. Each exchange

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6. The theory of optimal currency areas has suggested that the benefits of currency unions are the highest for those countries with the highest comovements in output (see, e.g., Alesina et al. 2002).

7. The executive summary of the five economic tests relating to the U.K. government policy on the euro states: "When in 1997 the Government committed the UK to the principle of joining the single currency, the Chancellor (Gordon Brown) stated that [...] the advantages are [...] lower transaction costs, [...] less exchange rate volatility, more incentives for cross-border trade and investment, and potentially lower long-term interest rates. [...] EMU entry could reduce the cost of capital for UK firms if membership of a larger financial market reduced the cost of finance. [...] But, on balance, [...] though the potential benefits of investment, trade, a boost to financial services, growth and jobs are clear, we cannot at this point in time conclude that there is sustainable and durable convergence or sufficient flexibility to cope with any potential difficulties within the Euro area."

introduces a new asset that maximizes the welfare of its participating agents (comprising the financial institutions and the retail investors local to that exchange). Trades in this new market affect the trades of financial institutions in other exchanges. This, in turn, endogenously affects the optimal structure of innovations by these other exchanges: Each such exchange maximizes the welfare of its own participating agents, taking as given the existing assets as well as the assets already introduced by other exchanges. It is exactly the possibility of such overlapping sets of participating agents in different exchanges that potentially leads to the market failure of uncoordinated financial innovations. The failure arises despite the alignment of each exchange's objective with the welfare of its participating agents.

This result has important implications for the social desirability of coordination or harmonization among innovating intermediaries. Specifically, it helps in understanding phenomena such as the recent trend toward pan-European exchanges, the alliances between exchanges and over-the-counter intermediaries for creating "hybrid markets," and various other instances of consolidation or collaboration partly motivated by the desire to coordinate on new products. An example of the last is the recent tie-up between the Chicago Board of Trade and Euronext-Liffe exchanges.

The remainder of the paper is organized as follows. Section II sets up the model. Section III introduces the welfare measure with which we compare financial-market structures. Section IV characterizes the optimality of financial-market structures. Section V studies the welfare effects of financial innovations, and Section VI examines their decentralizability. A reader not interested in our general equilibrium analysis of financial innovations can safely skip Section II. Section VII concludes. The proofs are in the appendix.

II.   The Economy

We examine financial innovations in 2-period pure-exchange Capital Asset Pricing Model (CAPM) economies in which financial markets are incomplete and traders' participation in financial markets is restricted (Willen 1997). Endowments and asset payoffs in these economies are normally distributed, and agents have negative exponential utility. Since agents consume in both periods, they trade to smooth wealth across time as well as to diversify risk. In turn, a real risk-free interest rate is well-defined, and financial innovations affect the equilibrium risk-free rate. We show later that financial innovations reduce the demand for precautionary savings and raise the interest rate in equilibrium.[8]

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8. The properties of competitive equilibria for the class of economies we examine have been extensively studied, notably by Willen (1997) and the references therein.

Formally, an economy is populated by H agents who live for 2 periods, 0 and 1.

Agent $h \in \mathcal{H} := \{1, \ldots, H\}$ has a safe endowment y_0^h of the unique consumption good in period 0 and a random endowment y_1^h in period 1.

ASSUMPTION 1.   Endowments are normally distributed: Let v denote an N-dimensional column vector of multivariate normal random variables with mean 0 and variance-covariance matrix I, the identity matrix; endowments y_1^h, for any h, are

$$y_1^h := Y^h v, \; Y^h \in \Re^N.$$

Note that y^h is an N-dimensional row vector with scalar entries.

Each agent is also endowed with Von Neumann-Morgernstern preferences.

ASSUMPTION 2.   The utility function of agent h is as follows:

1. Time and state are separable:

$$u^h(c_0, c_1) := u^h(c_0) + u^h(c_1).$$

2. Absolute risk aversion is constant (CARA) with identical coefficient of absolute risk aversion, $A > 0$, across agents:[9]

$$u^h(c) := -\frac{1}{A} e^{-Ac}.$$

In financial markets, a risk-free bond and J risky assets are traded. The bond, asset 0 in our notation, has a payoff $x_0 = 1$ (in units of the consumption good) with probability 1. On the other hand, asset j's payoff, denoted x_j is random if $j \in \mathcal{J} = \{1, \ldots, J\}$.

ASSUMPTION 3.   Assets' payoffs are normally distributed: Assets' payoffs $x := [x_j]_{j \in \mathcal{J}}$ are multivariate normal random variables with mean 0 and variance-covariance matrix I, the identity matrix.

Let $\mathcal{J}^h$ denote the set of assets that agent h is allowed to trade. Let $\mathcal{H}_j$ denote the set of agents allowed to trade asset j, H_j being the size of the set. In general, we allow for incomplete markets, $J < N$, and restricted participation, $\mathcal{H}_j \subset \mathcal{H}$, for some j. We impose the following assumption that cannot be relaxed without losing the closed-form characterization of the equilibrium.

ASSUMPTION 4.   All agents h are allowed to trade the risk-free bond:

$$\mathcal{H}_0 = \mathcal{H}.$$

Fixing the endowments, $[y_0^h, y_1^h]$, and the set of agents in the economy, $\mathcal{H}$, we parameterize a financial market structure by the list of tradable

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9. Only notational complications are added by allowing heterogeneity in absolute risk aversion parameters.

assets, their payoffs, and their participation sets: $[x_j, \mathcal{H}_j; j \in \mathcal{J}]$. To introduce general financial innovations, we say that a financial structure $F' = [x'_j, \mathcal{H}'_j; j \in \mathcal{J}']$ innovates on $F = [x_j, \mathcal{H}_j; j \in \mathcal{J}]$, if F' either adds assets or relaxes some participation restriction (or both) with respect to F; that is, when

$$\mathcal{J} \subseteq \mathcal{J}', x_j = x'_j \text{ if } j \in \mathcal{J}, \text{ and } \mathcal{H}_j \subseteq \mathcal{H}'_j \text{ for all } j,$$

with at least one of the subset relationships being that of a proper subset.

We associate a vector of "betas" to each financial structure. Specifically, for each risky asset, there is a set of "endowment beta" (β_j^h) of individual agents with respect to that asset, as well as an "average beta" (β_j), the average of the endowment betas of all agents not restricted from trading that asset:

$$\beta_j^h := \frac{\text{cov}(y_1^h, x_j)}{\text{var}(x_j)}; \quad \beta_j := \frac{\text{cov}\left(\frac{1}{H_j} \Sigma_{h \in \mathcal{H}_j} y_1^h, x_j\right)}{\text{var}(x_j)}, \tag{1}$$

where $h \in \mathcal{H}, j \in \mathcal{J}$.

The converse is not true: assets' payoffs x_j and agents' endowments y^h are characterized by the variance of y^h, the variance of x_j, and their covariance (means are normalized to zero by assumption 3), which cannot be recovered uniquely from the knowledge of betas.

III. Competitive Equilibria and Welfare

We study and characterize competitive equilibria for the class of economies just introduced.

Let $\pi_0 \in \mathcal{R}_+$ denote the price of the risk-free bond, and $\pi j \in \mathcal{R}^J$ the price of the asset j. The problem of each agent h is to choose consumption in period $0, c_0^h$, a random consumption in period 1, c_1^h, and portfolio position in the risk-free bond and in all tradable assets, $[\theta_0^h, \theta_j^h]_{j \in \mathcal{J}} \in \mathcal{R}^{J+1}$, to maximize expected utility

$$E[u^h(c_0^h, c_1^h)] := -\frac{1}{A} e^{-Ac_0^h} + E\left[-\frac{1}{A} e^{-Ac_1^h}\right] \tag{2}$$

subject to the budget constraints and the restricted participation constraints:

$$c_0^h = y_0^h - \pi_0 \theta_0^h - \sum_{j \in \mathcal{J}} \pi_j \theta_j^h, \tag{3}$$

$$c_1^h = y_1^h + \theta_0^h + \sum_{j \in \mathcal{J}} \theta_j^h x_j, \quad \text{and} \tag{4}$$

$$\theta_j^h = 0, \ j \notin \mathcal{J}^h. \tag{5}$$

DEFINITION 1. A competitive equilibrium is a consumption and portfolio allocation $(c_0^h, c_1^h, \theta_0^h, \theta_j^h$ for each $j)$, for all agents h, and a price vector $(\pi_0, \pi_j,$ for each $j)$, such that the consumption and portfolio allocations maximize (2) subject to (3)–(5) for each agent h, and consumption and financial markets clear:

$$\sum_h (c_0^h - y_0^h) \leq 0, \tag{6}$$

$$\sum_h (c_1^h - y_1^h) \leq 0, \quad \text{and} \tag{7}$$

$$\sum_h \theta^h = 0. \tag{8}$$

A. Characterization of Equilibria

Closed-form solutions for equilibrium allocations and prices are easily derived (see Willen 1997; we report the solution in Appendix A for completeness). It suffices here to note that, at the competitive equilibrium,

1. The price of any existing asset j, π_j, relative to the price of the bond, π_0, is independent of the set of assets traded in the economy. This follows from the property of the equilibrium that the relative price of asset j depends only on the covariance between asset j's payoff and the aggregate endowment of agents not restricted from trading asset j:

$$\frac{\pi_j}{\pi_0} = E(x_j) - A \operatorname{cov}\left(\frac{1}{H_j} \sum_{h \in \mathcal{H}_j} y_1^h, x_j\right), \quad j \in \mathcal{J}. \tag{9}$$

2. Each agent holds the bond, the market portfolio, and the unhedgeable component of his endowment:

$$c_1^h = y_1^h - \sum_{j \in \mathcal{J}^h} \beta_j^h x_j + \sum_{j \in \mathcal{J}^h} \beta_j x_j + \theta_0^h, \tag{10}$$

where β_j^h and β_j are as defined in equation (1).

B. Welfare

As a measure of the welfare associated with an arbitrary financial-market structure F, we consider the average welfare gains under structure F, relative to the autarkic financial-market structure in which only

the risk-free bond is traded. More precisely, the welfare of financial structure F is measured by the compensating aggregate transfer: the amount of time-0 consumption allocation whose lump-sum redistribution across all agents in autarky makes them indifferent between the consumption allocation under the financial structure F and the allocation under autarky. The compensating aggregate transfer is the appropriate measure of the welfare gains of a particular financial structure (with respect to autarky) for an economy in which it is possible to make lump-sum transfers across agents to redistribute welfare gains and losses.[10]

Using the closed-form competitive-equilibrium solution (Appendix A), it is straightforward to show that the compensating aggregate transfer of financial-market structure, F, μ_F, depends only on the equilibrium price of the risk-free bond under the financial structure F, π_0^F, and its counterpart under the autarkic structure, π_0^a; in particular (see Willen 1997):

$$\mu_F = -\frac{1}{A} \ln \frac{1 + \pi_0^F}{1 + \pi_0^a}. \tag{13}$$

The welfare associated with a financial-market structure F decreases with π_0, the equilibrium price of the risk-free asset and thus increases with the risk-free interest rate: the price of the risk-free asset is low when the agents' precautionary component of savings is relatively low, that is, when a large fraction of the risk in the economy is hedged.

