

Moral hazard and nonexclusive contracts

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and

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We study equilibria for economies with hidden action in environments in which the agents' contractual relationships with competing financial intermediaries cannot be monitored (or are not contractible upon). We fully characterize equilibrium allocations and contracts for such economies, as well as discuss their welfare properties. Depending on the parameters of the economy, either the optimal action choice is not sustained in equilibrium or, if it is, agents necessarily enter into multiple contractual relationships and intermediaries make positive profits, even under free-entry conditions. The main features and implications of these environments are consistent with several stylized facts of markets for unsecured loans.

1. Introduction

■ Models of contracts with asymmetric information are usually models of exclusive contractual relationships. In other words, it is assumed that a party in a contract can enforceably restrict the other party's participation in contractual relationships with other agents.¹ As a consequence, agents cannot undo the incentive effects of one contract by engaging in additional contractual relationships with other agents or institutions. In terms of informational requirements, exclusive contracts effectively require that the institutions which design the contracts are able to perfectly monitor the agents' trades with other institutions. Also, courts can enforce exclusive contracts only if agents' trades are observable and verifiable, which requires a rich institutional setting allowing for some centralized information about trades.

Enforceability of exclusive contracts is a strong assumption, and while it is a very useful

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¹ See, e.g., the survey of contract theory by Hart and Holmström (1987). Even general equilibrium analyses of economies with asymmetric information rely heavily on exclusivity assumptions; see, e.g., Prescott and Townsend (1984) and Townsend (1987) for theoretical foundations, and Atkinson and Lucas (1992) and Townsend (1994) for applied analysis.

benchmark, there are many interesting economic environments in which technologies to monitor trades are either quite costly or tightly regulated, and exclusive contracts are difficult to enforce. Only rarely do debt covenants in financial contracts include in fact explicit exclusivity clauses (Asquith and Wizman, 1990; Smith and Warner, 1979).

In particular, markets for unsecured or partially secured loans do not seem to operate even implicitly through exclusive contracts. Consumer credit markets are a clear example: for instance, in the United States, consumers hold several credit cards and are constantly solicited to open new accounts; also, consumers often finance the acquisition of many durable goods, like cars, furniture, and electronic appliances, with distinct debt contracts. Information sharing among lenders is frequently absent or imperfect in most countries for small business transactions (see Jappelli and Pagano, 2000).

Unsecured and partially secured loan markets constitute a relevant component of credit markets in general. In the United States, for instance, revolving consumer loans (mostly credit card loans) are unsecured, and they account for more than a third of outstanding consumer credit. Moreover, a consistent share of consumer credit in general—including, for example, automobile, mobile home, and education loans—is only partially secured (Ausubel, 1997).² What characterizes unsecured or partially secured credit contracts is, of course, default risk. In the credit card market, for instance, delinquency rates (the percentage of accounts 30 days or more past due) exhibit peaks of about 3.5%; similarly, chargeoff rates (the percentage of outstanding balances written off as uncollectible) peak at about 5% (Ausubel, 1997).

Other markets that could be appropriately modelled as characterized by nonexclusive contractual relationships include farm credit markets in less-developed countries, where private moneylenders and family-related informal financial transactions interact and compete with banks and other formal financial institutions. The success of microfinance programs like the Grameen Bank in Bangladesh is often explained in terms of their ability to partially relax the informational constraints on exclusivity by means of extensive monitoring of trades (see, e.g., Morduch, 1997). In addition, the market for private lending to governments and other international institutions shares some aspects of nonexclusivity: a lack of information sharing across banks has been reported after the Latin American debt crisis in the 1970s as well as after the Asian crisis in the 1990s (see Radelet and Sachs, 1998). Finally, nonexclusivity, in the specific form of the inability of large firms to monitor the portfolio positions of their managers, has possibly important effects on the managerial incentive structure induced by stock-based compensation schemes, as documented by Ofek and Yermack (2000).

In this article we study in particular economies with moral hazard in the form of hidden action, to capture some abstract features of credit markets in which the borrowers need to take actions to prevent insolvency, and private default provisions are either contractually specified or determined by courts. In such economies, agents choose a costly action to limit the probability of insolvency. The action is agents' private information and, more specifically, affects the probability distribution of the agent's future income. Default is not strategic, as we interpret an unsuccessful realization of the agent's future income shock as a state of insolvency and default. Financial intermediaries issue contracts whose default provisions insure agents on their outcome realization, without being able to condition on their action choice.

We analyze economies in which the action is dichotomous ("high" or "low"), although we briefly discuss some of the implications of the corresponding model with a richer domain for the action variable. For instance, when the agent is a consumer, the high action might consist of taking an extra job opportunity or, when the agent is an entrepreneur, of an indivisible investment.

For this class of economies we are able to characterize equilibria with nonexclusivity. We show that for an open set of economies, in particular those with a relatively high cost of action, there exist only inefficient equilibria in which the low action is implemented. For those economies the effects of nonexclusivity are quite severe: in equilibrium the agents choose the low action even

² Also, the fraction of unsecured loans for firms with fewer than 500 employees (such firms generate 51% of private GNP) amounts to about 31% of all loans (from the National Survey of Small Business Finances).

if both at the incentive-constrained optimal contract and at the autarchic (no-trade) allocation they would choose high action. These equilibria occur because of the intermediaries' incentive to fully insure agents conditionally on their undertaking the low action whenever other intermediaries provide insurance at more favorable terms for the agents. As a consequence, high action can only be sustained, if at all, as an equilibrium when contracts can be designed to prevent entry of other contracts that adversely affect incentives for the incumbents. We show that equilibria that implement high action can be sustained for an open set of economies, and require the presence of "latent contracts," i.e., contracts that are not actively traded in equilibrium but whose presence reduces the profitability of equilibrium deviations for potential entrants. Latent contracts serve the purpose of restricting the entry of other contracts that would have negative incentive effects on the incumbents and thereby moderate the effects of nonexclusivity. On the other end, latent contracts guarantee rents to the incumbents in equilibria that support agents' high action.

As a consequence, we show that the high action can be implemented in equilibrium, and intermediaries necessarily make positive profits. Furthermore, in equilibrium agents actually engage in multiple contractual relationships with different lenders. Multiple contracts are necessary to prevent the active intermediaries from deviating to contracts guaranteeing even higher profits. Latent contracts take the form of available lines of credit at high interest rates (fair conditionally on the agents' low action).

Some existing evidence suggests that both supranormal profits and multiple credit relationships with different lenders may characterize some markets for unsecured loans. The credit card market provides our best example, as data for it are readily available. In fact, agents in general hold several cards, on average more than seven per household in the United States (and, according to Evans and Schmalensee (1999), more than nine for those households that hold at least one). Also, as documented in detail by Ausubel (1991, 1997), since the deregulation of the credit card market in 1982, the profits of credit card companies in the United States range from three to five times higher than the standard profits in the banking industry (interest rates are about three times the cost of funds adjusted for default risks).³

Latent contracts, i.e., available credit lines at high interest rates, also seem to characterize credit card markets in the form of frequent mailing and telemarketing solicitations (in 1995 the number of direct-mail solicitations from credit card issuers totalled more than two per month per American household on average; Ausubel (1997)).

Several pieces of evidence also suggest that the main implications of our analysis might be consistent with the observed structure of unsecured credit markets other than credit card markets. Multiple credit sources, for instance, are documented by Petersen and Rajan (1994) for the U.S. market for unsecured loans to small businesses; that article also presents some evidence of a "pecking order" in credit sources that can be interpreted as evidence of the availability of credit lines at high interest rates. In Europe, multiple credit sources are more prevalent: Detragiache, Garella, and Guiso (2000) document in detail the Italian case. Also, Jappelli and Pagano (2000) provide a cross-country analysis of credit markets, bankruptcy institutions, private credit bureaus, and public credit registers. Multiple credit relationships for small business are common, especially, as our analysis implies, in those countries where information-sharing institutions among lenders (private credit bureaus and public credit registers) are either recent or only partially developed. Furthermore, default rates are about twice as high on average in such countries, which is also consistent with the implications of our analysis of economies with nonexclusive contracts.

Finally, we show that nonexclusivity has important welfare effects. When contractual relationships are nonexclusive, two forms of moral hazard arise. First, agents' choice of action is private information. Second, agents' choice of trades is also private information. We show that equilibrium allocations, not surprisingly, are inefficient from the point of view of a planner who does not control the effort choice but does control the agents' trade. On the other hand, consider

³ More conservative estimates of the profitability of the credit card market in the 1980s emerge from the case study of the Discover Card program in Lapuerta and Myers (1997). No consensus is reached in the literature on the profitability of the credit card industry in the long run; see Evans and Schmalensee (1999).

a planner facing the same observability constraints that intermediaries face; that is, the planner controls neither the effort choice nor the agents' trade. We show that equilibria are efficient from the point of view of such a planner, that is, there is no other feasible allocation (feasible with respect to the observability constraints) that is preferred to an equilibrium allocation by both the agents and the intermediaries. We say that equilibria are incentive-constrained inefficient but third-best efficient. This is true even though, as we noted, in equilibrium intermediaries are not perfect competitors and entry is prevented via latent contracts.

□ **Related literature.** The analysis of hidden-action economies in nonexclusive environments has been pioneered by Arnott and Stiglitz in a sequence of unpublished papers in the early 1980s (their work is now collected in Arnott and Stiglitz (1993)) and by the enlightening comments on their work by Hellwig (1983).⁴ Our article is mostly related to this line of work. We study the same class of economies as do Arnott and Stiglitz and Hellwig, but we postulate a larger contract space that allows for "negative insurance" contracts (i.e., insurance contracts that pay in the high income state). We see no compelling reason to exclude negative insurance from the contract space. Even though negative insurance contracts are not traded in equilibrium, in fact we show that their presence in the contract space substantially reduces the set of contracts that are sustained as an equilibrium of the game played by intermediaries. As a consequence, the equilibrium set has a simpler structure, providing for sharper predictions. After introducing our results in Section 3, we analyze more precisely the relationships between our article and those of Arnott and Stiglitz and Hellwig.

