

COOPERATION AS A TRANSMITTED CULTURAL TRAIT

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ABSTRACT

In this paper, we study an endogenous cultural selection mechanism for cooperative behavior in a setting where agents are randomly matched in a one-shot interaction Prisoner's Dilemma, and may or may not have complete information about their opponent's preferences. We focus on an endogenous socialization mechanism in which parents spend costly effort to transmit directly their trait to their offspring, taking into account the impact of (oblique) societal pressures on cultural transmission. For various ranges of parameter values, this mechanism generates a polymorphic population with a long-run presence of cooperative agents, even where replicator and *indirect evolutionary* mechanisms would bring about a monomorphic population with non-cooperation. Further, under some circumstances, the long-run fraction of cooperative agents is shown to be larger under incomplete than complete information.

KEY WORDS • cooperation • cultural transmission • endogenous preferences • evolutionary selection

1. Introduction

In various real-life economic and social environments in which collectively beneficial actions are not in the interest of self-regarding individuals, agents often do nonetheless cooperate and sustain collective benefits. Examples include the adherence to collective norms of behavior in families, tribes, firms, and most social groups at large. Cooperative behavior in various strategic games like the Prisoner's Dilemma or public good contribution games has been also frequently reproduced in experimental exercises.¹

While this observation is consistent with the functionalist tradition in sociology, it is not immediately consistent with methodological individualism. Consequently, explaining the emergence of

norms of behavior in general, and cooperative and altruistic behaviors in particular, has become one of the most fundamental problems addressed in economics and other social sciences. To be more specific, consider the Prisoner's Dilemma, which has become the paradigm to study cooperation in abstract settings of social interaction.² Since the early Folk Theorem results (Friedman 1971), it is well known that cooperative behavior can be enforced by self-interested optimizing agents engaging in infinitely repeated relationships.³

A complementary literature considers instead the problem of emergence of cooperation from the different perspective of sociology and evolutionary game theory. Hence, instead of asking, 'Why is it that cooperative behavior occurs when egoistic rational agents play non-cooperative games?', this literature postulates that some agents are encoded to play cooperatively even if it is individually costly for them to do so (in payoffs or fitness terms), and asks instead the question, 'How can agents who play cooperatively at their own cost survive evolutionary selection when they compete with agents who do not cooperate?' This is, in Gintis's (2003b) words, the *puzzle of pro-sociality*.

In fact, from a pure evolutionary perspective, in which (a) biological fitness coincides with material payoffs, (b) agents are matched randomly to play the Prisoner's Dilemma, and (c) the dynamics is determined by fitness via payoff monotonic reproductive success (like the Replicator), only non-cooperative behavior is evolutionary stable; see Weibull (1995), example 2.1, p. 39.

But the pure evolutionary perspective has been integrated to allow for various novel aspects of genetic interactions, ranging from genetic relatedness (kin altruism), reciprocity, group selection, to mutualism and signalling, in order to provide different explanations of the adaptiveness of cooperation, that is, solutions to the *puzzle of pro-sociality*.⁴

In particular, several authors have built on the work of Cavalli-Sforza and Feldman (1981) and Boyd and Richerson (1985, 1988), by focusing on the subtle interactions between cultural and genetic evolution (co-evolution processes). For instance, Gintis (2003a) shows that the capacity to internalize fitness-enhancing norms of behavior can be biologically adaptive. Cooperation and other altruistic norms (not personally fitness-enhancing) can then also be internalized by 'hitchhiking' on the general tendency of other fitness-enhancing norms to be internalized.⁵