IV. Optimal Financial Structures

We turn to the characterization of optimal financial-market structures and optimal financial innovations.

Consider the set of economies in which markets are not complete, that is, $J < N$, or the participation in financial markets is restricted, that is, $H_j < H$, for some j. The degree of market incompleteness as well as the degree of restricted participation is exogenously determined

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10. Formally, let $[c_0, c_1] := [c_0^h, c_1^h]_{h \in \mathcal{H}}$; let $U([c_0, c_1])$ denote the average welfare associated with the consumption allocation $[c_0, c_1]$:

$$U([c_0, c_1]) := \frac{1}{H} \sum_{h \in H} \left(-\frac{1}{A} e^{-Ac_0^h} + E\left[-\frac{1}{A} e^{-Ac_1^h} \right] \right). \tag{11}$$

Let $[c_0^F, c_1^F]$ be the equilibrium allocation of the economy with financial structure F; and let $[c_0^a, c_1^a]$ be the equilibrium allocation for the autarkic economy, in which no agent can trade any assets except the risk-free asset. The compensating aggregate transfer of F, μ_F, by definition, solves

$$U([c_0^a + \mu_F, c_1^a]) = U([c_0, c_1]). \tag{12}$$

(e.g., by the nature of transaction costs) and affects the set of feasible financial-market structures for the economy. We study the optimality of financial-market structures in such a restricted feasible set. In other words, we provide a characterization of the financial-market structure $F = [x_j, \mathcal{H}_j; j \in \mathcal{J}]$ that maximizes μ_F, given J and H_j, for all $j \in \mathcal{J}$.

The next lemma characterizes such a financial structure. It shows that the optimality of a financial structure can be determined by looking only at its associated vector of betas, rather than at the whole variance-covariance matrix of endowments and assets' payoffs: the optimal financial structure maximizes an appropriate measure of the "dispersion" of the betas in the population, specifically the sum of the squared distances between the betas of each pair of agents in each possible market.[11]

LEMMA 1 (BETA REPRESENTATION). The compensating aggregate transfer μ_F is maximal for the financial structure F whose betas maximize

$$\sum_{j \in \mathcal{J}} \sum_{h \in \mathcal{H}_j} \sum_{h' \in \mathcal{H}_j} \frac{1}{H_j} \left(\beta_j^h - \beta_j^{h'} \right)^2. \tag{14}$$

Such characterization can be substantially sharpened if we consider economies in which each agent's participation in financial markets is unrestricted; that is, $H_j = H$, for any j.

Suppose the economy's financial market structure is restricted to be composed of J assets that all agents can trade. Which are the J optimal assets for such an economy, that is, the J assets whose betas maximize (14)? The following proposition answers this question.

PROPOSITION 1 (PRINCIPAL-COMPONENTS CHARACTERIZATION). Suppose that market participation is not restricted in any asset:

$$\mathcal{H}_j \equiv \mathcal{H}, \quad \text{for all } j \in \mathcal{J}.$$

Then, a financial structure F maximizes the compensating aggregate transfer μ_F if the asset payoffs, $[x_j]$, are a linear combination of the agents' endowments:

$$x_j := R_j v, R_j \in \Re^{\mathrm{N}};$$

and the columns of $R' := [R_j]'$ are spanned by the J principal components associated with the J largest eigenvalues of the matrix

$$M = \sum_{h \in \mathcal{H}} (Y^h - Y)'(Y^h - Y),$$

where $Y = 1/H \sum_{h \in \mathcal{H}} Y^h$.

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11. We assume in what follows that $H_j \geq 2$ for any j, without loss of generality. Since assets are in zero-net supply, an asset j such that $H_j = 1$ is not traded in equilibrium.

The optimal financial structure is the set of J assets with payoffs that are particular linear combinations of endowments: those that produce a maximum dispersion of betas across the agents (lemma 1). If the participation in any asset is unrestricted, then the optimal assets are the eigenvectors corresponding to the J largest eigenvalues of the matrix M. Matrix M represents the dispersion in the individual agents' endowment processes. The optimal assets are thus the principal "factors" driving this dispersion, and they capture as much of the risk-sharing opportunities of the economy as possible. In other words, the optimal assets are composed of as many such factors as possible, in the order of their importance for risk-sharing opportunities, given the level of market incompleteness.[12]

This characterization of optimal financial structures coincides with that derived by Demange and Laroque (1995), even though our economy differs in an important way from theirs: we allow for consumption in period 0 and, hence, for welfare effects through changes in the risk-free rate at the competitive equilibrium. The intuition of this result is as follows.[13] Risky assets are in zero-net supply in our setup; and prices of risky assets, relative to the price of the risk-free asset, therefore depend only on the covariances of the second-period endowments with risky asset payoffs. In particular, this implies that prices of risky assets relative to each other are identical under the two setups. In our model, the risk-free asset is traded by agents as a precautionary motive against unhedged second-period risks. Thus, the price of the risk-free asset captures precisely the average risk-sharing available to agents in the economy (as revealed by equation [13]). Demange and Laroque also measure the welfare in their 1-period economy in terms of this average risk sharing. Since the welfare under the two setups is identical (up to constant terms), it follows that the optimal financial structures are identical as well.

The characterization of optimal financial-market structures of lemma 1 can also be specialized to study the optimal composition of the agents trading a given asset. Consider an arbitrary asset j, with payoff x_j. Suppose that, due to diseconomies of scale, there is a maximal number H_j of agents who are allowed to participate in trading asset j. In this context, what is the optimal composition of the set of agents who are allowed to trade; that is, which set $\mathcal{H}_j$ maximizes aggregate welfare

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12. We implicitly assume that the intermediaries or the planner introducing financial assets knows precisely the endowment processes. Athanasoulis and Shiller (2000) analyze innovation in a similar CAPM model where the innovator has imprecise knowledge about endowments. They show that, in this case, the optimal asset corresponds to a "market portfolio" that puts an appropriate weight on each of the principal components that drive the dispersion in agents' endowments. While this result is interesting in its own right, we conjecture that our results on the integration of financial markets and the decentralizability of optimal financial structures are qualitatively robust to such an extension.

13. We are grateful to an anonymous referee for providing this insight.

(14)? Diseconomies of scale can be due to the difficulty of policy coordination in a currency union or to the costs of social and economic policies in culturally heterogeneous societies. More specifically, the issue at hand is related to the optimal composition for integration of economies with a focus on financial integration rather than policy co-ordination or trade liberalization (as studied in Alesina, Spolaore, and Wacziarg 2000).

The next proposition addresses this issue.

PROPOSITION 2 (TRADERS' COMPOSITION). Assume that agents' betas for asset j are distinct, and agents: are ordered such that

$$\beta^N > \cdots > \beta^h > \cdots > \beta^1.$$

There exists an $r < H_j$ such that the optimal composition of agents trading asset j consists of the first r agents and the last $H_j - r$ agents; that is, $\mathcal{H}_j = \{1, \ldots, r, H - (H_j - r) + 1, \ldots, H\}$.[14]

The optimal composition of agents allowed to trade an arbitrary asset j in an economy with restricted participation consists of two sets of agents, the sets being at the two extremes of the ranked betas of the agents with respect to the asset. The distribution of agents in the two extreme sets depends, in general, on the structure of the endowments. Once again, this characterization suggests that optimal financial-market structures maximize a measure of the dispersion of betas across the agents affected by the innovations.

To better evaluate the implications of this analysis, consider the case of international financial integration of a collection of countries with endowment processes driven by a common series of factors. Each country's endowment differs in terms of the magnitude and the sign of its factor loadings. Allowing only a subset of these countries to participate in financial-market integration, the union will achieve maximal risk sharing and welfare by selecting a particular subset: those countries with the greatest variation in dependence on the factors. An example of such a set are those countries with endowments that are positively correlated to the maximal extent with a common factor, along with those countries with endowments that are maximally negatively correlated with that factor. Countries with segmented financial markets, for example, because of the investors' home bias or the lack of harmonization of market regulations, gain maximally from financial integration if their endowment processes are negatively correlated. This is a natural consequence of the risk-sharing motive for financial integration and our general characterization of the optimality of financial-market structures (lemma 1). It should be noted that this result has a parallel in the

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14. See lemma 3 in Appendix B, where the proposition is proved.

literature on international trade: the gains from trade in a Heckscher-Ohlin setting are also greater for economies with negatively correlated output shocks (see Dixit and Norman 1989, chaps. 3 and 4).

However, our result is at odds with the prescriptions derived by the literature on the theory of optimal currency areas; such literature abstracts from financial integration and instead stresses the ease of monetary policy coordination and its associated benefits. These benefits grow in accordance with the comovements in the member countries' outputs, that is, when there exists a positive correlation in the income processes of the different economies (see, e.g., Alesina et al. 2002). Since there is no money in our model, it is difficult to directly compare the present analysis with the models of optimal currency areas. Our analysis is nonetheless relevant in evaluating the benefits of currency areas between countries whose financial markets are not fully integrated. In fact, to determine the optimal composition of a currency union when member countries might experience financial integration, the analysis must trade off gains due to the coordination of monetary policies with those due to financial integration. As we argued in the Introduction, the evidence shows that financial integration is a relevant component of welfare gains even for the European Monetary Union (EMU),[15] and its effects on the cost of capital have been crucial to the cost-benefit analysis made by the U.K. Treasury regarding joining the EMU.[16]

Importantly, our analysis implies that the gains from financial integration potentially bear even greater relevance if the EMU is enlarged to include economies with income processes driven by largely different factors. We argue that the cost-benefit analysis for enlarging the euro zone should not consider only the difficulties associated with conducting a common monetary policy in economies that have very different income processes: such an analysis would underestimate the welfare gains from the enlargement of the union by disregarding the gains due to financial integration, which increase precisely when income processes are less correlated.[17]

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15. See the wealth of data, in this respect, contained in Danthine et al. (2000), Hardouvelis et al. (2000), and Bris et al. (2002).

16. See the discussion on the five economic tests and the cost of capital in the reports by Her Majesty's Treasury (2003a, 2003b).

17. A group of 11 countries of Central and Eastern Europe, together with Cyprus, Malta, and Turkey, have applied for full membership of the European Union. From the standpoint of economic output, this block of countries differs substantially from the block of countries in continental Europe that are currently members of the EMU. The conditions and criteria for membership of these applicant countries are based primarily on factors that help assess the difficulties in harmonization of monetary policy. These are the Copenhagen Criteria of 1993 and the Maastricht Criteria of 1998, which depend on inflation, interest rate, exchange rate, public deficit level, and independence of the central bank. However, there has been little discussion of welfare gains through the likely financial integration of continental Europe with Central and Eastern Europe, markets that to date remain largely segmented (Europarl Report 1999).