Our article also contributes to the literature started by Arnott and Stiglitz and Hellwig by studying the question of constrained efficiency of equilibria. Although nonexclusivity clearly imposes on the economic environment additional constraints with respect to incentive compatibility, and hence equilibria in general will not be incentive-constrained Pareto efficient, it is not clear whether equilibria are third-best Pareto efficient once the constraints due to the nonexclusivity are explicitly considered. We will answer the question affirmatively, in the sense that a planner facing the same observability constraints that intermediaries face at equilibrium cannot find an allocation that is preferred to an equilibrium allocation by both the agents and the intermediaries.

Kahn and Mookherjee (1998) also study nonexclusive contracts in moral hazard economies with hidden action. The structure of the game intermediaries and agents play in their model, though, is quite different from ours (and from Arnott and Stiglitz's and Hellwig's as well); in particular, in Kahn and Mookherjee's economy, agents design their own contracts (intermediaries can only either accept or reject the agents' offers), they make contractual decisions sequentially, and their contractual portfolios are observable, even if not contractible upon. Such differences in the strategic interactions of agents and intermediaries have a crucial effect on equilibrium allocations and contracts. In our economy, because intermediaries in equilibrium exploit the rents due to the presence of "latent contracts," intermediaries necessarily make positive profits and agents face distorted insurance prices at equilibrium. On the contrary, intermediaries make zero profits and agents face fair insurance prices in equilibrium in Kahn and Mookherjee's environments. In Section 3 we will identify the modelling aspects that are responsible for the different results we obtain with respect to Kahn and Mookherjee.

Parlour and Rajan (2001) independently study a model of strategic default in unsecured credit market economies with nonexclusive contractual relationships.⁵ Our analysis has many elements in common with theirs, including the existence of equilibria with positive profits. Their model, in contrast to our hidden-action model, concentrates on strategic default and thus has the property that in equilibrium default is never observed, thereby contradicting the evidence from most unsecured credit markets and, most important, making it impossible to derive any implications linking the

⁴ But see also Bizer and De Marzo (1992), Helpman and Laffont (1975), Jaynes (1978), and Pauly (1968). The analysis of equilibria with nonexclusivity is also related to the analysis of "common agency" in the contract theory literature; see, e.g., Bernheim and Whinston (1986).

⁵ We thank a referee for this reference.

institutional characteristics of unsecured credit markets (e.g., regulatory constraints, the existence of information-sharing institutions) with default rates.⁶ The characterization of equilibria is also substantially different. For example, no latent contracts arise in their analysis, as the possible out-of-equilibrium deviations by intermediaries are limited by the fact that if default is induced, both the incumbents' and the entrants' profits are negatively affected. By contrast, in our setup a richer strategy space of intermediaries allows entrants to offer contracts that negatively affect only the incumbents (by inducing low action on the part of the agents), as the entrants can control the implicit rate of return required on the contracts so as to guarantee themselves nonnegative profits.

Helpman and Laffont (1975) (see also Bisin and Gottardi, 1999) study competitive equilibria in economies with hidden action. In their setup, linearity of prices captures a strong form of nonexclusivity: each intermediary has no control over agents' trades, not even over trades of its own contracts. In the setup of the present article, intermediaries instead control agents' trades in the contracts they themselves issue.

Finally, our article is also related in part to the literature on Cournot convergence to competitive equilibria; in Bisin, Gottardi, and Guaitoli (1999) we develop the analysis of the present article to study the issue in detail (see also Hellwig (2000) and Segal and Whinston (2003)).

2. The economy

■ The general economy we study lasts two periods, $t \in \{0, 1\}$. It is populated by a continuum of *ex ante* identical agents, indexed by $i \in I$ with total measure 1, and by a finite number of financial intermediaries, indexed by $h \in H$; we will in general think of H as large as we want to model intermediation markets in which entry is free (requiring H to be finite is only for the purpose of avoiding technical difficulties of no substantial relevance in the analysis). Agents are risk averse. They value consumption in period 0 and 1, c_0 and c respectively, and action e :

$$u(c_0) + u(c) - e$$

(units are chosen so that e is measured in utils, without loss of generality). We assume that $u : \mathbb{R}_+ \rightarrow \mathbb{R}$ is twice differentiable, strictly increasing, strictly concave, and $\lim_{c \rightarrow 0} u(c) = -\infty$. The action is chosen in $t = 0$, is private information, and can take two values, $e \in \{a, b\}$ (but we will be careful in discussing which results are robust to the introduction of a richer support of the action variable). Without loss of generality, $a > b$. The choice of action affects the probability distribution of the uncertain income of the agents at time 1, a random variable w^i that is i.i.d. across agents $i \in I$, whose realization is publicly observable, and which takes values w_H, w_L , with $w_H > w_L$ (from now on we drop the index i whenever confusion should not arise). Let π_a (respectively π_b) denote the probability of income w_H given action a (respectively b). Assume $\pi_a > \pi_b$. The reader will have noticed that H (respectively a) takes the interpretation of the "high income state" (respectively "high action"). We think of the high action as an action that helps prevent insolvency, that is, it reduces the probability of the low income state.

As is standard in moral hazard environments, we use the properties of large economies. In particular, the Law of Large Numbers allows us to identify π_e with the fraction of agents that observe the realization w_H when producing action e (see Al-Najjar (1995), Sun (1998)).

Prior to the beginning of time, intermediaries strategically design contracts. Each intermediary h can design and issue J^h contracts, and J is the set of contracts issued overall. A contract prescribes a set of transfers from the intermediary to the buyer (possibly negative) conditional on publicly observable variables. Formally, a contract $j \in J$ is a vector d^j representing the payoff at each date, 0 and 1, and state, H and L . Intermediaries can also make a contract divisible, allowing agents to buy fractions $\lambda_j \in [0, 1]$. A complete specification of contract j is represented

⁶ Some implications of our analysis for default rates are discussed in the conclusions.

by $D^j = \{d^j, \Lambda_j\}$, where Λ_j (the set of admissible λ_j) is either $\{0, 1\}$ (indivisible) or $[0, 1]$ (divisible).⁷ Let $D^h = (D^j)_{j=1}^h$ denote the set of contracts issued by intermediary h .

Intermediaries maximize profits. Contractual relationships are nonexclusive, as intermediaries cannot condition payoffs to the agents' trading positions.

In such an economy, contracts have both an insurance and a credit component that interact with nonexclusivity. This is indeed the case in markets for unsecured and partially secured credit, which motivate our analysis. Consider for instance the credit card market (see Ausubel, 1997). In this market, because contracts are nonexclusive, agents accumulate credit by engaging in multiple contractual relationships (multiple credit cards). Agents are in general exposed to private income risks and, as a consequence, frequently let credit card accounts go delinquent. Credit card companies often simply charge off the delinquent accounts, thereby extending a form of private insurance provision to their debtors. Credit limits and implicit insurance provisions are designed by credit card companies for an environment in which contracts are nonexclusive and agents hold multiple credit accounts.

To simplify the analysis, while capturing the fundamental properties of unsecured and partially secured credit markets, we consider two special cases and deal separately with credit limits and private insurance provisions.

In the first economy, which we call the *pure insurance* economy, private default provisions are contractually specified, but we restrict credit limits at time $t = 0$ to be exogenous. In fact, without further loss of generality, we normalize the agents' borrowing position at zero.⁸ Therefore, such an economy is simply a standard insurance economy with moral hazard. In the second economy we analyze, which we call the *credit* economy, we restrict instead state L to represent the default state, in which agents consume an exogenous amount w_L , that is, their endowment or a consumption provision exogenously determined by bankruptcy laws. On the other hand, in this economy credit limits are determined endogenously, agents borrow to finance consumption at time 0, and repay their debt at time 1 in state H .

In fact, the insurance and the credit economy turn out to be essentially equivalent in terms of our analysis and results. Therefore we proceed first with the analysis of the pure insurance economy because it is this economy that has been studied in the literature to which we mean to compare our results: Arnott and Stiglitz (1993), Hellwig (1983), and Kahn and Mookerjee (1998). In Section 4 we will then show how all of our results extend to the unsecured credit economy. We will also provide some simulations to show that the results extend to the general economic environment introduced in this section in which both the insurance and the credit dimension are jointly analyzed.

3. Insurance economy

■ Consider the pure insurance economy, in which agents consume only at time 1. We first introduce the definition of equilibrium we shall use in this article: *equilibrium with nonexclusivity*. Given the set of contracts issued by intermediaries, agents choose which contracts to buy. This determines their consumption allocation. Agents also choose an action. Anticipating the choices of agents, as a function of the set of contracts they are allowed to trade, intermediaries strategically choose which contracts they issue, to maximize profits.⁹

The problem solved by agents can be formally described as follows. Each agent chooses an action $e \in \{a, b\}$, portfolio choices $\lambda = \{\lambda_j \in \Lambda_j\}_{j \in J}$, and consumption $c = (c_L, c_H)$, to

⁷ As it turns out, only one type of contract will be made divisible in equilibrium (the latent contracts in the high-action equilibria); to save on notation, we will often refer to a contract only by its transfers d^j , intended to be indivisible unless specified otherwise.

⁸ The normalization in fact amounts to letting w denote income net of repayment of the amount borrowed at time 0, and similarly to let d^j denote contract's j payoff net of repayment of the amount borrowed at time 0.