Other authors have instead studied environments in which evolutionary selection, rather than acting directly on behavioral rules, actions, and strategies, acts indirectly on preferences (*indirect evolutionary selection*); see Guth and Yaari (1992), Guth and Kliemt (1994), Bester and Guth (1998), Possajennikov (1999), Guttman (2001a, b). Preferences for cooperation, which induce agents to play cooperatively in the Prisoner's Dilemma by associating a psychological payoff with cooperative actions, might be selected according to the fitness implications of the strategies they induce in the game played.⁶ For instance, Guttman (2000, 2003) studies the indirect evolution of preferences for cooperation in the context of finitely repeated Prisoner Dilemma interactions in which some agents prefer cooperation to exploiting their opponents. Under different forms of incomplete information, these papers show that a positive fraction of agents with preferences for cooperation is in general evolutionary stable, and even agents without preferences for cooperation play cooperatively at times to establish a reputation. In this paper we study instead a purely cultural transmission mechanism of preferences for cooperation, where socialization is the result of actions and decisions of parents (vertical transmission) as well as the result of cultural characteristics of the social environment in which children are raised (oblique transmission).⁷

Our approach is related to various strands of the economic literature on endogenous preferences. Gintis (1974) was perhaps the first to introduce endogenous preferences in the economic literature (in his case via education decisions). Akerlof's paper (1983) on 'Royalty Filters' is another seminal analysis of children's preferences and parental socialization.⁸ Coleman (1994) and, more systematically, Becker (1993, 1996) extend the utility maximizing framework to analyze the effect of individual experiences (e.g. habitual behavior) and social forces (e.g. advertising, peer pressures, parental influences) on preferences, in a wide variety of economic problems. An early formulation of Becker's theory of socialization and preference formation is contained in Stigler and Becker (1977).

The specific class of cultural transmission models we study in this paper was first introduced by Cavalli-Sforza and Feldman (1981). Our main point of departure is that we model cultural transmission processes that are affected by purposeful (and therefore endogenous) socialization decisions taken by parents to influence the cooperative behavior of their children; see also Bisin and Verdier (2000). For instance, parents spend time with their children and

invest resources (e.g. in the form of private school's tuition) to socialize them to their preferred social norms; some parents favor cooperative norms, prescribing caring socially conscious behavior, while others might favor more competitive individualistic norms.⁹

As in the *indirect evolutionary selection* models, we consider a context in which agents may play cooperatively, in the Prisoner's Dilemma, because they receive a psychological payoff from doing so. Given that preferences for cooperation can be internalized by purposeful socialization, we then ask the question: 'How can these agents survive an endogenous *cultural* evolutionary selection mechanism when they interact strategically with agents who do not cooperate, and also socialize their children not to?'

We assume that parents who socialize their children do so motivated by altruism, but we assume altruism is not perfect in the sense that parents evaluate their children payoffs with their own preferences (i.e. parents that gain subjective psychological utility when cooperating also gain subjective psychological utility out of their children's cooperative behavior, even though their children might not); in Bisin and Verdier (2000) we defined this as *imperfect empathy*.¹⁰

Regarding the social interaction, and in particular the information structure in matchings, we distinguish two cases, complete and incomplete information. In the first case preferences and payoffs of each player in a match are common knowledge. This setting therefore captures in an abstract manner social relationships which are well established, such as relationships with friends, peers, and family members (but, recall, we do not consider infinitely repeated relationships). When information is incomplete, instead, players do not know their matches' relevant traits, like preferences and payoffs, and must infer those from the aggregate population distribution of such traits. This setting is meant to capture, again abstractly, more casual relationships across agents in society.

A first contribution of the paper, therefore, is to provide plausible microfoundations for an explicit cultural selection mechanism of preferences. We then proceed to investigate how such a mechanism affects the evolution of cooperation.