V.   The Welfare Effects of Financial Innovations

We turn now to study the implications of the characterization of optimal financial structures derived in Section IV. Specifically, we study separately the welfare implication of financial innovations consisting of (1) the introduction of new assets and (2) the integration of existing but segmented markets for the same financial asset. It follows from equation (13) that the measure of the incremental welfare gains associated with a financial structure F' that innovates on F is $\mu_{F'} - \mu_F$. The following is then a simple implication of lemma 1.

PROPOSITION 3.   If a financial structure F' innovates on another financial structure F, no welfare losses are possible, $\mu_{F'} \geq \mu_F$. Moreover, strictly positive welfare gains are realized, $\mu_{F'} > \mu_F$, if the innovation involves the integration of agents with different betas or if the innovation involves the introduction of an asset tradable by agents with different betas; that is, if for some $j \in \mathcal{J}'$, there exist $h, h' \in \mathcal{H}'_j$ such that $\beta^h_j \neq \beta^{h'}_j$ and either $j \notin \mathcal{J}$ or $h' \notin \mathcal{H}_j$.

Thus, financial innovations invariably have positive welfare effects as measured by the aggregate compensation transfer. However, a subset of the agents might have to be compensated by lump-sum transfers after a financial innovation since they might experience a loss due to the change in the risk-free asset's price. This is a feature of the model with consumption in period 0. The 1-period CAPM economy studied by Demange and Laroque (1995) has no price effects due to financial innovations.

We consider first financial innovations consisting of the introduction of an asset, given a set of preexisting assets in the economy.

PROPOSITION 4 (INTRODUCTION OF A NEW ASSET).   Suppose a financial innovation consists of introducing a new asset j'; that is, F' is like F except that $\mathcal{J}' = \mathcal{J} \cup \{j'\}$. The welfare gain of such innovation, $\mu_{F'} - \mu_F$, is increasing in

$$\sum_{h \in \mathcal{H}_{j'}} \sum_{h' \in \mathcal{H}_{j'}} \frac{1}{H_{j'}} \left(\beta^h_{j'} - \beta^{h'}_{j'} \right)^2. \tag{15}$$

The welfare gains due to the introduction of a new asset j' depend only on the dispersion of the betas (relative to asset j') of agents allowed to trade the asset. In other words, the resulting welfare gains are independent of the betas relative to all assets traded before the innovation. An innovation that consists of the introduction of a single asset with unlimited participation is optimal if its betas are maximally dispersed across the agents, that is, if the extent of risk sharing introduced by the asset is maximal. In the context of the innovation of an asset market in which financial institutions from different countries will participate, resulting welfare gains increase if the endowments (cash flows) of these

participating financial institutions are dispersed in terms of their co-variance with the new asset's cash flows.[18]

We now address the welfare effects of innovations that consist of the integration of two distinct markets for the same financial asset.

PROPOSITION 5 (INTEGRATION OF TWO DISTINCT MARKETS).   Suppose the financial structure F has the property that assets j and j' have the same payoff, $x_j = x_{j'}$ but are traded in distinct markets, $\mathcal{H}_j \cap \mathcal{H}_{j'} = \emptyset$. A financial innovation that integrates such markets, that is, a financial structure F', which is like F except that $\mathcal{H}'_j = \mathcal{H}_j \cup \mathcal{H}_{j'}$, has a welfare gain $\mu_{F'} - \mu_F$ that is increasing in

$$\frac{H_j H_{j'}}{H_j + H_{j'}} (\beta_j - \beta_{j'})^2.$$

The welfare gains of innovations that consist of the integration of markets increase in the number of agents integrated. Most important, given the number of agents in each group, the welfare gains of market integration depend on only the difference between the average betas of the two groups, β_j and $\beta_{j'}$, and not on the individual betas of the agents in the groups. Keeping constant the size of the markets, the integration of two distinct markets is optimal when no other pair of markets exists with a greater average difference in betas. For example, the welfare gain of allowing agent h' to trade asset j is increasing in $(\beta_j - \beta_j^{h'})^2$, the difference between the beta of agent h' with respect to asset j and the average of the betas of agents trading asset j before the integration; whereas the integration in some market j of agents whose beta matches the average of the betas of the traders in that market has no welfare effects.

This result shows in a particularly succinct manner that to estimate the risk-sharing benefits from financial integration of member countries in a currency union, it suffices to examine only the average endowment processes of economies that will participate in the integrated financial arena. Furthermore, the benefits from financial integration are minimal if these average endowment processes of member countries are close to each other. In contrast, if the average endowment processes of member countries are dispersed in terms of their loadings on traded financial assets, then the benefits of financial integration can be significant. This underscores once again the difference between our result and that of the optimal currency areas literature, which ignores the integration of financial

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18. In a related setup but with no restricted participation, Duffie and Jackson (1989) show that the optimal innovation in the financial asset market maximizes the total transaction volume. It can be shown (Appendix B, lemma 2) that our optimality criterion is equivalent to maximizing the sum of squared transaction volume. This difference in the characterization of the optimal asset arises from a difference in the operational definition of optimality: Duffie and Jackson study the Pareto optimality of a financial innovation in a setting where lump-sum transfers are not allowed.

markets and concludes that the benefits from harmonization are maximized if member countries are similar in their endowment processes.

VI. The Decentralizability of Financial Innovations

For an economy in which markets are not complete or participation in financial markets is restricted, Section V characterized the optimal financial-market structure. But can such a financial-market structure be decentralized as an equilibrium of economies in which financial intermediaries or exchanges introduce new securities and integrate segmented markets in an uncoordinated fashion?

We do not explicitly model the process by which innovations enter the financial markets. This would require the sources of market incompleteness to be explicitly modeled, along with the direct or indirect costs associated with any specific financial innovation. Further, it would entail a strategic analysis of financial institutions, intermediaries, and exchanges.[19] In other words, our analysis is silent on the costs associated with financial innovation and integration, costs that might exceed the advantages for a specific innovation or integration. Nevertheless, our analysis sheds light on the "decentralizability" of optimal financial structures through financial innovations that are introduced sequentially by different intermediaries.

Suppose that, given J, the number of asset markets, and H_j, the size of each market, $j \in \mathcal{J}$, each innovation is introduced into financial markets independent of the others, so as to satisfy the orthogonality of asset payoffs and maximize the optimality criterion, (14), for the existing financial market structure. Innovations might consist of the introduction of a new asset, the integration of two markets, or both. Does the financial-market structure resulting from such a sequence of financial innovations necessarily coincide with the optimal structure? If so, we say that the optimal financial-market structure is *decentralizable*.[20]

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19. See for instance the economic analysis in Duffie and Jackson (1989), Ross (1989), Madan and Soubra (1991), Cuny (1993), Pesendorfer (1995), and Bisin (1998) for economies with transaction costs; Boot and Thakor (1993), Nachman and Noe (1994), Glaeser and Kallal (1997), DeMarzo (1999), and Duffie and DeMarzo (1999) for economies with asymmetric information; and Athanasoulis and Shiller (2000) for economies with imprecise information about the endowment processes of agents.

20. Hara (1997) asks a related question: does there exist a sequence of financial innovations such that, when introduced sequentially, each is Pareto improving and leads to the completion of financial markets? He answers the question in the affirmative. Our analysis of decentralizability is conceptually different from his analysis, because we require each financial innovation in the sequence to satisfy an optimality requirement that is stricter than Pareto improvement and we are interested in the optimality of resulting financial-market structures that are still incomplete, rather than in the possibility of completing the markets. Also, in the sequence of financial innovations Hara constructs, asset prices are not affected by the introduction of a new asset, while this is not the case in our model (as implied by equation [8]). Unlike Hara, we consider only those economies that already have a traded risk-free bond (assumption 4). Finally, Hara does not consider innovations in the form of market integration.

A precise definition of *decentralizability,* which applies generally to optimal asset structures as well as to optimal compositions of traders, is as follows.

DEFINITION 2. Let the optimal financial structure of an economy with J orthogonal financial assets and market participation structure $\mathcal{H}_j, j = 1, \ldots, J$, be denoted F. The betas associated with F maximize $\Sigma_{j \in \mathcal{J}} \Sigma_{h \in \mathcal{H}_j} \Sigma_{h' \in \mathcal{H}_j} (1/H_j)(\beta_j^h - \beta_j^{h'})^2$ (by lemma 1). Consider then another financial structure, $\hat{F}$, with J orthogonal financial assets and market participation structure $\mathcal{H}_j, j = 1, \ldots, J$. Suppose $\hat{F}$ satisfies the following:

1. The betas associated with each asset j maximize $\Sigma_{h \in \mathcal{H}_j} \Sigma_{h' \in \mathcal{H}_j} (1/H_j) \times (\beta_j^h - \beta_j^{h'})^2$, given $\mathcal{H}_j$.
2. For any asset j, the components of $\mathcal{H}_j$ can be ordered so that, without loss of generality,

$$h := \arg \max_{h' \notin \{1, \ldots, h-1\}} \sum_{h''=1}^{h-1} (\beta^{h''} - \beta^{h'})^2.$$

We say that the optimal financial structure F is decentralizable if F does not strictly dominate $\hat{F}$ in welfare terms; that is, $\mu_F = \mu_{\hat{F}}$.[21]

We explore the concept of decentralizability to understand whether the method of introducing a set of innovations (e.g., sequentially of in an uncoordinated manner) might influence the optimality of resulting financial structure.[22] If decentralizability fails to hold, it suggests that there are costs from having decentralized exchanges or intermediaries that introduce innovations independently or in an uncoordinated fashion. The efficiency gains (from the standpoint of financial optimality) that result from a harmonization of the innovation process thus might be significant for market structures that lack decentralizability. Such gains in fact are a lower bound, since decentralizability is a relatively weak requirement for optimality of innovation process: it requires only that each innovation introduced into financial markets be optimal given the existing financial market structure.[23]

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21. Since F is optimal, obviously, $\mu_F \geq \mu_{\hat{F}}$.

22. The issue of the optimal order of financial innovation has been largely overlooked in the literature. A notable exception is Dow (1998), who considers the opening up of a new market where trading occurs only as a way of hedging by arbitrageurs exploiting private information in another existing market (that is correlated with the new market). The new market thus may give rise to greater informed trading in the existing market, reducing the existing market's liquidity and potentially reducing the average welfare of the economy. In contrast, he shows that, in the reverse sequence of market introductions, each sequential market introduction improves welfare. See also Cuny (1993).

23. In general, allowing for assets with correlated payoffs and innovations to maximize trading volume (as in Duffie and Jackson 1989) or intermediation profits (as in Pesendorfer 1995), rather than an optimality criterion, in fact might introduce other inefficiencies in the design of assets.