⁹ We restrict the definition of equilibrium to the symmetric case in which all agents behave identically. This is just for the sake of notation, and we do not in fact make such an assumption in the analysis.

maximize

$$\pi_e u(c_H) + (1 - \pi_e)u(c_L) - e \tag{1}$$

subject to

$$c = w + \sum_{j \in J} \lambda_j d^j, \text{ given } D^j, \quad j \in J. \tag{2}$$

Note that if $\Lambda_j = \{0, 1\}$ agents can either buy or not buy contract j ; they cannot buy just a fraction, although they can buy multiples if the same contract is issued by more than one intermediary. In other words, while we model *nonexclusivity* as a form of inability of each intermediary to observe agents' trades with other intermediaries, we assume that each intermediary does observe her own aggregate trades with each agent.

The problem solved by intermediaries can be described as follows. Intermediary $h \in H$ chooses $D^h = (D^j)_{j=1}^{J^h}$ to minimize¹⁰

$$\sum_{j \in J^h} \left(\pi_e d_H^j + (1 - \pi_e) d_L^j \right) \lambda_j \tag{3}$$

subject to

$$e, \lambda \text{ solve (1)–(2), } (D^{h'})_{h' \neq h} \text{ given.} \tag{4}$$

Note that intermediaries effectively play a simultaneous game by choosing the structure of contracts they trade.

Definition 1 (equilibrium). An equilibrium with nonexclusivity is an array

$$\langle e, \lambda, c, (D^h)_{h \in H} \rangle$$

such that

- (i) $\langle e, \lambda, c \rangle$ maximize (1) subject to (2) given $D = (D^h)_{h \in H}$;
- (ii) D^h minimizes (3) subject to (4), for any $h \in H$.

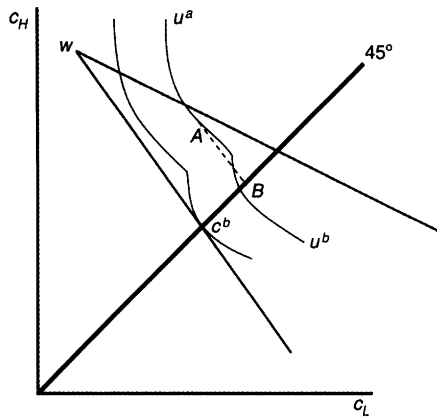
This definition of equilibrium (with a strategic component in the intermediaries' choices) has been used in different contexts by Rothschild and Stiglitz (1976), Arnott and Stiglitz (1993), and many others.

□ **Characterization.** We are now ready for the characterization of equilibria with nonexclusivity.¹¹ We will first derive conditions under which equilibria implement the low action (Proposition 1). We will then show that in general, whenever equilibria with nonexclusivity implement the high action, (i) agents trade multiple contracts at the equilibrium; (ii) such trades are necessarily sustained as an equilibrium by other contracts that are issued but not traded (and are therefore called *latent* contracts, following Hellwig's terminology); and (iii) latent contracts operate as a barrier to entry and hence guarantee positive profits to intermediaries (Propositions 2 and 3). We illustrate our analysis and most of our results graphically. Consider Figure 1. Point w is the no-trade allocation at which consumption coincides with income in each state. The straight lines from w represent the fair-price lines (equivalently, the zero-profit lines): the steeper line has slope $-(1 - \pi_b)/\pi_b$, the (negative of the) fair price of insurance conditionally on action b , while the less steep line has slope $-(1 - \pi_a)/\pi_a$, the (negative of the) fair price of insurance conditionally on action a . Let u^e denote the indifference curve conditional on action e being chosen; we have drawn them so

¹⁰ We implicitly assume that intermediaries have large-enough income to avoid bankruptcy issues. Note also that they do not need to be risk neutral: profits in fact are deterministic because of the Law of Large Numbers.

¹¹ All proofs are in the Appendix.

FIGURE 1



that u^a and u^b have the same expected utility, and hence the unconditional indifference curve is represented by the lower envelope of u^a and u^b . Since $(1 - \pi_b)/\pi_b > (1 - \pi_a)/\pi_a$, the marginal rate of substitution at any point is higher on u^b than on u^a .

Let C denote the subset of consumption allocations $(c_L, c_H) \geq 0$ that satisfy

$$c_L \geq w_L$$

$$\frac{1 - \pi_a}{\pi_a} \leq \left| \frac{c_H - w_H}{c_L - w_L} \right| \leq \frac{1 - \pi_b}{\pi_b}.$$

It contains all allocations that involve positive insurance on the part of the agents at prices that are not more expensive than the fair prices when the agents choose action b , and not cheaper than the fair prices when action a is chosen. In Figure 1, C coincides with the area contained between the two (zero-profit) lines from the no-trade allocation w . Any equilibrium allocation must be contained in C (otherwise either profits are negative or there is always room for a profitable contract to be issued).

Proposition 1. Define $\Delta = a - b$. If

$$\pi_a u(c_H) + (1 - \pi_a)u(c_L) - u(\pi_b c_H + (1 - \pi_b)c_L) - \Delta < 0, \quad \text{for any } c \in C, \quad (5)$$

then there exists a unique equilibrium allocation with nonexclusivity characterized by

$$c_H = c_L = \pi_b w_H + (1 - \pi_b)w_L \quad \text{and} \quad e = b.$$

We call these *low-action* equilibria. With the help of Figure 1 we can provide an illustration of the proposition. Consider an allocation like A in the figure, at which agents choose action a . Such allocation can never be sustained as an equilibrium for the preferences represented in the figure. Rather than consuming allocation A and choosing action a , agents would strictly prefer to buy additional insurance at the price $(1 - \pi_b)/\pi_b$, reaching allocation B , and choose action b . But Proposition 1 requires this to hold for any consumption point in C (condition (5)). Under this condition, no allocation in C with high action can represent an equilibrium, because at such an allocation any intermediary could make positive profits with a contract selling insurance at a price slightly higher than $(1 - \pi_b)/\pi_b$. On the other hand, insurance contracts at the price $(1 - \pi_b)/\pi_b$ can never make losses. We conclude that under the conditions of Proposition 1, an equilibrium is necessarily represented by action b and the consumption allocation that is preferred by agents facing a fair insurance price conditional on action b , $(1 - \pi_b)/\pi_b$. Such an allocation is obviously c^b in the figure, the full-insurance allocation conditional on action b .

Condition (5) is satisfied for a robust set of economies, which therefore have only low-action equilibria. To see this, consider an economy with logarithmic preferences, $u(c) = \ln(c)$. For such an economy, condition (5) can be written as

$$\exp\{-\Delta\} \left(\frac{c_H}{c_L}\right)^{\pi_a} < (1 - \pi_b) + \pi_b \left(\frac{c_H}{c_L}\right), \quad \text{for all } c \in C.$$

It can be immediately shown that such a condition is in fact satisfied for Δ high enough. More precisely, a Δ can be found that satisfies (5) without removing the first-best efficiency of high effort. Also, locally perturbing preferences around the logarithmic formulation, condition (5) can be satisfied for a robust set of economies.

Proposition 2. Suppose that, for an open subset of consumption allocations $(c_L, c_H) \in C$,

$$\pi_a u(c_H) + (1 - \pi_a)u(c_L) - u(\pi_b c_H + (1 - \pi_b)c_L) - \Delta \geq 0; \quad (6)$$

then any equilibrium allocation in pure strategies satisfies

$$\pi_a u(c_H) + (1 - \pi_a)u(c_L) - u(\pi_b c_H + (1 - \pi_b)c_L) - \Delta = 0 \quad (7)$$

with $e = a$.¹²

We call these *high-action equilibria*. Equation (7) defines the locus of allocations such that, when associated with the high action, agents are as well off as they would be buying the optimal level of additional insurance at price $(1 - \pi_b)/\pi_b$ and switching to $e = b$. The properties of high-action equilibria can be illustrated with the help of Figure 2, in the case of logarithmic preferences.

With logarithmic preferences, property (7) becomes

$$\exp\{-\Delta\} \left(\frac{c_H}{c_L}\right)^{\pi_a} = (1 - \pi_b) + \pi_b \left(\frac{c_H}{c_L}\right).$$

For Δ small enough, such an equation has two solutions in c_H/c_L , represented in the figure by line (7) and line (7).¹³ We proceed now to show that any high-action equilibrium must lie on (7). From any allocation in the interior of the cone, in fact, another allocation on (7) preferred by agents can be reached with a contract making nonnegative profits. The same is obviously true for allocations on (7). Finally, any allocation lying outside of the cone satisfies $\pi_a u(c_H) + (1 - \pi_a)u(c_L) - u(\pi_b c_H + (1 - \pi_b)c_L) - \Delta < 0$ and hence cannot be supported as an equilibrium with high action.

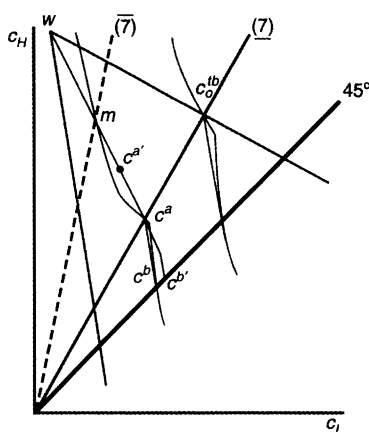
An allocation such as c^a on (7) may be supported as an equilibrium even if divisible contracts offering positive or negative insurance at price $(1 - \pi_b)/\pi_b$ are also issued. At any allocation on (7), consider again c^a for instance—agents are by construction indifferent between choosing action a and moving to allocation c^b with action b . But since allocation c^b is the most preferred allocation that can be reached from c^a with a contract that allows the agents to buy any amount of insurance (a divisible contract) at price $(1 - \pi_b)/\pi_b$, we can assume that such contracts would remain untraded if issued (latent); moreover, we can assume that such contracts are in fact issued, as they guarantee nonnegative profits to the issuer.