When financial innovations are restricted to the introduction of new assets, decentralizability requires that the optimal financial structure can be obtained equivalently by introducing a first financial asset x_1 that maximizes

$$\sum_{h \in \mathcal{H}_1} \sum_{h' \in \mathcal{H}_1} \frac{1}{H_1} \left(\beta_1^h - \beta_1^{h'} \right)^2,$$

a second asset whose payoff is orthogonal to x_1, and maximizes

$$\sum_{h \in \mathcal{H}_2} \sum_{h' \in \mathcal{H}_2} \frac{1}{H_2} (\beta_2^h - \beta_2^{h'})^2,$$

and so on sequentially until the Jth asset. Similarly, for the case of market integration in a given asset, decentralizability requires that a sequential strategy of adding an optimal agent, given the existing participation in the asset, gives rise to the overall optimal composition of traders in that asset.

We examine the decentralizability of the optimal financial-market structures for three sets of economies. First, we study economies for which the number of assets is fixed, there is unrestricted participation in all assets, and innovations consist of designing the payoffs of the assets. Second, we examine economies in which the asset structure (the number of assets and the payoffs of assets) is fixed, and innovations consist of relaxing participation constraints. Finally, we consider economies in which the number of assets is fixed, participation in assets is restricted but fixed, and innovations consist of designing the payoffs of the assets.

Consider first an economy in which the optimal financial structure consists of J assets with no restriction in participation. The following is a simple implication of the principal-components characterization (proposition 1).

PROPOSITION 6. The financial structure F which is optimal in the class of financial structures with J assets and with no restricted participation in any asset, $\mathcal{H}_j \equiv \mathcal{H}$ for all $j \in \mathcal{J}$, is decentralizable.

The optimal financial asset structure defined by the principal-component characterization has the property that the nth asset is chosen so that the asset's payoff equals the eigenvector corresponding to the nth largest eigenvalue of the matrix M defined in proposition 1. The nth asset in a sequence of financial innovations is equivalently chosen.

We next examine the decentralizability of the optimal financial asset structure of an economy in which the payoff of each tradable asset is exogenously determined and the number of agents allowed to trade each particular asset is limited.

PROPOSITION 7. The financial structure *F*, which is optimal in the class of financial structures with *J* assets that have exogenously given payoffs and with asset *j* being traded by no more than $n < H$ agents, is decentralizable.

The result can be extended to the case in which the payoffs of financial assets are exogenous and the restriction on the participation in financial market *j* depends on *j* itself, that is, in which no more than H_j agents are allowed to trade asset *j*, with $H_j < H$ but H_j not necessarily equal to $H_{j'}$ if $j \neq j'$. This is not the case, however, when we consider financial structures with limited participation but assets whose payoffs are endogenous or optimally designed.

The next proposition shows that, if the restricted market participation structure is different across different assets but nevertheless overlapping for some of the assets, then the sequential introduction of financial assets can produce a financial-market structure that is not optimal among all structures with the same number of financial assets and the given market participation structure. In other words, the order of innovation of financial assets can affect aggregate welfare.

PROPOSITION 8. The financial structure $\tilde{F}$, which is optimal in the class of financial structures with *J* assets and given restricted participation structure $\mathcal{H}_j$, with $\mathcal{H}_j \subset \mathcal{H}$ for some *j*, is not necessarily decentralizable.

Proof. We prove the proposition by introducing an example economy in which the optimal financial structure, in fact, can strictly dominate the financial structure that results from the sequential introduction of optimal innovations.

Consider an economy with $H = 3$; that is, $\mathcal{H} = \{0, 1, 2\}$. Also the dimension of endowment space is $N = 3$. The agents' endowments are given by

$$Y_0 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}; \quad Y_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}; \quad Y_2 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}.$$

Consider $J = 2$ financial assets with an exogenously given restricted market participation structure of $\mathcal{H}_1 = \{0, 1\}$ and $\mathcal{H}_2 = \{1, 2\}$.[24] The

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24. Note that the matrix *M* in the principal components characterization (proposition 1) is

$$M = \frac{1}{3} \begin{bmatrix} 2 & -1 & -2 \\ -1 & 6 & 4 \\ -2 & 4 & 8 \end{bmatrix},$$

solution to the optimization problem for the overall financial structure yields the optimal assets as

$$x_1 = \begin{bmatrix} 0.10 \\ -0.28 \\ -0.96 \end{bmatrix}; \quad x_2 = \begin{bmatrix} 0.48 \\ 0.86 \\ -0.20 \end{bmatrix}.$$

The corresponding betas of the participating agents are $\beta_1^0 = -3.32$, $\beta_1^1 = -1.13$, and $\beta_2^1 = 1.14, \beta_2^2 = 3.33$. The welfare measure for this structure is

$$\ln(\mu_F) = k_1 + k_2 \cdot \sum_{j=1}^{2} \sum_{h \in \mathcal{H}_j} \sum_{h' \in \mathcal{H}_j} \frac{1}{2}(\beta_j^h - \beta_j^{h'})^2$$

$$= k_1 + k_2 \cdot \frac{1}{2}(4.791 + 4.791) = k_1 + k_2 \cdot 4.791,$$

where k_1 and k_2 are positive constants irrelevant to the analysis.

On the other hand, for the case of sequentially optimal asset introduction, solutions to the optimization problems are given as

$$x_1 = \begin{bmatrix} 0.00 \\ -0.45 \\ -0.89 \end{bmatrix}; \quad x_2 = \begin{bmatrix} 0.49 \\ 0.78 \\ -0.39 \end{bmatrix}.$$

The corresponding betas of the participating agents are given as $\beta_1^0 = -3.58, \beta_1^1 = -1.34$, and $\beta_2^1 = 0.88, \beta_2^2 = 2.93$. Thus, the welfare measure can be computed as

$$\ln(\mu_F) = k_1 + k_2 \cdot \sum_{j=1}^{2} \sum_{h \in \mathcal{H}_j} \sum_{h' \in \mathcal{H}_j} \frac{1}{2}\left(\beta_j^h - \beta_j^{h'}\right)^2$$

$$= k_1 + k_2 \cdot \frac{1}{2}(5.000 + 4.1999) = k_1 + k_2 \cdot 4.599.$$

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whose eigenvalues are $\lambda_1=11.62$, $\lambda_2=3.00$, and $\lambda_3=1.38$, with the corresponding eigenvectors:

$$x_1 = \begin{bmatrix} -0.27 \\ -0.80 \\ 0.53 \end{bmatrix}; \quad x_2 = \begin{bmatrix} 0.22 \\ -0.59 \\ -0.78 \end{bmatrix}; \quad x_3 = \begin{bmatrix} -0.94 \\ 0.09 \\ -0.34 \end{bmatrix}.$$

Thus, in the absence of any restricted market participation, for J = 1, the optimal financial structure is (x_1), and for J = 2, the overall optimal financial structure is (x_1, x_2).

Note that the welfare measure is smaller than that with overall optimal structure (k_1 and k_2 are the same for a given economy). Q.E.D.

This example illustrates that a sequentially optimal structure might not do as well in welfare terms as would an overall optimal structure in the presence of restricted market participation, due to a lack of coordination in the innovation process.

To motivate the example, consider two exchanges in two economies. Each economy has its set of local retail agents who participate only in markets of local exchanges. These agents are numbered 0 and 2, respectively, in the example. Across the two economies, there are financial institutions that participate in all exchanges. These agents bear the number 1 in the example. The exchanges in the two economies innovate sequentially, taking into account that the participation in the market introduced by an exchange consists only of the financial institutions (agent 1) and the respective retail investors (agent 0 or agent 2). The example thus maps into a natural market participation structure, where due to transaction costs or geographical and technological distance, retail investors display a home bias.

The intuition behind the lack of decentralizability in the example can be understood as follows. Notice that the first sequential asset produces a welfare change greater than each of the two assets under the overall optimal structure. However, the first sequential asset, that is, effectively the first innovating exchange, does not take into account the restricted market participation of the second asset (which is different from that of the first asset). As a result, the second sequential optimal asset introduced by the other innovating exchange is inferior in welfare terms to both of the overall optimal assets. In fact, it is sufficiently inferior that the sequential structure is strictly dominated by the overall structure.

Furthermore, the construction in the example is robust. A careful inspection of the endowments of different agents and the market participation structures reveals the following. From an overall efficiency standpoint, the goal of the first financial asset should be to produce risk sharing between agent 0 and agent 1 and simultaneously produce a posttrading risk profile for agent 1 that provides sufficient risk sharing with agent 2. This would indirectly generate risk sharing between agent 0 and agent 2, even though they do not participate in a common asset market. However, from a sequential efficiency standpoint, the goal of the first asset is to simply produce maximal risk sharing between agent 0 and agent 1.

In the example, agent 0 and agent 1 do not need to share risk along the first dimension of risk. Thus, the sequentially optimal asset x_1 has no loading on this dimension of risk. However, the overall optimal asset x_1 does have a loading on this dimension to facilitate risk sharing between agent 0 and agent 2, indirectly through trading of asset x_2 between agent 1 and agent 2. Thus, if introducing sequentially optimal assets restricts

risk sharing between a set of participating agents and another set of agents who cannot trade directly with the first set, then the sequentially optimal design is likely to produce a less-than-optimal overall financial structure. The intuition being adequately compelling, we do not delve into a more rigorous analysis of precise financial structures where decentralizability fails to hold.[25]

We showed in propositions 6 and 7 that optimal financial structures are decentralizable whenever the financial innovation consists of either the introduction of new assets in an economy without restricted participation constraints or the relaxation of restricted participation constraints for an existing asset. In contrast, proposition 8 shows that optimal financial structures are not decentralizable when the innovation consists of the introduction of new assets into economies with given restricted participation: the introduction of new assets and the integration of segmented markets interact such that even the weak notion of optimality of financial intermediation, guaranteed by decentralizability, is not satisfied. If the nature of participation restriction varies across markets for different financial assets, but if these markets are not completely distinct in terms of participating agents, then the optimal innovation of financial assets requires coordination among innovating intermediaries. In this case, the order in which financial assets are introduced could affect welfare. We conjecture that decentralizability also fails to hold for financial structures that are optimal when the number of tradable assets and the number of agents allowed to trade each asset are fixed, but the asset payoffs and the agents allowed to trade each asset are optimally designed.[26]

Our analysis suggests that the coordination of the financial-innovation process, for instance, in the form of consolidation of exchanges, should favor the production of socially desirable financial innovations. Conversely, lack of coordination should render financial innovations less

25. It is important to note that, in our model, each sequential asset is restricted to be orthogonal to the set of existing assets. This is for the sake of tractability in our CARA-normal setup. The assumption, however, is not very far from the observed practice. Silber (1981) and Black (1986) document empirically that futures contract innovations from 1960 and 1980 have succeeded at exchanges (measured using the induced trading volumes) primarily when these have been new contracts, that is, when they provide risk sharing along dimensions of risk that are residual given the existing contracts. Attempts of exchanges to compete with existing contracts on other exchanges have usually failed. Tufano (1989) also documents such a first-mover advantage in the financial innovations undertaken by investment banks over the period 1974–86. Cuny (1993) demonstrates theoretically that, in a sequential innovation process, each successive contract, in fact, is orthogonal to all the existing ones whenever there is a fixed cost to exchanges (traders) for innovation (participation).