Latent contracts are a necessary component of equilibrium. Consider an agent trading two contracts to consume c^a in equilibrium: the first takes him from w to m , and the second from m

¹² The case in which (6) is satisfied for a zero-measure subset of C is nongeneric in the parameters of our economy, as it is immediately demonstrated by locally perturbing Δ . Refer to the Appendix for a discussion of this case.

¹³ A condition on the parameters is needed to guarantee that the cone from the origin generated by these two lines does not include the no-trade point, w , as in the figure; such a cone, however, always lies northwest of the 45° line.

FIGURE 2



to c^a . Without the latent contract, another intermediary could offer a contract that takes the agent from m to some point $c^{a'}$, which the agent prefers to c^a , and possibly to the allocation he can reach buying all three contracts offered. But with the latent contracts, as the agent reaches a point outside the area defined by (7), he can do even better by getting more insurance and switching to action b , reaching a point like $c^{b'}$. If, and only if, the latent contracts are issued, then the intermediary offering the deviation contract makes negative profits.

Let J_1 denote the subset of the set of contracts J that contains contracts actively traded in equilibrium, i.e., contracts j such that $\lambda_j > 0$. Our main characterization result regarding high-action equilibria is the following.

Proposition 3. At a high-action equilibrium, each agent actively trades multiple contracts:

$$\text{the cardinality of } J_1 \text{ is } > 1,$$

all of which guarantee positive profits to the intermediaries offering them:

$$\pi_a d_H^j + (1 - \pi_a) d_L^j < 0, \quad \forall j \in J_1.$$

To illustrate the first result of Proposition 3, note that for an allocation like c^a in Figure 2 to represent an equilibrium, it is necessary that agents actually trade multiple contracts; there are two in this case: the first takes agents from w to m , the second from m to c^a . Such contracts are constructed, in fact, to prevent the two intermediaries who are active in equilibrium from deviating and charging a higher price for their contract. They have the property that agents are indifferent between buying either one or both of them. Therefore, a deviation that worsens the terms of trade of one of the contracts would have the effect that such a contract is not traded. The crucial property that prevents the deviations of the incumbent intermediaries is that the indifference curve for action a through c^a cuts the implicit insurance price line, connecting the no-trade allocation point w with the consumption allocation c^a , twice: at c^a and at m . A single intermediary offering both contracts and allowing agents to consume at the same allocation c^a , with no other active intermediary, would not sustain the equilibrium, as he could deviate and increase profits by charging a higher price for insurance.¹⁴ As for the latent contracts, there must be at least two intermediaries, each selling a divisible contract for a large-enough quantity of insurance: agents can then always buy the optimal

¹⁴ In general there can be many equilibria with different numbers of active intermediaries: in the limit as such number goes to infinity, each intermediary offers a negligible quantity of insurance and the price of insurance equates the (negative of the) tangent to the indifference curve at the equilibrium allocation.

amount in response to any entry, and the intermediaries selling the latent contracts do not have profitable deviations.

We can finally see on Figure 2 why positive profits are necessary in equilibrium. The only allocation sustained by contracts making zero profits and lying on (7) is c_0^{tb} . To be an equilibrium for some number of active intermediaries $n > 1$, as we just argued, the indifference curve should cut the fair-price line, with slope $(1 - \pi_a)/\pi_a$, twice: at $c_0^{tb} = w + d$ and at $w + [(n - 1)/n]d$.¹⁵ In the limit, as $n \rightarrow \infty$, the indifference curve should be tangent to the fair-price line. But with action $e = a$, indifference curves have a slope $(1 - \pi_a)/\pi_a$ at full insurance and a steeper slope in the region of underinsurance, as at c_0^{tb} . In this region, then, they can only cut the zero-profit line once from above. Therefore, c_0^{tb} cannot be supported as a high-action equilibrium with any number n of active intermediaries. The same argument used to show that multiple contractual relationships are necessary at high-action equilibria implies that c_0^{tb} cannot be supported by only one active intermediary; such an intermediary would deviate to a contract with positive profits, along (7).

High-action equilibria do exist. We can easily show this by considering an economy with logarithmic preferences, $u(c) = \ln c$. By Proposition 2, a high-action equilibrium necessarily lies on (7). In the logarithmic case, we have repeatedly argued, (7) is the lower edge of the cone defined by

$$\exp\{-\Delta\} \left(\frac{c_H}{c_L}\right)^{\pi_a} \leq (1 - \pi_b) + \pi_b \left(\frac{c_H}{c_L}\right).$$

We will first construct a limit equilibrium, in which the number of active intermediaries, n , tends to infinity.¹⁶ In this case, each agent's indifference curve at the equilibrium allocation must be tangent to the price of insurance. To demonstrate the existence of such an equilibrium, therefore, it suffices to show that there exists an allocation on (7) with this property, that is, such that the indifference curve through the allocation is tangent to the price line from the endowment w to the allocation. But, as a consequence of homotheticity of preferences, the marginal rate of substitution is constant along any ray from the origin, in particular along (7). Moreover, the marginal rate of substitution along (7) is greater in absolute value than $(1 - \pi_a)/\pi_a$, since (7) is steeper than the full-insurance line. It is also smaller than $(1 - \pi_b)/\pi_b$ whenever the cone defined by (7) has a nonempty interior (the generic case). This is the case because when the cone has a nonempty interior, (7) is less steep than the line $c_H = (1 - \pi_b)\pi_a/\pi_b(1 - \pi_a)c_L$, the singular solution of (7) when the cone collapses into a line, along which the marginal rate of substitution is $(1 - \pi_b)/\pi_b$. We conclude that it is always possible to find a price line from the endowment w , with slope p in absolute value, $(1 - \pi_a)/\pi_a < p < (1 - \pi_b)/\pi_b$, which cuts (7) tangentially at the indifference curve; therefore a high-action equilibrium exists in the limit case with $n \rightarrow \infty$.

Equilibria with a finite number of active intermediaries, finite n , can now be constructed for lower insurance prices, with the only caveat that integer constraints must be satisfied.

Note also that our analysis of the logarithmic economy demonstrates that in this case, aggregate equilibrium profits for intermediaries increase with the number of intermediaries that are active at equilibrium.

We turn now to a discussion of how our characterization of equilibria for economies with dichotomous action extends to economies with continuous-action variables, in which for instance the action variable takes the interpretation of costly effort in the success of an investment. Our analysis of low-effort equilibria is immediately extended to economies with a continuous hidden-action variable and nonexclusive contractual relationships. If the support of the action variable is continuous and connected, e.g., the interval $[b, a]$, and the dependence of the probability

¹⁵ We are here implicitly assuming that each of the n intermediaries offers an identical contract to the agents. This is the case since, in equilibrium, agents must be indifferent between buying the n contracts or any $n - 1$ of them.

¹⁶ This example is essentially taken from Hellwig (1983). We report it here for completeness, as Hellwig's article is not published and might not be easily available to the reader.

distribution of income on the action is smooth, low-effort equilibria, characterized by the full-insurance allocation and the minimal action b , in fact arise precisely under condition (5).

The characterization of equilibria in which an action strictly higher than b is supported is difficult. Our analysis of the dichotomous case shows, though, that high-effort equilibria are supported when a discontinuity arises in the agents' choice of action as their allocation is parametrically varied. The construction of high-effort equilibria in the dichotomous case in fact requires latent contracts, which operate by inducing "large" discontinuous shifts in the agents' behavior as a consequence of the entry of a contract that deviates from the equilibrium. Such discontinuities follow in general from the nonconvexity of the agents' choice problem. More precisely, our analysis indicates that high-effort equilibria could be supported, for economies in which the unobservable action lies in a continuous domain, provided that assumptions on the map from the action to the probability distribution of income (concavity of the preference index is naturally maintained) are made which guarantee that the agents' objective function lacks concavity in the appropriate range of consumption allocations (the lack of connectedness of the domain of the action variable would also do, as would the lack of continuity of the probability distribution of income as a function of the action).

□ **Welfare.** We are now ready to study the welfare properties of equilibria with nonexclusivity. In the class of economies we study in this article, two forms of moral hazard arise. First, the choice of action e on the part of the agents is private information. Second, the choice of trades on the part of the agents is also private information. We therefore introduce two different definitions of optimality.

At an *incentive-constrained* optimum the planner does not control the effort choice but does control the agents' trade. If equilibria with nonexclusivity were incentive-constrained optimal, then the unobservability of trades, the nonexclusivity, would not matter in equilibrium. We will show that in general, nonexclusivity matters; it introduces an externality in equilibrium, and equilibria are not incentive-constrained optimal.

At a *third-best* optimum, instead, the planner controls neither the effort choice nor the agents' trade. The planner faces in fact the same observability constraints that intermediaries face in equilibrium.