26. The result on the lack of decentralizability has implications for the recent evolutionary approach to financial innovations (see, e.g., Bettzuge and Hens 2000). In this literature, financial innovations are introduced sequentially, and their survival depends on the extent of market participation (trade volume). Our results suggest that, if the innovation process stops before the completion of markets (say, due to trade frictions), then the evolutionary process might fail to reach an optimal financial-market structure.

efficient, possibly even inducing financial institutions to bypass socially desirable innovations. While the optimality of innovations, or the lack thereof, is difficult to identify directly in the real world, we present indirect, anecdotal evidence supporting these implications of our analysis. In particular, we discuss instances where financial innovations are induced by cross-border mergers between exchanges and instances where financial institutions operating in segmented markets are unable to innovate integrated financial products that would provide insurance against different financial risks.

Consolidation among exchanges. With the formation of the EMU, financial institutions and large international banks started participating in all markets of member nations. However, a large number of the retail household investors have access only to their domestic markets, giving rise to market segmentation or restricted participation in many financial markets. The consolidation of exchanges in Europe to create a pan-European stock exchange, as documented in McAndrews and Stefanadis (2002), therefore might have desirable consequences in terms of the efficiency of induced innovations. The presence of a similar market segmentation between the financial markets of the United States and Europe points toward likely efficiency gains from the collaboration between exchanges in the United States and Europe, such as the one between Chicago Board of Trade (CBOT) and Euronext-Liffe.

To illustrate the important role of coordinating innovations when markets are segmented, consider in some detail the collaboration of CBOT and Euronext-Liffe. At the beginning of 2003, CBOT announced a tie-up with Euronext-Liffe to share trading platforms and capitalize on synergies in financial innovation, especially in swap products. The Chicago Board of Trade specializes in 10-year U.S. Treasury bond futures, whereas the main interest rate futures product of Euronext-Liffe is the 3-month Euribor contract. That is, CBOT has the "long end" of the dollar yield curve and Euronext-Liffe has the "short end" of the euro yield curve. There has been a growing demand from investors on both sides of the Atlantic to be able to trade derivatives denominated in both U.S. and European currencies and at both ends of the corresponding yield curves. As part of the merger talks, CBOT and Euronext-Liffe have focused on development of the short-term dollar-denominated swap futures, a product that is to be made available mutually, that is, for participants trading in each exchange. The contract would meet the residual demand of investors for dollar swap futures on short-dated interest rates. Since the clientele for short-dated interest rate derivatives is currently with Euronext-Liffe but any dollar-denominated clearing is easier for CBOT to "net" and manage, the proposed product is attractive to the collaborating exchanges but not as much to the stand-alone exchanges. While the integration of technologies has also motivated the collaboration, both exchanges also cite "exploiting possible synergies

in product development" as a driving factor.[27] It is difficult to disentangle from the proposed synergies the exact role played by the integration of segmented markets. The example is nevertheless suggestive of gains from coordination in innovations when exchanges merge.

Absence of integrated insurance products. Talks with industry practitioners reveal a relative dearth in the financial markets of products that enable risk-sharing along different dimensions of risk in an integrated or a compound manner. Examples of such products are (1) loans where interest rates are tied to commodity prices and (2) compound insurance products, for example, a protective instrument whereby political risk and foreign exchange risk in project finance loans can be simultaneously hedged. The lack of innovation in such products at least partially derives from segmentation in the markets for provision of insurance against individual financial risks. A variety of types of insurance provided by different financial intermediaries results in a partial segmentation of their clienteles. In turn, this yields an incomplete internalization of benefits from the design of integrated products.

To illustrate the economic desirability of such products, we consider the following example. Over the past few years, Ashanti Goldfields (Ghana) has faced stringent margin requirements on the gold-price hedging programs implemented using gold forwards and futures. Ashanti's project finance providers required the firm to hedge its market risk to gold-price fluctuations. Ashanti responded by undertaking short positions in exchange-traded commodity futures and over-the-counter commodity forwards. The possibility of a spike in the gold price left Ashanti vulnerable to significant margin calls (on futures as well as forwards, as discussed later). Indeed, the sudden rise in the gold price in late September and early October 1999 forced Ashanti to post in excess of USD280 million in cash as margin requirements. The interest rate burden on project-finance loans, however, remained unaffected by these gold price fluctuations, putting a severe strain on the balance sheet of Ashanti.

When gold prices rise, Ashanti's underlying economic position is positive, as it is a supplier of gold. However, the entire value of its economic position may not be realizable in cash. Thus, hedging in this scenario can lead to ex-post regret if the size of margin calls exceeds the available increase in liquidity. A more attractive borrowing-cum-hedging vehicle for Ashanti would be an "indexed" project finance loan, whereby the amount of repayment increases in the price of gold. However, one does not usually see such products offered in the markets. Why? Large banks constitute the primary intermediaries in the project-finance market and corporations in emerging markets constitute

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27. See "Capital Markets: CBOT, Euronext discuss link up," *Financial Times* (January 28, 2003).

their clientele, with a subset comprising those corporations that are commodity suppliers. Similarly, futures contracts are traded on organized exchanges such as the New York Mercantile Exchange (NYMEX), the clientele of which reaches far beyond the set of commodity suppliers. Thus, innovation of the proposed indexed loan alone would fail to satisfy either the banks or the commodity exchanges.

In principle, one might argue that banks could write such indexed loans and then hedge the market risk in secondary markets. But, in case of Ashanti, an additional complication in the contract arises from political risk (and its possible correlation with gold-price risk). This brings us to another compound product that would be attractive in the context of Ashanti but is not witnessed. Typical forward contracts do not require daily marked-to-market margin requirements from counterparties, although Ashanti's hedge counterparties did require such margins before 2000. The rationale is this: the Ghana government had a "golden share" (veto rights) in Ashanti's management structure, and the margin requirements were a partial protection against this political risk.[28] An ideal product here would be a forward contract with embedded political-risk insurance. But the hedge counterparties that supply forward contracts and the typical providers of political-risk insurance are typically different institutions, reducing the individual institutional attraction of innovating such a product. From Ashanti's standpoint, the substitution of political-risk insurance by margin requirements in fact resulted in an inferior hedging product.

Although these specific products do not necessarily map precisely into the risk-sharing innovations we examine in our paper, the reason these products are not introduced corresponds well to the motive we identified for the potential suboptimality of decentralized innovations. We believe coordination between intermediaries and exchanges that provide products on individual financial risks would lead to efficient investments in the innovation of products that integrate these risks. This, in turn, would lead to better risk sharing between intermediaries and financial corporations. Consistent with this view, many experts in futures markets stress the need and the benefits of consolidation of futures exchanges and over-the counter (OTC) intermediaries to create integrated markets (Melamed 2002).[29] The recent consolidation of

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28. Ashanti claims it had a crisis only because it was required to show more margin on its contracts than other, often smaller mining companies headquartered in the United States, United Kingdom, Canada, South Africa, and Australia. It blames the unequal treatment on being a Ghana-based concern, with the Ghanaian government as a key shareholder. "Many, such smaller companies were not required to put up anything," said one adviser of Ashanti Goldfields (based on "The golden share – Should it go or stay?" by Dr. O. A. Kwapong, *Ashanti*—Company News Archive and research articles from CIBC World Markets).

29. Leo Melamed, chairman emeritus and senior policy advisor to the Chicago Mercantile Exchange, observes that "[T]he days of narrow-based niche market capabilities are limited. Strategies pertaining to equity, debt, indexing, foreign exchange, futures, forwards, options,

national exchanges and the derivatives exchanges in practically all of Europe (Cappon 1998) and the CBOT-Prebon and Cantor-Fitzgerald-New York Board of Trade alliances promise to deliver the integration of cash markets, OTC derivatives, repo markets, and exchange-traded futures and options.[30]

VII.   Conclusions

The theoretical results of this paper provide two important normative prescriptions: (1) financial innovations that generate a higher level of risk-sharing, as characterized by dispersion across agents of betas (covariances of their endowments with traded financial assets), are more desirable than others from an overall welfare standpoint; and (2) some form of harmonization or coordination of the innovation process of decentralized financial intermediaries is desirable when asset markets in the integrating economies are segmented with different but overlapping sets of participating agents.

   While this paper analyzes the optimality of financial structures only for economies that permit lump-sum transfers across agents, many interesting issues surround the individual welfare effects of financial innovations. These pertain to circumstances when lump-sum transfers cannot be implemented, and in fact, the welfare effect of financial innovations can be negative on a subset of the agents. In particular, the relative price effects of innovation are exhibited in a lower demand for precautionary savings and hence for the risk-free asset. The risk-free rate of interest in the economy consequently increases, and the agents, who need to borrow in equilibrium and benefit little from risk-sharing provided by financial innovation, are relatively hurt. Unfortunately, only few clear implications of optimal financial innovations for individual welfare can be derived analytically (see Willen 1999).

   We repeatedly noted that our analysis of financial market integration has different, in fact, opposite, welfare implications from those derived in the theory of optimal currency areas, which stresses monetary policy

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swaps, and cash, are all interdependent and interchangeable. The futures exchange of to-morrow must be able to provide comprehensive risk management in every sense of the word. . . . There is little doubt that the ongoing trend of blurring distinctions between the instruments of futures and securities is continuing . The recent Joint Venture between the Chicago Mercantile Exchange (CME), Chicago Board of Trade (CBOT) and Chicago Board of Options Exchange (CBOE) is a giant step in that direction.''

   30. Andre Cappon, the president of the CBM Group Inc., a management consulting firm that specializes in financial services, notes that ''A 'hybrid model' of interdealer broker-cum-exchange can provide a highly flexible market model which can accommodate a broad variety of investors, large and small, professional or individual . . . Interdealer brokers are typically active in the cash market, OTC derivatives and financing markets, i .e., repo for fixed income instruments. When 'married' to a derivatives exchange, the possibility of integrating cash, OTC derivatives and financing with futures and options becomes compelling'' (Cappon 1997).

integration. Since money is absent from our model, our opposing results strongly suggest the importance of a joint analysis of the welfare benefits of financial and trade integration, on the one hand, and a coordinated monetary policy, on the other. Developing a dynamic model of financial integration with multiple consumption goods and money appears a fruitful goal. This would enable the empirical application of the present analysis. Further, such a model would spur the study of the welfare benefits of currency unions and optimal currency areas when the integration process also involves financial markets and international commerce. The recent work of Sutherland (2003), which introduces financial risk sharing in the monetary policy integration model of Obstfeld and Rogoff (2002), constitutes an important first step in this direction.