More precisely, at an incentive-constrained optimum the planner chooses the agents' action, the consumption allocation, and the set of contracts to be traded to maximize agents' utility subject to the following: the definition of consumption, equation (9); the condition that guarantees a given aggregate amount k of profits to intermediaries, equation (10); and the incentive-compatibility constraint, equation (11). We are interested only in the nontrivial case in which the high action, $e = a$, can be supported at an incentive-constrained optimum. We restrict attention to this case: the incentive constraint guarantees that agents have no incentives, at the prescribed consumption allocation, to deviate and choose instead $e = b$.¹⁷

Definition 2. An *incentive-constrained* optimum supporting $e = a$ is an array

$$\langle c, d \rangle$$

that maximizes

$$\pi_a u(c_H) + (1 - \pi_a)u(c_L) - a \tag{8}$$

subject to

$$c = w + d, \tag{9}$$

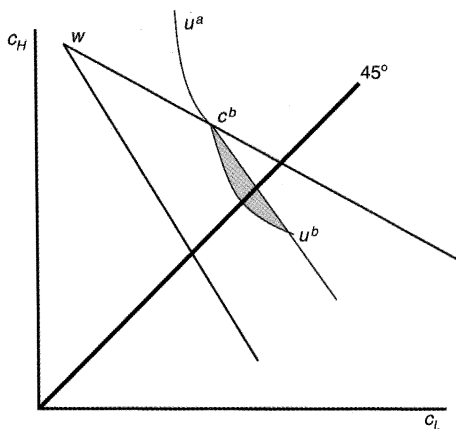
$$\pi_a d_H + (1 - \pi_a)d_L = -k, \tag{10}$$

and

$$\pi_a u(c_H) + (1 - \pi_a)u(c_L) - \pi_b u(c_H) + (1 - \pi_b)u(c_L) - \Delta \geq 0. \tag{11}$$

¹⁷ A sufficient condition for the existence of incentive-constrained optima supporting $e = a$, for instance, is the following: $\pi_a u(w_H) + (1 - \pi_a)u(w_L) - u(\pi_b w_H + (1 - \pi_b)w_L) \geq \Delta$.

FIGURE 3



Note that the amount of aggregate intermediaries' profits, k , parameterizes the frontier of incentive-constrained optima.

An incentive-constrained optimum supporting action $e = a$ cannot be decentralized by an equilibrium with nonexclusivity. In other words, nonexclusivity has relevant welfare effects, because it introduces an externality. Figure 3 illustrates graphically this point for the incentive-constrained optimum associated with zero profits, but the argument extends to $k > 0$. Again, u^e denotes the indifference curve conditional on action e being chosen; u^a and u^b have the same expected utility. With c^{ic} we denote the incentive-constrained optimum allocation. Since $\frac{1-\pi_b}{\pi_b} > \frac{1-\pi_a}{\pi_a}$, the marginal rate of substitution at any point is higher on u^b than on u^a . Any point in the shaded area in the figure can then be reached from c^{ic} with a contract making positive profits and is preferred by agents to c^{ic} . This proves that c^{ic} cannot be an equilibrium with nonexclusivity.

It may be interesting to characterize the payoffs $\{d^{j'}\}$ of the contract that makes positive profits when coupled with the incentive-constrained optimal contract. Since agents when buying only $\{d^j\}$ in equilibrium are indifferent between $e = a$ and $e = b$ (the incentive constraint is easily shown to be binding), there exists a contract $d^{j'}$ with the following properties:

- (i) $d_L^{j'} > 0$, $d_H^{j'} < 0$, and $\pi_b d_H^{j'} + (1 - \pi_b) d_L^{j'} = -\varepsilon$ (small enough),
- (ii) agents prefer $d^j + d^{j'}$ to d^j .

The first property states that contract $d^{j'}$ offers positive insurance and makes profits ε (i.e., the price of the insurance is less than fair for the agents). The second property guarantees that agents nonetheless prefer to buy the combination of contracts d^j and $d^{j'}$ rather than the incentive-constrained optimal contract d^j by itself. As a consequence, if both $\{d^j, d^{j'}\}$ are issued, agents choose action $e = b$, and the intermediary issuing d^j makes negative profits while the one issuing $d^{j'}$ makes positive profits.

Equilibria with nonexclusivity, then, are not in general incentive-constrained optimal for economies in which constrained optimal allocations support the high action, $e = a$. Nonexclusivity introduces an externality that is distinct from the externality caused by the nonobservability of the action e .

At a third-best optimum, the planner does not control agents' trades. Therefore the incentive-compatibility constraint is more restrictive in this case: it must guarantee that agents do not have an incentive to deviate both from the prescribed action and from the prescribed consumption allocation. We continue restricting attention to the case in which the high action, $e = a$, can be supported at an optimum; the condition in footnote (17) is sufficient also for this case of third-best optimum. When contemplating a deviation to $e = b$, agents anticipate being able to supplement

the prescribed consumption allocation with any feasible trade at fair odds; that is, they anticipate facing insurance opportunities at price $(1 - \pi_b)/\pi_b$.¹⁸

Definition 3. A *third-best* optimum supporting $e = a$ is an array

$$\langle c, d \rangle$$

that maximizes (8) subject to equation (9), equation (10), and

$$\pi_a u(c_H) + (1 - \pi_a)u(c_L) - u(\pi_b c_H + (1 - \pi_b)c_L) - \Delta \geq 0. \tag{12}$$

Note that the incentive constraint, equation (12), coincides with the condition that guarantees existence with high-action equilibria, equation (6). Therefore, for those economies for which low-action equilibria arise, no third-best optimum supporting the high action, $e = a$, exists. In that case, any third-best optimum can only support action b , and the planner cannot do any better than offering full insurance at fair prices. We conclude that low-action equilibria are third-best optimal. Note that even though third-best optimal, the inefficiency associated with *low-action* equilibria is quite severe. For a robust set of economies, for instance, the following three properties hold simultaneously: (i) all equilibria are low action (hence they implement $e = b$); (ii) at the incentive-constrained optimum allocation, $e = a$; and (iii) if no contract were offered (i.e., at autarchy), agents would choose $e = a$.¹⁹

The following result, the main one of this section, concerns third-best optimal allocations that support the high action.

Proposition 4. High-action equilibria with nonexclusivity are third-best optimal.

To understand the third-best optimality of high-action equilibria, it is convenient to characterize the third-best optimum frontier. The frontier is parameterized by the aggregate profits of intermediaries, $k \geq 0$. Consider Figure 4 (the figure is drawn for logarithmic preferences, but the argument extends). It is straightforward to see from the definition that third-best allocations supporting the high action must belong to C and lie on (7). Furthermore, moving along (7) northeast of A , toward B , one moves along the third-best frontier in the direction of allocations preferred by agents. In other words, allocation B in Figure 4 is a third-best optimum associated with smaller profits k than allocation A (c_0^{tb} is the third-best optimal allocation preferred by agents, associated with zero profits for intermediaries, $k = 0$). We conclude that high-action equilibria are third-best optimal, as, by Proposition 2, they must necessarily lie on (7).

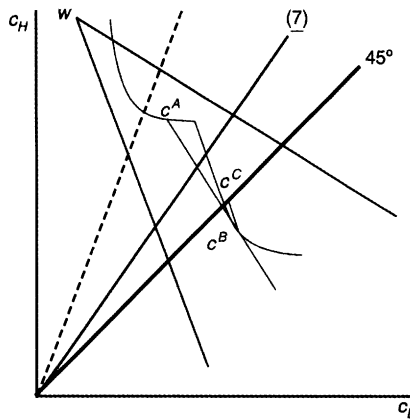
This result may require an explanation. Proposition 2 shows that high-action equilibria are supported by latent contracts and require positive profits for the intermediaries. Therefore, high-effort equilibria never decentralize c_0^{tb} . As a consequence, high-effort equilibria induce a distortion of the equilibrium price of insurance; more specifically, agents face in equilibrium less-than-fair insurance odds. Consider Figure 4, for instance, an equilibrium allocation such as B is supported by an implicit insurance price $q > (1 - \pi_a)/\pi_a$. How can third-best optimality be maintained despite this price distortion? The answer is that relative prices do not determine the agents' consumption allocation at equilibrium, the incentive constraint does (the incentive constraint (12) is binding at any high-action equilibrium allocation). In other words, prices do not matter, as agents' insurance positions are "rationed" in equilibrium.

We can illustrate this point with the help again of Figure 4. The implicit insurance price agents face at allocation B is q , and intermediaries make profits k (in the figure, k is the length

¹⁸ The implicit price of insurance in case of deviation is arbitrary. While this is a delicate issue, it has no impact on our analysis.

¹⁹ We leave the details of the proof of this statement to the reader. It follows from noticing that (ii) and (iii) hold generically for economies with logarithmic preferences for which (5) (and hence (i)) are satisfied, and that the result is maintained under small perturbations of preferences.

FIGURE 5



all surplus, thereby exploiting both the barriers to entry—which, as our analysis has demonstrated, endogenously arise in markets for nonexclusive contracts—and the bargaining power due to her position in the sequence of offers.

In other words, our and KM’s analysis of markets for nonexclusive contracts can be interpreted to show that: (i) competition of intermediaries in the design of contracts (and implicitly in their prices) is not sufficient to drive profits to zero, but (ii) if (and only if) agents are endowed with the whole bargaining power in the contractual relationships, intermediaries in equilibrium cannot exploit any market power by constructing barriers to entry.

As we argued in the Introduction, ASH study the same economy as we do, with intermediaries simultaneously designing contracts. They restrict the strategy space of intermediaries to contracts that offer positive insurance, i.e., to contracts whose payoff in state L is nonnegative, $d_L \geq 0$. Even though contracts offering negative insurance are never traded in equilibrium, the restriction of the strategy space in ASH is not without loss of generality. We argue that such restriction expands the set of equilibria that can be supported by latent contracts. Suppose that some allocation, e.g., c^A in Figure 5, is supported by a latent contract that, added to the allocation c^A , induces an allocation c^B (by construction the agent is indifferent between consuming c^A with action a and consuming c^B with action b).