Appendix A

Competitive Equilibrium (Willen 1997)

The competitive equilibrium of the 2-period CAPM economy, defined by equations (1)–(7), is characterized by prices of assets (π_j) portfolio choices (θ_j^h), and consumption allocations (c_t^h), given here. Note that $j=0$ denotes the risk-free asset:

$$\pi_0 = \exp\left\{ A(y_0 - Ey_1) + \frac{A^2}{2H} \sum_{h \in H} \left[(1 - R_h^2)\mathrm{var}(y_1^h) + \sum_{j \in J^h} \mathrm{var}(\beta_j x_j) \right] \right\} \quad (17)$$

where

$$R_h^2 := \frac{\Sigma_{j \in J^h} (\beta_j^h)^2 \mathrm{var}(x_j)}{\mathrm{var}(y_1^h)} \quad (18)$$

$$\frac{\pi_j}{\pi_0} = E(x_j) - A \mathrm{cov}\left(\frac{1}{H_j} \sum_{h \in H_j} y_1^h, x_j \right), \, j \in \mathcal{J} \quad (19)$$

$$\theta_j^h = \beta_j - \beta_j^h, \, j \in \mathcal{J}^h, \quad \text{and } \theta_j^h = 0, \, j \in (\mathcal{J}^h)^c \quad (20)$$

$$\theta_0^h = \frac{1}{1 + \pi_0} \left[y_0^h - E(y_1^h) - \sum_{j \in \mathcal{J}^h} \pi_j \theta_j^h + \frac{A}{2} \mathrm{var}(c_1^h) - \frac{1}{A} \ln(\pi_0) \right] \quad (21)$$

$$c_j^h = y_1^h - \sum_{j \in \mathcal{J}^h} \beta_j^h x_j + \sum_{j \in \mathcal{J}^h} \beta_j x_j + \theta_0^h \quad (22)$$

$$\mathrm{var}(c_1^h) = \mathrm{var}(y_1^h) - \sum_{j \in \mathcal{J}^h} (\beta_j^h)^2 + \sum_{j \in \mathcal{J}^h} \beta_j^2. \quad (23)$$

Consumption allocations c_0^h can be solved by using (17)–(22) and the budget constraint (3):

$$c_0^h = -\frac{1}{A} \ln \frac{1}{\pi_0} + E(y_1^h) + \theta_0^h - \frac{A}{2} \mathrm{var}(c_1^h). \tag{24}$$

Appendix B

Proof of Lemma 1

From Willen (1997),

$$\mu_F = -\frac{1}{A} \ln \frac{1 + \pi_0}{1 + \pi_0^a} = -\frac{1}{A} \ln \frac{\dfrac{1}{\pi_0^a} + \dfrac{\pi_0}{\pi_0^a}}{1 + \dfrac{1}{\pi_0^a}},$$

where for simplicity, we denote π_0^F as simply π_0. Thus, aggregate welfare is maximized when the ratio, π_0 / π_0^a, is minimized. However,

$$\pi_0^a = \exp \left\{ A(y_0 - Ey_1) + \frac{A^2}{2H} \sum_{h \in H} \mathrm{var}(y_1^h) \right\}, \quad \text{and}$$

$$\pi_0 = \exp \left\{ A(y_0 - Ey_1) + \frac{A^2}{2H} \sum_{h \in H} \left[(1 - R_h^2) \mathrm{var}(y_1^h) + \sum_{j \in J^h} \mathrm{var}(\beta_j x_j) \right] \right\}.$$

Using $R_h^2 \mathrm{var}(y_1^h) = \Sigma_{j \in J^h} (\beta_j^h)^2 \mathrm{var}(y_1^h)$,

$$\frac{\pi_0}{\pi_0^a} = \exp \left\{ \frac{A^2}{2H} \sum_{h \in H} \left[-\sum_{j \in J^h} (\beta_j^h)^2 + \sum_{j \in J^h} (\beta_j)^2 \right] \mathrm{var}(x_j) \right\},$$

which, in turn, can be written as

$$\frac{\pi_0}{\pi_0^a} = \exp \left\{ \frac{A^2}{2H} \sum_{j \in J} \sum_{h \in H_j} \left[\beta_j^2 - (\beta_j^h)^2 \right] \mathrm{var}(x_j) \right\}. \tag{25}$$

Then, using $\beta_j = 1/H_j \Sigma_{h \in H_j} \beta_j^h$, we get

$$\sum_{h \in H_j} \left[\beta_j^2 - (\beta_j^h)^2 \right] = \frac{1}{H_j} \left(\sum_{h \in H_j} \beta_j^h \right)^2 - \sum_{h \in H_j} (\beta_j^h)^2 = -\frac{1}{H_j} \sum_{h \in H_j} \sum_{h' \in H_j} (\beta_j^h - \beta_j^{h'})^2,$$

and, hence, given $\text{var}(x_j) = 1$, we have the result

$$\frac{\pi_0}{\pi_0^a} = \exp\left\{ \frac{A^2}{2H} \sum_{j \in J} \sum_{h \in H_j} \sum_{h' \in H_j} -\frac{1}{H_j} \left(\beta_j^h - \beta_j^{h'} \right)^2 \right\}. \tag{26}$$

Q. E. D.

Proof of Proposition 1

We first derive another characterization of the optimality of financial structures in terms of portfolio volumes.

LEMMA 2 (PORTFOLIO REPRESENTATION). The compensating aggregate transfer μ_F is maximal for the financial structure F whose equilibrium trading portfolios, $[\theta_j^h]_{j \in J}^{h \in H}$, maximize

$$\sum_{j \in \mathcal{J}} \sum_{h \in \mathcal{H}_j} \sum_{h' \in \mathcal{H}_j} \frac{1}{H_j} (\theta_j^h - \theta_j^{h'})^2. \tag{27}$$

This is equivalent to maximizing

$$\sum_{j \in \mathcal{J}} \sum_{h \in \mathcal{H}_j} \frac{1}{H_j} (\theta_j^h)^2. \tag{28}$$

Proof. Equation (27) follows from lemma 1, equation (26), and $\theta_j^h = \beta_j - \beta_j^h$, for $j \in \mathcal{J}^h$. The second representation is a result of the fact that $\Sigma_{h \in \mathcal{H}_j} \theta_j^h = 0$.

Assume $H_j = H$, for any $j \in \mathcal{J}$. In an abuse of notation, we use μ_F to represent the measure of beta dispersion that maximizes the compensating aggregate transfer as defined in note 10 and specified in equation (13) (see the beta representation of lemma 1). This, of course, is innocuous as far as the following proof is concerned. From lemma 2, using $\theta_j^h = \beta_j - \beta_j^h$,

$$\mu_{F'} - \mu_F = \sum_{j \in J} \sum_{h \in H_j} \sum_{h' \in H_j} \frac{1}{H_j} (\beta_j^h - \beta_j^{h'})^2.$$

But, using the assumption $H_j = H$,

$$\mu_{F'} - \mu_F = \sum_{h \in H} \sum_{j \in J} (\beta_j^h - \beta_j^{h'})^2 = \sum_{h \in H} \sum_{j \in J} [\text{cov}(y_1^h, x_j) - \text{cov}(y_1, x_j)]^2.$$

Let $y_1^h = Y^h v$, and $Y = 1/H \Sigma_{j \in J} Y^h$; that is, $y_1 = Yv$. Since assets' payoffs are normally distributed (assumption 3), we can write $x = R'v + \varepsilon$ with $\text{cov}(\varepsilon, v) = 0$. We first derive the result for $\varepsilon = 0$, that is, assets' payoffs are linear in the

agents' endowments; and we then show that such linearity actually must hold if the asset structure is optimal:

$$
\text{Let } R = \begin{bmatrix} R_1 \\ \vdots \\ R_J \end{bmatrix}, \quad \text{and let } R^T \text{ denote the transpose of R. Then,}
$$

$$
\mu_{F'} - \mu_F = \sum_{h \in H} \sum_{j \in J} \mathrm{cov}(y_1^h - y_1, x_j)^2 = \sum_{h \in H} \sum_{j \in J} \mathrm{cov}(Y^h v - Yv, R_j v)^2
$$

$$
= \sum_{h \in H} \sum_{j \in J} (Y^h - Y) R_j^T R_j (Y^h - Y)^T.
$$

Since $R_j R_{j'}^T = 0$, for any $j \neq j'$, and $R_j R_j^T = 1$, for all j, it follows that RR^T is an identity matrix. Then,

$$
\mu_{F'} - \mu_F = \sum_{h \in H} \sum_{j \in J} (Y^h - Y) R_j^T R_j R_j^T R_j (Y^h - Y)^T
$$

$$
= \sum_{h \in H} (Y^h - Y) R^T R (Y^h - Y)^T.
$$

The rest of the proof for the case $x = R'v$ follows proposition 2.3 of Demange and Laroque (1995, p. 226). The payoff of assets, however, generally takes the form $x = R'v + \varepsilon$. It remains to be shown that $\varepsilon = 0$ is required by optimality. If $x = R'v + \varepsilon$, $\beta^h = [\mathrm{cov}(R'v + \varepsilon, Y^{h'}v)/\mathrm{var}(x)] = R'Y^h/(R'R + \varepsilon^2)$ (since $\mathrm{var}(v) = 1$). Since the welfare criterion, (15), is homogeneous of degree 2 in betas, the optimal asset structure has $\varepsilon = 0$. Q.E.D.

Proposition 2 is implied by lemma 3, and is stated and proved in proposition 7 later.

Proof of Proposition 3

From $\mu_F = -1/A \ln[(1 + \pi_0)/(1 + \pi_0^a)]$,

$$
\mu_{F'} - \mu_F = -\frac{1}{A} \ln \frac{1 + \pi'_0}{1 + \pi_0} = -\frac{1}{A} \ln \frac{\dfrac{1}{\pi_0} + \dfrac{\pi'_0}{\pi_0}}{1 + \dfrac{1}{\pi_0}}.
$$

Thus, as in lemma 1, aggregate welfare is maximized when the ratio, π'_0/π_0, is minimized. We fix H_j, for any $j \in J$. Then, from lemma 1,

$$
\frac{\pi'_0}{\pi} = \exp\left\{ \frac{A^2}{2H} \sum_{j \in J} \sum_{h \in H_j} \sum_{h' \in H_j} -\frac{1}{H_j} \left(\beta_j^h - \beta_j^{h'} \right)^2 \right\}.
$$

It follows that $\mu_{F'} > \mu_F$ if the innovation consists of introducing an asset $j' \notin \mathcal{J}$, such that there exist $h, h' \in H_{j'}$ for which $\beta_{j'}^h \neq \beta_{j'}^{h'}$.