Suppose also that at c^B the agent is overinsured, i.e., $c_H^B < c_L^B$, as is the case in Figure 4, where c^B is below the 45° line. Such a latent contract is never part of an equilibrium if negative insurance contracts are allowed, because such a contract can be profitably introduced that would be added to c^B by the agents, (i) inducing action b on the part of the agents and thereby (ii) negative profits for the intermediary trading the latent contract (in Figure 5, e.g., such a contract supports the allocation c^C when added to c^B). In ASH’s environment, on the contrary, many equilibria are supported by such latent contracts (see Hellwig (1983), in particular, for a characterization).

4. Credit economy

■ Consider now the credit economy, in which agents in state L consume w_L . The problem solved by agents can be formally described as follows. Each agent chooses an action $e \in \{a, b\}$, portfolio choices $\lambda = \{\lambda_j \in \Lambda_j\}_{j \in J}$, and consumption $c = (c_0, c_H)$ to maximize

$$u(c_0) + \pi_e u(c_H) + (1 - \pi_e)u(w_L) - e \tag{13}$$

subject to

$$c_0 = w_0 + \sum_{j \in J} \lambda_j d_0^j \tag{14}$$

$$c_H = w_H + \sum_{j \in J} \lambda_j d_H^j. \tag{15}$$

Intermediaries offer contracts characterized by a loan at time 0 in amount d_0^j , requiring a repayment d_H^j when the agent is not in default, that is, in state H . Formally, intermediary $h \in H$ chooses $D^h = (D^j)_{j=1}^{J^h}$ to minimize²¹

$$\sum_{j \in J^h} \left(d_0^j + \pi_e d_H^j \right) \lambda_j \tag{16}$$

subject to

$$e, \lambda \text{ solve (1)-(2), } (D^{h'})_{h' \neq h} \text{ given.}$$

We assume $w_0 < (1/\pi_b)w_H$ to guarantee that agents will borrow in equilibrium rather than lend.

We argue that the pure insurance economy studied in the previous section and the credit economy introduced here are equivalent. More specifically, for the credit economy as well, equilibria are either low or high action; high-action equilibria are sustained by latent contracts, which guarantee positive profits for the intermediaries, and by multiple active contractual relationships. Moreover, equilibria are incentive-constrained inefficient but third-best efficient.

All this should become apparent to the reader by studying, for the credit economy, the condition analogous to (6), that is, the incentive constraint associated with third-best optima. We will derive here this condition and then limit our analysis to pointing out the formal equivalence between the pure insurance and the credit economy.

For the credit economy, latent contracts are contracts that provide the agent with credit at a price $1/\pi_b$. Such contracts necessarily guarantee nonnegative profits for the intermediaries offering them. An equilibrium allocation with high action therefore has the property that it must be (weakly) preferred to the allocation an agent could reach by choosing action $e = b$ and buying additional credit at price $1/\pi_b$. Formally,

$$u(c_0) + \pi_a u(c_H) \geq \max u(c_0 + d_0) + \pi_b u \left(c_H - \frac{1}{\pi_b} d_0 \right) - \Delta',$$

where $\Delta' = a - b + (\pi_a - \pi_b)u(w_L)$ denotes the relative costs associated with the high action in this economy, an exogenous constant as in the case of the pure insurance economy. It is also straightforward to show that the optimal deviation when choosing the low action $e = b$, called d_0 , is such that $c_0 + d_0 = c_H - (1/\pi_b)d_0$; that is, following a deviation to the low action, agents use latent contracts to perfectly smooth consumption at time 0 and at time 1 in state H . It follows that the incentive-compatibility constraint, i.e, the condition analogous to (6), for the credit economy can be written as

$$u(c_0) + \pi_a u(c_H) - (1 + \pi_b)u \left(\frac{1}{1 + \pi_b} c_0 + \frac{\pi_b}{1 + \pi_b} c_H \right) - \Delta' \geq 0. \tag{17}$$

The formal analysis of equilibrium characterization now proceeds essentially as in the case of the insurance economy, but in the space of consumption allocations (c_0, c_H) rather than in the space (c_L, c_H) . Let C' denote the set of allocations $(c_0, c_H) \geq 0$ that satisfy

$$\begin{aligned} c_0 &\geq w_0 \\ \frac{1}{\pi_a} &\leq \left| \frac{c_H - w_H}{c_0 - w_0} \right| \leq \frac{1}{\pi_b}. \end{aligned}$$

Then, for parameter configurations such that there exists no allocation $(c_0, c_H) \in C'$ satisfying (17), the only equilibrium is a low-action equilibrium with perfect smoothing: $e = b, c_0 = c_H$.

²¹ We maintain the assumption that intermediaries have large-enough income to avoid bankruptcy issues (see footnote 10).

Otherwise, as in the pure insurance case, we have high-action equilibria. Such equilibria must satisfy conditions similar to those derived for the pure insurance economy. First, the incentive constraint (17) will be binding (otherwise another profitable credit contract can be introduced with agents still choosing the high action). Furthermore, to limit the market power of active intermediaries, multiple contracting is needed such that a single intermediary does not contribute a net surplus to the agent (the local satiation or double intersection between the price line and the indifference curve that we have seen in Proposition 3). At any such equilibrium, intermediaries necessarily make positive profits. In fact, the marginal rate of substitution between c_0 and c_H is always greater than $1/\pi_a$ above the 45° line, i.e., at all incentive-compatible allocations. Therefore the zero-profit point on the third-best frontier cannot satisfy the “local satiation” condition.

Finally, it is straightforward to show that high-action equilibria are third-best but not second-best efficient.

5. The general economy with credit and insurance

■ As we argued in the Introduction, contractual relationships in markets for unsecured credit have both relevant credit and insurance components. We have studied separately an insurance and a credit economy for the sake of simplicity. Analytical results for the economy with both credit and insurance, that is, for the general economy introduced in Section 2, are hard to derive. In this section we briefly report on some computations we have performed for specific parameter values, with the aim of showing that our analysis carries through for this more complex class of economies.

Consider logarithmic preferences, $u(c) = \ln c$, and fix arbitrary parameter values: $(w_0, w_H, w_L, \pi_a, \pi_b, a - b) = (5.0, 12.0, 2.0, .5, .2, .1)$.

We then proceed as follows.²² We first solve for the agent’s equilibrium choice of borrowing at time $t = 0$, d_0 , as a function of (c_L, c_H) , and conditionally on the agent also choosing the high action $e = a$. We then substitute the solution into the incentive constraint, the constraint analogous to equation (6), and into preferences. By looking at such indirect utility functions and incentive constraints, effectively we reduce the consumption space to (c_L, c_H) . This procedure has the advantage that results can be directly compared to those of the insurance economy, analyzed in detail in Section 3. The incentive-constrained region in the space (c_L, c_H) , for our parametric example, is shown in Figure 6. In the insurance economy, with logarithmic preferences, this set is a cone from the origin; in the general economy here it is still a cone, but translated from the origin. Also, indirect indifference curves on the space (c_L, c_H) are regularly shaped and quasi-concave.

Finally, we have computed an equilibrium, in the limit case when $n \rightarrow \infty$ (the number of active intermediaries), following the example by Hellwig reported in our analysis of the insurance economy.

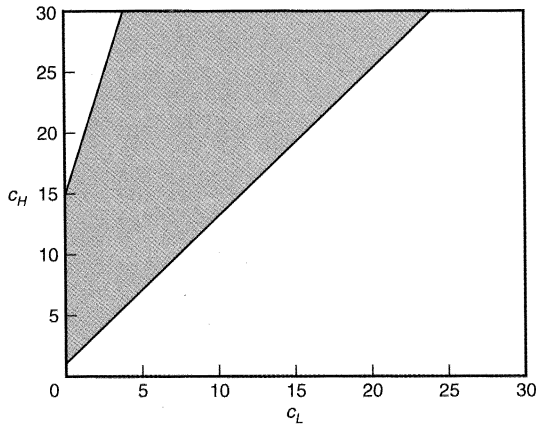
At the equilibrium (see Figure 7) the implicit insurance price, the slope of the line from $w - d_0$ to the equilibrium allocation c^* , is less than $(1 - \pi_a)/\pi_a$ in absolute value, and therefore intermediaries make positive profits. Also, as in the pure insurance economy (and the credit economy), there is no equilibrium at which intermediaries make zero profits.

6. Conclusions

■ Our analysis of the hidden-action model in economies in which contractual relationships are nonexclusive has shown that equilibria in such environments have several stylized properties that have been associated with unsecured credit markets. In particular, when the high action is supported, agents enter in equilibrium multiple contractual relationships and intermediaries make positive profits.

²² We only sketch our procedure here. Details are available in an Appendix posted at <http://www.econ.nyu.edu/user/bisina>.

FIGURE 6

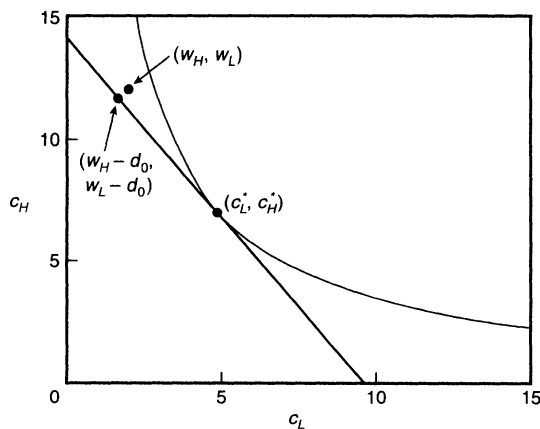


We conclude here by discussing some possible extensions with the intent of better understanding the implications of this class of models with respect to default rates and the size of the market.