To study the case of market integration, suppose that there exist $j, j' \in J$ such that $x_j = x_{j'}$ and $H_j \cup H_{j'} = \emptyset$, and consider a new economy in which $H'_j = H'_{j'} = H_j \cup H_{j'}$. Then,

$$\frac{\pi'_0}{\pi} = \exp\left\{\frac{A^2}{2H} \sum_{h \in H'_j} \sum_{h' \in H'_j} \left[\frac{\left(\beta^h_j - \beta^{h'}_j\right)^2}{H_j + H'_j} - \frac{\left(\beta^h_j - \beta^{h'}_j\right)^2}{H_j} - \frac{\left(\beta^h_j - \beta^{h'}_j\right)^2}{H'_j}\right]\right\}.$$

Using (25),

$$\frac{\pi'_0}{\pi_0} = \exp\left\{\frac{A^2}{2H}\left[\sum_{j \in H'_j}(\beta'_j)^2 - (\beta^h_j)^2 - \sum_{h \in H_j}(\beta_j)^2 - (\beta^h_j)^2 - \sum_{h \in H'_j}(\beta_{j'})^2 - (\beta^h_{j'})^2\right]\right\}$$

$$= \exp\left\{\frac{A^2}{2H}\left[\sum_{h \in H'_j}(\beta'_j)^2 - \sum_{h \in H_j}(\beta_j)^2 - \sum_{h \in H_{j'}}(\beta_{j'})^2\right]\right\}.$$

But, since $\beta'_j = (H_j\beta_j + H_{j'}\beta_{j'})/(H_j + H_{j'})$, we have

$$\frac{\pi'_0}{\pi} = \exp\left\{\frac{A^2}{2H} \cdot \frac{-H_j H_{j'}}{(H_j + H_{j'})}(\beta_j - \beta_{j'})^2\right\}. \tag{29}$$

It follows that, if the innovation consists of integrating assets j and j', then $\mu_{F'} > \mu_F$ if $\beta_j \neq \beta_{j'}$. The necessary condition for $\beta_j \neq \beta_{j'}$ is that there exist $h \in H_j$ and $h' \in H_{j'}$ such that $\beta^h_{j'} \neq \beta^{h'}_{j'}$ (but $\beta^{h'}_j = \beta^{h'}_{j'}$, since $x_j = x_{j'}$). Q.E.D.

Proposition 4 and proposition 5 follow directly from proposition 3; proposition 6 follows immediately from proposition 1.

Proof of Proposition 7

In the notation that follows, the index referring to an arbitrary security j is dropped for simplicity.

Let H denote the number of agents in the population. Let n denote the maximum number of agents allowed to trade an arbitrary security. We study the optimal choice of the n agents. Clearly, the only interesting case to consider is when $2 \leq n < H$. Also, we restrict ourselves to the case in which agents' betas (for the arbitrary asset) are distinct. (The proof can be extended to account for identical betas with only notational complications.) This allows us to order agents, without loss of generality, so that

$$\beta^H > \cdots > \beta^h > \cdots > \beta^1.$$

Two definitions of optimality are compared, which we call *optimality* and *sequential optimality* in the following. Let $O^n \subset H$ denote the optimal set of agents, while $S^n \subset H$ denotes the sequential optimal set of agents (both sets have cardinality n by construction).

In particular, if H^n denotes any arbitrary n-dimensional subset of H and

$$\bar{\beta}^n := \frac{1}{n} \sum_{h \in H^n} \beta^h,$$

$$O^n := \arg \max_{H^n} \sum_{h \in H^n} (\beta^h)^2 - n(\bar{\beta}^n)^2.$$

Also, S^n is defined recursively as follows: $S^n = S^{n-1} \cup h'$, where

$$h' := \arg \max_{h' \notin S^{n-1}} \sum_{h \in S^{n-1}} (\beta^h)^2 + (\beta^{h'})^2 - n \left(\frac{1}{n} \left(\sum_{h \in S^{n-1}} \beta^h + \beta^{h'} \right) \right)^2,$$

and $S^2 := O^2$. Note that, equivalently,

$$h' := \arg \max_{h' \notin S^{n-1}} \sum_{h \in S^{n-1}} (\beta^{h'} - \beta^h)^2. \tag{30}$$

LEMMA 3. There exists an $r_o^n < n$ such that O^n consists of the first r_o^n agents and the last $n - r_o^n$ agents; that is, $O^n = \{1, \ldots, r_o^n, H - (n - r_o^n) + 1, \ldots, H\}$. Moreover, if we let $\bar{\beta}_o^n = 1/n \Sigma_{h \in O^n} \beta^h$, then

$$\beta^{r_o^n} < \bar{\beta}_o^n < \beta^{H-(n-r_o^n)+1}. \tag{31}$$

Proof. By contradiction. Suppose the statement does not hold. Pick a couple $s_1, s_2 \in O^n$ such that $\beta^{s_1} \leq \bar{\beta}_o^n < \beta^{s_2}$. Then, one of the following holds:

Case 1. $\{\beta^h \mid h \in O^n, h \leq s_1\}$ are not the r smallest betas, for some $r \leq s_1$.
Case 2. $\{\beta^h \mid h \in O^n, h > s_2\}$ are not the $n - r$ largest betas, for some $r \leq s_1$ (i.e., $(n - r) > s_2$).
Case 3. Finally, both cases hold.

We now show that, in each of these cases, it is possible to pick an agent k and substitute him with another agent k' and improve welfare.

The change in welfare due to the substitution of k with k' can be calculated to be

$$n \sum_{h \in O^n - \{k\} \cup \{k'\}} (\beta^h)^2 - \left(\sum_{h \in O^n - \{k\} \cup \{k'\}} \beta^h \right)^2 - n \sum_{h \in O^n} (\beta^h)^2 + \left(\sum_{h \in O^n} \beta^h \right)^2,$$

which, in turn, after some algebra, can be written as

$$(\beta^{k'} - \beta^k) \left[(n - 1)(\beta^{k'} - \beta^k) + 2n(\beta^k - \bar{\beta}_O^n) \right].$$

By construction, $\beta^{s_1} \leq \bar{\beta}_O^n$.

In case 1, taking $k = s_1$ and k' such that $\beta^{k'} < \beta^{s_1}$, we can improve welfare. Similarly, in case 2, taking $k = s_2$ and k' such that $\beta^{k'} > \beta^{s_2}$, we can improve welfare. In case 3, welfare can be improved as in case 1 (and also as in case 2).

Moreover, (31) is implied by our construction. In fact, we picked s_1, s_2 such that $\beta^{s_1} \leq \bar{\beta}_O^n < \beta^{s_2}$, and we have just showed that $s_1 = r_o^n$ and $s_2 = H - (n - r_o^n) + 1$. Q.E.D.

LEMMA 4.   There exists an $r_s^n < n$ such that S^n consists of the first r_s^n agents and the last $n - r_s^n$ agents; that is, $S^n = \{1, \ldots, r_s^n, H - (n - r_s^n) + 1, \ldots, H\}$. Moreover, if we let $\bar{\beta}_s^n = 1/n \Sigma_{h \in S^n} \beta^h$, then

$$\beta^{r_s^n} < \bar{\beta}_s^n < \beta^{H-(n-r_s^n)+1}. \tag{32}$$

Proof.   By induction. The case $n = 2$ is trivial. Assume the statement holds for $n - 1$. Let

$$\tilde{h} := \arg \max_{h' \notin S^{n-1}} \sum_{h \in S^{n-1}} (\beta^{h'} - \beta^h)^2.$$

But note that $\Sigma_{h \in S^{n-1}} (\beta^{h'} - \beta^h)^2 = (n-1)(\beta^{h'} - \bar{\beta}_s^{n-1})^2$; and hence $\tilde{h}$ is either $r_s^{n-1} + 1$ or $H - (n - r_s^{n-1})$.

To prove (32), we first write

$$\bar{\beta}_s^n = \frac{1}{n} \left[(n-1)\bar{\beta}_s^{n-1} + \beta^{\tilde{h}} \right] \tag{33}$$

and consider three cases.

Case 1. $\beta^{\tilde{h}} < \bar{\beta}_s^{n-1}$. Then, by the induction hypothesis, $\beta^{H-(n-r_s^{n-1})+1} > \bar{\beta}_s^{n-1}$, hence, using (33), $\bar{\beta}_s^{n-1} > \bar{\beta}_s^n > \beta^{\tilde{h}}$, which implies (32).

Case 2. $\beta^{\tilde{h}} > \bar{\beta}_s^{n-1}$. Then, by the induction hypothesis, $\beta^{r_s^{n-1}} < \bar{\beta}_s^{n-1}$, hence, using (33), $\bar{\beta}_s^{n-1} < \bar{\beta}_s^n < \beta^{\tilde{h}}$, which implies (32).

Case 3. $\beta^{\tilde{h}} = \bar{\beta}_s^{n-1}$. This case can occur only if $n = H$, which is excluded. Q.E.D.

Let $W_o^n := \Sigma_{h \in O^n} (\beta^h)^2 - n(\bar{\beta}_o^n)^2$, and $W_s^n := \Sigma_{h \in S^n} (\beta^h)^2 - n(\bar{\beta}_s^n)^2$.

LEMMA 5. $W_O^n = W_S^n$.

Proof.   By induction. By definition, $O^2 = S^2$. Assume the statement holds for $n - 1$. Let $O_{-1}^n \subset O^n$ denote a possible $(n - 1)$-dimensional subset of O^n.

By the induction hypothesis,

$$\sum_{h,h' \in S^{n-1}} \left(\beta^h - \beta^{h'} \right)^2 \geq \sum_{h,h' \in O_{-1}^n} \left(\beta^h - \beta^{h'} \right)^2, \quad \text{for all } O_{-1}^n \subset O^n. \tag{34}$$

Also, by the definition of S^n, either $S^n = S^{n-1} \cup \{r_s^{n-1} + 1\}$ or $S^n = S^{n-1} \cup \{H - (n - r_s^{n-1})\}$. We consider only the first case; the second is symmetric and left to the reader: $r_s^n = r_s^{n-1} + 1$. Then again, by the definition of S^n and (30),

$$\sum_{h \in S^{n-1}} (\beta^{r_s^n} - \beta^h)^2 \geq \sum_{h \in S^{n-1}} \left(\beta^{h'} - \beta^h \right)^2, \quad \text{for all } h' \notin S^{n-1}. \tag{35}$$

Let $O^n := O^n_{-1} \cup \{\hat{h}\}$. We now write

$$W^n_s = \sum_{h,h' \in S^{n-1}} \left(\beta^h - \beta^{h'}\right)^2 + \sum_{h \in S^{n-1}} \left(\beta^{r^n_s} - \beta^h\right)^2, \quad \text{and} \tag{36}$$

$$W^n_O = \sum_{h,h' \in O^n_{-1}} \left(\beta^h - \beta^{h'}\right)^2 + \sum_{h \in O^n_{-1}} \left(\beta^{\hat{h}} - \beta^h\right)^2. \tag{37}$$

But, by (34), the first term of (36) is greater than the first term of (37). We now pick $\hat{h} \in O^n$ such that the second term of (36), also, is greater than the second term of (37).