A model with endogenous borrowing and lending positions can generate in equilibrium a negative correlation between the amount borrowed and the action chosen by the agents, as a consequence of exogenous changes in the rate of return on borrowing and the insurance price. An exogenous reduction in the rate of return and in the price of insurance induces higher borrowing and possibly higher insurance in equilibrium. In such a model, in fact, agents perfectly smooth their consumption over time: nonexclusivity implies that any equilibrium contract must be immune to deviations in the form of pure borrowing and lending contracts, whose rate of return is constant and hence independent of the agents' action choice. This in turn might tighten the agents' incentive constraint and make it more difficult to sustain a high action. In an economy with a continuous-action variable and with enough nonconvexity of the agents' choice problem to support in equilibrium actions higher than the minimal action, the agents' choice of action at equilibrium might be reduced. Since default rates are in our interpretation measured by the probability of the low state, this would imply higher default rates.

The other fundamental stylized fact in the credit card industry is indeed the contemporaneous increase of the size of the market and of default rates. Delinquency rates on credit cards exhibited cyclical peaks of about 2.75% before 1982 and of about 3.5% after; similarly, chargeoff rates exhibited peaks of about 3.5% before 1982 and of about 5.0% after (Ausubel, 1997). Such a

FIGURE 7



behavior of default rates in the credit card market, as a consequence of the burgeoning of the market after deregulation, could then be explained in the context of this model. The direct effect of deregulation has in fact been to reduce the interest rates of credit card borrowing from usury rates, thereby increasing the amount borrowed by agents in equilibrium, from essentially an autarchic equilibrium in which credit cards operated as means of payment rather than a credit instrument.

Similarly, such a model can explain the observed correlation between default rates and the lack of information sharing institutions (Jappelli and Pagano, 2000). Such institutions in fact better support quasi-exclusive contractual relationships and as a consequence can more easily support high-action choices, by restricting the set of contracts available, and hence lower default rates.

Appendix

■ Proofs of Propositions 1–4 follow.

We first sketch here a proof of existence in mixed strategies for completeness. The proof follows the lines of Bisin (1998).

We let for simplicity $\Lambda_j = \{0, 1\}$, for all j , and leave the general case to the reader. The agents’ optimal choice in problem (1)-(2) is described by a mapping from $\{d^h\}_{h \in H}$ into (e, λ, c) . Let this mapping be denoted ψ . Then clearly, under the assumptions on preferences, ψ is upper hemicontinuous.

We can now restrict the set of feasible contracts, without loss of generality, as follows: $d_L^j \in [-w_L, +w_L]$, $d_H^j \in [-w_H, +w_H]$, $\forall j \in J$. The strategy space of the game played by intermediaries is then compact, and their payoff function is continuous in (e, λ, c) . Intermediaries rationally anticipate the map ψ of $\{d^h\}_{h \in H}$ into (e, λ, c) . In fact, because there is a continuum of agents $i \in I$, intermediaries rationally anticipate the convex hull of ψ (see Bisin, 1998). Intermediaries’ profits are then effectively an upper hemicontinuous convex valued correspondence, since they are a continuous function of ψ . The main theorem in Simon and Zame (1990) allows us finally to show that, for some selection of the intermediaries’ profit correspondence, a Nash equilibrium in mixed strategies exists.

Proof of Proposition 1. Consider the allocations (c_L, c_H) satisfying

$$\pi_a u(c_H) + (1 - \pi_a)u(c_L) < \pi_b u\left(c_H - \frac{1 - \pi_b}{\pi_b} \tilde{d}\right) + (1 - \pi_b)u(c_L + \tilde{d}) - \Delta, \tag{18}$$

where

$$\tilde{d} = \operatorname{argmax} \left\{ \pi_b u\left(c_H - \frac{1 - \pi_b}{\pi_b} d\right) + (1 - \pi_b)u(c_L + d) \right\}.$$

By concavity of preferences, \tilde{d} is the full-insurance contract (conditionally on action b); substituting in (18), we obtain condition (5). No allocation (c_L, c_H) with high action satisfying (5) can be sustained as an equilibrium because, for some small positive ε , d' s.t. $\pi_b d'_H + (1 - \pi_b)d'_L = -\varepsilon$ is a strict best reply to any contract d such that $c = w + d$. Only the full-insurance allocation, $c_H = c_L = \pi_b w_H + (1 - \pi_b)w_L$, with action b can be sustained in equilibrium. The equilibrium contracts $D = \{D^j\}_{j \in J}$ sustaining such allocation satisfy $\pi_b d_H^j + (1 - \pi_b)d_L^j = 0$ and $\Lambda_j = \{0, 1\}$, $\forall j \in J$. A contract D' such that (i) $\pi_b d_H^j + (1 - \pi_b)d_L^j = 0$, $\Lambda^j = \{0, 1\}$ and (ii) $d_L^j \geq 0$ (“positive insurance”) is in fact a weak best reply to D . *Q.E.D.*

Proof of Proposition 2. If an open set of consumption allocations (c_L, c_H) in C satisfies (6), there exists a nonempty set of allocations for which (7) is satisfied (by continuity, since (5) is always satisfied when $c_L = c_H$). No allocation with high action for which (5) is satisfied can be an equilibrium (from Proposition 1). Suppose (6) is satisfied with strict inequality at some allocation (c_L, c_H) and sustained by contracts

$$c = w + \sum_{j \in J} d^j.$$

In the region of underinsurance the marginal rate of substitution is greater than the zero-profit rate, i.e.,

$$\frac{1 - \pi_a}{\pi_a} \frac{u'(c_L)}{u'(c_H)} > \frac{1 - \pi_a}{\pi_a} \quad \text{if } c_L < c_H.$$

There is always then another profitable contract, d' , such that $U(c + d', a) > U(c, a)$ and $U(c + d', a) > U(c + d', b)$.

However, an allocation (c_L, c_H) that satisfies (7) could be sustained by a set of contracts $\{d^j\}$ including not only

those needed to reach the consumption point, i.e., $\{d^j\}_{j \in J_1}$ s.t.

$$c = w + \sum_{j \in J_1} d^j,$$

but also “latent” contracts that deter entry, i.e., $\{d^j\}_{j \in J_2=J-J_1}$ s.t.

$$\max_{\lambda_j \in [0,1]} U(c + \sum_{j \in J_2} \lambda_j d^j, b) = U(c, a).$$

If $U(c + d', a) > U(c, a)$, then

$$\max_{\lambda_j \in [0,1]} U(c + d' + \sum_{j \in J_2} \lambda_j d^j, b) > U(c + d', a).$$

Here the following lemma applies.

Lemma A1. The set of contracts sustaining an equilibrium with $e = a$ must include “latent” contracts $\{d^j\}_{j \in J_2}$ s.t. $\lambda_j = 0, \forall j \in J_2$; moreover, any latent contract must satisfy $\pi_b d_H^j + (1 - \pi_b)d_L^j = 0, \forall j \in J_2$.

Proof of Lemma A1. If no latent contracts are issued, there always exists another profitable contract, for any proposed allocation c . Suppose latent contracts provide additional insurance at a price $|d_H^j/d_L^j| < (1 - \pi_b)/\pi_b$, for any $j \in J_2$. Agents will be indifferent between the candidate equilibrium allocation c , with high action a , and the best allocation they can reach with the latent contracts, $\hat{c} = \operatorname{argmax} U(c', b)$ s.t. $c' = c + \sum_{j \in J_2} \lambda_j d^j$, which will be in the region of overinsurance (where the marginal rate of substitution with low action is less than $(1 - \pi_b)/\pi_b$). But then there exists a contract d' selling negative insurance (i.e., $d_L' < 0, d_H' > 0$) with $|d_H'/d_L'| < |d_H^j/d_L^j| < (1 - \pi_b)/\pi_b$, such that $U(\hat{c} + d', b) > U(\hat{c}, b) = U(c, a)$ (agents strictly prefer the low-action allocation with latent contracts and the negative insurance). Also, agents prefer this to adding the negative insurance to c with high action, i.e., $U(\hat{c} + d', b) > U(c + d', a)$. If such a contract is introduced, agents will then buy all the contracts and choose low action. Any contract selling positive insurance with a slope less than $(1 - \pi_b)/\pi_b$ will make losses, and d' , being the negative of such a contract, will make positive profits. Suppose now the aggregate quantity of insurance offered by such latent contracts is rationed so that agents cannot reach the region of overinsurance, but only allocations either of underinsurance or of full insurance. Then, in the first case, additional positive insurance at a price slightly higher than $(1 - \pi_b)/\pi_b$ is a profitable deviation; in the second case, any intermediary selling part of the latent can unilaterally deviate to a profitable contract, since the remaining quantity of the latent contracts will be insufficient to trigger the agents’ reaction (even in the limit with an infinite number of intermediaries selling each an infinitesimal amount of the latent contracts, given that the full-insurance point is not a point of tangency, i.e., of local satiation). A profitable deviation exists, then, for any candidate equilibrium supported by latent contracts at a price less than $(1 - \pi_b)/\pi_b$. The only equilibrium contracts that survive are those supported by latent contracts at a price $|d_H^j/d_L^j| = (1 - \pi_b)/\pi_b$, for any $j \in J_2$. To trigger agents’ reaction for any possible entry, these contracts will be available in any quantity (i.e., they have to be divisible) up to a large-enough maximum. To be a best reply for intermediaries, the number of intermediaries selling the latent contracts must be, say, $m \geq 2$, with the aggregate quantity offered by any $m - 1$ intermediaries large enough to satiate agents for any possible deviations (e.g., $d_H^j = -w_H$ for any $j \in J_2$). *Q.E.D.*

Any equilibrium allocation sustained by latent contracts (according to Lemma A1) must then satisfy (7), by definition: more precisely, it must belong to (7), the subset of points c such that $c_L > c'_L$ for any c' in (7) with $\pi_b c'_H + (1 - \pi_b)c'_L = \pi_b c_H + (1 - \pi_b)c_L$.

Proof of Proposition 3. Latent contracts deter entry, but also intermediaries that are active in equilibrium must be prevented from deviating and charging a higher price for their part of the aggregate insurance. If $U(c, a) > U(c - d^j, a)$ for some $j \in J_1$, there is some contract d^j (with a higher price) that is more profitable than d^j . The equilibrium requires $U(c, a) = U(c - d^j, a)$ for all $j \in J_1$. With n active intermediaries, such a condition implies $d^j = (1/n)|c - w|$: isoprofit lines have a slope $(1 - \pi_a)/\pi_a$, less steep than the indifference curve, so no deviation is profitable.

We will now show that positive profits are necessary in equilibrium: a point of intersection of (7) with the zero-profit line, as c_0^{tb} in Figure 2, cannot be an equilibrium with nonexclusivity. To be an equilibrium for some number of active intermediaries $n > 1$, the indifference curve should cut the zero-profit line twice, at $c_0^{tb} = w + d$ and at $w + [(n - 1)/n]d$ (or, in the limit as $n \rightarrow \infty$, be tangent). But with action $e = a$, indifference curves have a slope $(1 - \pi_a)/\pi_a$ at full insurance and a steeper slope in the region of underinsurance. In this region, then, they can only cut the zero-profit line once from above. Therefore c_0^{tb} cannot be supported as a high-action equilibrium with n active intermediaries. Nor can it be supported with $n = 1$: a configuration in which only one intermediary sells the active zero-profit contract and others offer latent contracts is not an equilibrium configuration, since the active intermediary could do better, given the equilibrium choice of the others, by charging a higher price to reach a point along (7), associated with positive profits. *Q.E.D.*

Notice that the arguments in the proofs of Propositions 2 and 3 require that for an open subset of consumption

allocations $(c_L, c_H) \in C$,

$$\pi_a u(c_H) + (1 - \pi_a)u(c_L) - u(\pi_b c_H + (1 - \pi_b)c_L) - \Delta \geq 0.$$

In the case in which, on the contrary, this condition holds for a zero-measure subset of allocations $c \in C$ (and therefore holds necessarily with equality, by continuity), equilibria that do not satisfy $d^j = (1/n)|c - w|, \forall j \in J_1$ might exist. Moreover, in this case, equilibria that support low action and equilibria that support high action, with or without positive profits, might both exist. This set of economies is nongeneric, as it can immediately be proved using perturbations of the parameter Δ .

Proof of Proposition 4. Since we have already proved that high-effort equilibria lie on (7) (Proposition 2), we just need to show that the incentive constraint (12) is binding at any third-best optimum and the third-best frontier coincides with (7). From Definition 3 we see that a third-best optimum is a point in the incentive-compatible set (12) of maximum expected utility for the agents along a given isoprofit line. Isoprofit lines under high effort have a slope (in absolute value) $(1 - \pi_a)/\pi_a$ (parallel to the zero-profit line, see (9)–(10)). Agents' marginal rate of substitution (the slope of indifference curves) is always greater than $(1 - \pi_a)/\pi_a$ in the region of underinsurance (hence at all incentive-compatible points). Therefore, along any isoprofit line, the point preferred by agents is the one closest to full insurance, so the third-best optimum must lie on the boundary of the incentive-compatible set (12) closer to the full-insurance set. That boundary obviously coincides with (7). *Q.E.D.*

References

- AL-NAJJAR, N. "Decomposition and Characterization of Risk with a Continuum of Random Variables." *Econometrica*, Vol. 63 (1995), pp. 1195–1224.
- ARNOTT, R. AND STIGLITZ, J. "Equilibrium in Competitive Insurance Markets with Moral Hazard." Mimeo, Boston College, 1993.
- ASQUITH, P. AND WIZMAN, T. "Event Risk, Covenants, and Bondholders Returns in Leveraged Buyouts." *Journal of Financial Economics*, Vol. 27 (1990), pp. 195–213.
- ATKESON, A. AND LUCAS, R. "On Efficient Distribution with Private Information." *Review of Economic Studies*, Vol. 59 (1992), pp. 427–453.
- AUSUBEL, L. "The Failure of Competition in the Credit Card Market." *American Economic Review*, Vol. 81 (1991), pp. 50–81.
- . "Credit Card Defaults, Credit Card Profits and Bankruptcy." *American Bankruptcy Law Journal*, Vol. 71 (1997), pp. 249–270.
- BERNHEIM, B. AND WHINSTON, M. "Common Agency." *Econometrica*, Vol. 54 (1986), pp. 923–942.
- BERTINI, B. *Manuale delle Cessioni Stipendio*. Grafiche Artigianelli, 1967.
- BISIN, A. "General Equilibrium with Endogenously Incomplete Financial Markets." *Journal of Economic Theory*, Vol. 82 (1998), pp. 19–45.
- AND GOTTARDI, P. "General Competitive Analysis with Asymmetric Information." *Journal of Economic Theory*, Vol. 87 (1999), pp. 1–48.
- , ———, AND GUAITOLI, D. "A Note on the Convergence to Competitive Equilibria in Economies with Moral Hazard." In P.J.J. Herings, G. van der Laan, and A.J.J. Talman, eds., *Theory of Markets*. Amsterdam: North-Holland, 1999.
- BIZER, D. AND DE MARZO, P. "Sequential Banking." *Journal of Political Economy*, Vol. 100 (1992), pp. 41–61.
- DETRAGIACHE, E., GARELLA, P., AND GUIISO, L. "Multiple Versus Single Banking Relationships." *Journal of Finance*, Vol. 55 (2000), pp. 1133–1161.
- EVANS, D.S. AND SCHMALENSEE, R. *Paying with Plastic: The Digital Revolution in Buying and Borrowing*. Cambridge, Mass.: MIT Press, 1999.
- GUIISO, L., JAPPELLI, T., AND TERLIZZESE, D. "Why Is Italy's Savings Rate So High?" Discussion Paper no. 572, CEPR, 1991.
- HART, O. AND HOLMSTRÖM, B. "The Theory of Contracts." In T. Bewley, ed., *Advances in Economic Theory: Fifth World Congress*. New York: Cambridge University Press, 1987.
- HELLWIG, M. "On Moral Hazard and Non-price Equilibria in Competitive Insurance Markets." Mimeo, University of Bonn, 1983.
- . "Nonlinear Incentive Contracting in Walrasian Markets: A Cournot Approach." Discussion Paper no. 00-42, Sonderforschungsbereich, University of Mannheim, 2000.
- HELPMAN, E. AND LAFFONT, J.-J. "On Moral Hazard in General Equilibrium Theory." *Journal of Economic Theory*, Vol. 10 (1975), pp. 8–23.
- JAPPELLI, T. AND PAGANO, M. "Information Sharing, Lending and Defaults: Cross-Country Evidence." Mimeo, Università di Salerno, 2000.
- JAYNES, G. "Equilibria in Monopolistically Competitive Insurance Markets." *Journal of Economic Theory*, Vol. 19 (1978), pp. 394–422.

- KAHN, C. AND MOOKHERJEE, D. "Competition and Incentives with Nonexclusive Contracts." *RAND Journal of Economics*, Vol. 29 (1998), pp. 443–465.
- LAPUERTA, C. AND MYERS, S.C. "Measuring Profitability in the Credit Card Business." Mimeo, 1997.
- MORDUCH, J. "The Microfinance Revolution." Mimeo, Harvard University, 1997.
- OFEK, E. AND YERMACK, D. "Taking Stock: Equity-Based Compensation and the Evolution of Ownership." *Journal of Finance*, Vol. 55 (2000), pp. 1367–1384.
- PAULY, M. "The Economics of Moral Hazard." *American Economic Review*, Vol. 58 (1968), pp. 31–58.
- PARLOUR, C. AND RAJAN, U. "Competition in Loan Contracts." *American Economic Review*, Vol. 91 (2001), pp. 1311–1328.
- PETERSEN, M. AND RAJAN, R. "The Benefits of Lending Relationships: Evidence from Small Business Data." *Journal of Finance*, Vol. 49 (1994), pp. 3–37.
- PRESCOTT, E. AND TOWNSEND, R. "Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard." *Econometrica*, Vol. 52 (1984), pp. 21–45.
- RADELET, S. AND SACHS, J. "The Onset of the East Asian Financial Crisis." NBER Working Paper no. 6680, 1998.
- ROTHSCHILD, M. AND STIGLITZ, J. "Equilibrium in Competitive Insurance Markets." *Quarterly Journal of Economics*, Vol. 90 (1976), pp. 629–649.
- SEGAL, I. AND WHINSTON, M. "Robust Predictions for Bilateral Contracting with Externalities." *Econometrica*, Vol. 71 (2003), pp. 757–791.
- SIMON, L. AND ZAME, W. "Discontinuous Games and Endogenous Sharing Rules." *Econometrica*, Vol. 58 (1990), pp. 861–872.
- SMITH, C. AND WARNER, J. "On Financial Contracting: An Analysis of Bond Covenants." *Journal of Financial Economics*, Vol. 7 (1979), pp. 117–161.
- SUN, Y. "A Theory of Hyperfinite Processes: The Complete Removal of Individual Uncertainty via Exact LLN." *Journal of Mathematical Economics*, Vol. 29 (1998), pp. 419–503.
- TOWNSEND, R. "Arrow-Debreu Programs as Microfoundations of Macroeconomics." In T. Bewley, ed., *Advances in Economic Theory: Fifth World Congress*. New York: Cambridge University Press, 1987.
- . "Risk and Insurance in Village India." *Econometrica*, Vol. 62 (1994), pp. 539–591.