We can first simplify

$$\sum_{h \in S^{n-1}} \left(\beta^{r^n_s} - \beta^h\right)^2 - \sum_{h \in O^n_{-1}} \left(\beta^{\hat{h}} - \beta^h\right)^2 \text{ to}$$

$$(n-1)\left[\left(\beta^{\hat{h}} - \bar{\beta}^{n-1}_s\right)^2 - \left(\beta^{r^n_s} - \frac{1}{n-1}\sum_{h \in O^n_{-1}}\beta^h\right)^2\right].$$

Then, using (35),

$$\sum_{h \in S^{n-1}} \left(\beta^{r^n_s} - \beta^{\hat{h}}\right)^2 - \sum_{h \in O^n_{-1}} \left(\beta^{\hat{h}} - \beta^h\right)^2$$

$$\geq (n-1)\left[\left(\beta^{\hat{h}} - \bar{\beta}^{n-1}_s\right)^2 - \left(\beta^{\hat{h}} - \frac{1}{n-1}\sum_{h \in O^n_{-1}}\beta^h\right)^2\right], \quad \text{if } \hat{h} \notin S^{n-1}.$$

Finally,

$$(n-1)\left[\left(\beta^{\hat{h}} - \bar{\beta}^{n-1}_s\right)^2 - \left(\beta^{\hat{h}} - \frac{1}{n-1}\sum_{h \in O^n_{-1}}\beta^h\right)^2\right]$$

$$= (n-1)\left[\left(\frac{1}{n-1}\sum_{h \in O^n_{-1}}\beta^h - \bar{\beta}^{n-1}_s\right)\left(2\beta^{\hat{h}} - \frac{1}{n-1}\sum_{h \in O^n_{-1}}\beta^h\right)^2 - \bar{\beta}^{n-1}_s\right]. \tag{38}$$

We are now ready to choose $\hat{h} \notin S^{n-1}$ so as to show that the quantity in equation (38) is always positive. There are several cases:

Case 1. $\beta^{r^n_o} = \beta^{r^n_s}$. In this case, trivially, $O^n = S^n$.

Case 2. $\beta^{r^n_s} < \beta^{r^n_o} \leq \bar{\beta}^{n-1}_s$. In this case, we pick $\hat{h} = r^n_o$ (note that $r^n_o \notin S^{n-1}$). Then,

$$\beta^{r^n_o} < \bar{\beta}^n_o < \frac{1}{n-1}\underset{h \in O^n_{-1}}{\Sigma}\beta^h \leq \bar{\beta}^{n-1}_s,$$

which directly implies that (38) > 0.

Case 3. $\beta^{r_o^n} < \beta^{r_s^n}$. By the definition of S^n, then, $\beta^{r_o^n} < \beta^{r_s^{n-1}}$. In this case, we pick $\hat{h} = H - (n - r_o^n) + 1$.

We first show by contradiction that $H - (n - r_o^n) + 1 \notin S^{n-1}$. Suppose $H - (n - r_o^n) + 1 \in S^{n-1}$. Then, $r_o^n < \hat{h} = H - (n - r_o^n) + 1 \leq r_s^{n-1}$. But then, (34) and (35) imply that $\bar{\beta}_o^n < \beta^{\hat{h}} < \beta^{r_s^n} < \bar{\beta}_s^n$, this is a contradiction, since in such construction O^n contains some elements in common with S^n and all other elements that are greater than the remaining elements in S^n. The choice of $\hat{h} = H - (n - r_o^n) + 1$ implies that

$$\beta^{\hat{h}} > \bar{\beta}_o^n > \frac{1}{n-1} \sum_{h \in O_{-1}^n} \beta^h \geq \bar{\beta}_s^{n-1},$$

which in turn implies that (38) > 0.

Case 4. $\beta^{r_o^n} > \beta^{r_s^{n-1}}$. This case is not possible. Otherwise, by lemma 1, $\bar{\beta}_o^n > \beta^{r_o^n}$. But $r_o^n \geq r_s^n$, hence, $\bar{\beta}_o^n > \bar{\beta}_s^n$, which is not possible, since in such construction O^n contains some elements in common with S^n and all other elements that are smaller than the remaining elements in S^n. Q.E.D.

Proposition 8 is proved in the text.

References

Alesina, A., R. J. Barro, and S. Tenreyro. 2002. Optimal currency areas. Working paper 9072, National Bureau of Economic Research, Cambridge, MA.

Alesina, A., E. Spolaore, and S. Wacziarg. 2000. Economic integration and political disintegration. *American Economic Review* 90:1276–96.

Allen, F., and D. Gale. 1994. *Financial innovation and risk sharing.* Cambridge, MA: MIT Press.

Athanasoulis, S., and R. Shiller. 2000. The significance of the market portfolio. *Review of Financial Studies* 13:301–29.

———. 2001. World income components: Measuring and exploiting risk-sharing opportunities. *American Economic Review* 91, no. 4:1031–54.

Bettzuge, M., and T. Hens. 2000. An evolutionary approach to financial innovation. Working paper, Institute for Empirical Research in Economics, Universitat Zurich.

Bisin, A. 1998. General equilibrium with endogenously incomplete financial markets. *Journal of Economic Theory* 82:19–45.

Black, D. 1986. *Success and failure of futures contracts: Theory and empirical evidence.* Monograph Series in Finance and Economics. Salomon Brothers Center for the Study of Financial Institutions, New York University, Monograph 1986–1.

Blume, M., and I. Friend. 1975. The demand for risky assets. *American Economic Review* 65:900–22.

Blume, M., and S. Zeldes. 1993. The structure of stock ownership in the U.S. Manuscript, Wharton School, University of Pennsylvania.

Boot, A., and A. Thakor, A. 1993. Security design. *Journal of Finance* 48:1349–78.

Bris, A., Y. Koskinen, and M. Nilsson. 2002. The euro is good after all: Corporate evidence. Working paper no. 02–26, International Finance Corporation, Yale University.

Cappon, A. 1997. *The future of futures exchanges.* New York: The CBM Group, Inc.

———. 1998. *Reinventing exchanges.* New York: The CBM Group, Inc.

Ching, S., and M. B. Devereux. 2000. *Risk sharing and the theory of optimal currency areas: A re-examination of Mundell 1973.* Hong Kong: Hong Kong Institute for Monetary Research.

Cuny, C. J. 1993. The role of liquidity in futures market innovations. *Review of Financial Studies* 6:57–78.

Danthine, J., F. Giavazzi, and E. von Thadden. 2000. European financial markets after EMU: A first assessment. Discussion paper no. 2413, Center for Economic Policy Research. London.

Davis, S., J. Nalewaik, and P. Willen. 2001. On the gains to international trade in risky financial assets. Working paper, Graduate School of Business, University of Chicago.

Demange, G., and G. Laroque. 1995. Optimality of incomplete markets. *Journal of Economic Theory* 65:218–32.

DeMarzo, P. 1999. The pooling and tranching of securities. Working paper, Haas School of Business, University of California—Berkeley.

Dixit, A. K., and V. Norman. 1989. *Theory of international trade.* Cambridge, UK: Cambridge University Press.

Dow, J. 1998. Arbitrage, hedging, and financial innovation. *Review of Financial Studies* 11:739–55.

Duffie, D., and P. DeMarzo. 1999. A liquidity-based model of security design. *Econometrica* 67:65–99.

Duffie, D., and M. Jackson. 1989. Optimal innovation of futures contracts. *Review of Financial Studies* 2:275–96.

Duffie, D., and R. Rahi. Financial market innovation and security design. *Journal of Economic Theory* 65:1–42.

Europarl Report. 1999. Briefing No 34: Economic and Monetary Union (EMU) and enlargement. European Parliament. Luxembourg.

French, K., and J. Poterba. 1991. Investor diversification and international equity markets. *American Economic Review* 81:222–26.

Glaeser, E., and H. Kallal. 1997. Thin markets, asymmetric information, and mortgage-backed securities. *Journal of Financial Intermediation* 6:64–86.

Hara, C. 1997. A sequence of Pareto-improving financial innovations. Working paper, Churchill College, Cambridge University.

Hardouvelis, G., D. Malliaropoulos, and R. Priestley. 2000. EMU and European stock market integration. Discussion paper no. 2124, Center for Economic Policy Research. London.

Her Majesty's Treasury. 2003a. EMU and the cost of capital. London: H.M. Treasury Public Enquiry Unit.

———. 2003b. Government policy on EMU and the five economic tests: Executive summary. London: H.M. Treasury Public Enquiry Unit.

Helpman, E., and A. Razin. 1982. A comparison of exchange rate regimes in the presence of imperfect capital markets. *International Economic Review* 23, no. 2:365–88.

Kang, J., and R. Stulz. 1997. Why is there a home bias? An analysis of foreign portfolio equity ownership in Japan. *Journal of Financial Economics* 46:3–28.

Kendall, L., and M. J. Fishman. 1988. *A primer on securitization.* Cambridge, MA: MIT Press.

Lewis, K. 1999. Trying to explain home bias in equities and consumption. *Journal of Economic Literature* 37:571–608.

Madan, D., and B. Soubra. 1991. Design and marketing of financial products. *Review of Financial Studies* 4:361–84.

Mankiw, N., and S. Zeldes. 1991. The consumption of stockholders and non-stockholders. *Journal of Financial Economics* 29:97–112.

Mann, C. L., and E. E. Meade. 2002. Home bias, transaction costs, and prospects for the Euro: A more detailed analysis. Working paper, London School of Economics and Political Science.

McAndrews, J., and C. Stefanadis. 2002. The consolidation of European stock exchanges. *Current Issues in Economics and Finance* 8, no. 6:1–11.

Melamed, L. 2002. Futures exchanges in a changed world order. Derivatives Special Report. Profit and Loss: Digital FX, 4th quarter, http://www.leomelamed.com/speeches/02-china.htm.

Nachman, D., and T. Noe. 1994. Optimal design of securities under asymmetric information. *Review of Financial Studies* 7:1–44.

Naik, N., and P. Yadav. 1999. The effects of market reform on trading costs of public investors: Evidence from the London Stock Exchange. Working paper, Institute of Finance and Accounting, London Business School.

Neumeyer, P. A. 1998. Currencies and the allocation of risk: The welfare effects of a monetary union. *American Economic Review* 88:246–59.

Obstfeld, M., and K. Rogoff. 2002. Global implications of self-oriented national monetary rules. *Quarterly Journal of Economics* 117:503–36.

Pesendorfer, W. 1995. Financial innovation in a general equilibrium model. *Journal of Economic Theory* 65:79–116.

Ross, S. 1989. Institutional markets, financial marketing, and financial innovation. *Journal of Finance* 44:541–56.

Silber, W. 1981. Innovation, competition, and new contract design in futures markets. *Journal of Futures Markets* 1, no. 2:123–55.

Sutherland, A. 2003. International monetary policy coordination and financial market integration. Working paper, Center for Economic Policy Research. London.

Tufano, P. 1989. Financial innovation and first-mover advantages. *Journal of Finance* 25:213–40.

Wei, S. 1996. Intra-national versus international trade. How stubborn are nations in global integration? Working paper 5531, National Bureau of Ecenomic Research, Cambridge, MA.

Willen, P. 1997. Some notes on CAPM. Mimeo, Princeton University.

———. 1999. Distribution of welfare effects of new financial markets. Working paper, Princeton University.

Wincoop, E. 1994. Welfare gains from international risksharing. *Journal of Monetary Economics* 34:175–200.