Endogenous cultural transmission generally leads to long-run polymorphic populations of preferences in which cooperation can be sustained (a positive fraction of the steady-state population of agents will play cooperatively in equilibrium). Two crucial aspects

of cultural transmission processes account for such a result. First, the selection criterion is not based on objective (or purely material) payoffs, but on payoffs as perceived by cultural parents according to their own preferences and values: hence an 'imperfect empathy bias' in the cultural transmission process. Though sometimes counter-balanced by the logic of material payoffs, this imperfect empathy bias induces cultural parents to take actions to transmit preferably their own preferences to the next generation. In particular, it induces cooperative parents to value per se the transmission of the cooperative trait to their children. Second, cultural parents are optimizing on their socialization decisions and therefore react optimally to changes in the cultural environment in which their children are immersed and may be socialized. The implication is that parents in minority cultural groups will have, everything else being equal, higher incentives to spend resources to socialize their children to their own preferences than parents in majority cultural groups. These two features introduce therefore a persistence effect which allows for the stability of minority cultural groups and the existence of polymorphic populations. In the present context, it allows cooperative preferences to survive culturally, even though they could be associated generally with lower purely material payoffs.

We characterize the conditions under which cooperation is favored when matching occurs under complete, as well as under incomplete, information. Interestingly, we show that matching under incomplete information may in some circumstances promote more cooperation in the long run than matching under complete perfect information.

Closely related to our work, Guttman (2001a, b) also studies a cultural transmission mechanism where parents take purposeful socialization decisions, and which supports polymorphic populations and cooperation. His environment, however, is very different from ours. First, rather than being motivated by *imperfect empathy*, as in our case, socialization to preferences for cooperation is motivated in Guttman (2001a, b) as a 'parental investment': children with preferences for cooperation take better care of their parents. As a consequence, the modelling of strategic interactions is also different: in Guttman (2001a, b), within each generation agents play a repeated finite time Prisoner's Dilemma with asymmetric information and noisy signals on individuals' types. Finally, the cultural transmission process studied in Guttman (2001a, b) is

additive, and hence the parents' socialization decisions do not internalize the potential impact of oblique cultural transmission – a crucial aspect of our setting.

Finally, as already mentioned earlier, our work is closely related to a series of papers by Gintis and co-authors investigating the evolution of *pro-sociality* and reciprocity in human societies (Bowles and Gintis 1998, 2002, Gintis 2000, 2003a, b, Gintis et al. 2001, Bowles 2001, Boyd et al. 2003). In particular, Gintis (2003a) investigates the population dynamics of preferences for altruism under several selection mechanisms (e.g. biologically adaptive dynamics and learning dynamics), and under a cultural transmission mechanism with vertical and oblique transmission, as in the present paper. A major conclusion of Gintis (2003a) is that a strong enough oblique cultural transmission is necessary to support preferences for altruism and hence pro-social behavior. While we reach similar conclusions in the present paper, the rationale for the sustainability of preferences for cooperation is different: socialization is modelled as an exogenous process in Gintis (2003a), whereas in our context it is precisely the substitution between vertical and oblique socialization, induced by the purposeful socialization choices of parents, that helps support preferences for cooperation.

The paper is organized as follows; Section 2 presents the basic model of endogenous cultural transmission in the strategic context of the Prisoner's Dilemma game. Section 3 analyzes the evolution of preferences for cooperation in a complete information setting. Section 4 considers the case with incomplete information. Finally, Section 5 provides a short conclusion. All proofs are relegated to the appendix.

2. Cultural Transmission and Evolution of Cooperation

Consider overlapping generations of two period lived (young and adult) agents. Reproduction is asexual and fertility exogenous: each adult at time t has a child who is young at time $t + 1$. Adult agents are randomly matched to play the standard Prisoner's Dilemma with 'objective' payoffs represented by the following symmetric matrix:

Table 1. Prisoner's dilemma: NC player's payoffs

		Pl. 2	
		<i>c</i>	<i>nc</i>
Pl.1	<i>c</i>	<i>T, T</i>	<i>-R, T + V</i>
	<i>nc</i>	<i>T + V, -R</i>	<i>0, 0</i>

with $T, R, V > 0$.

Since the payoffs when both players do not cooperate, play *nc*, are normalized to zero, the parameter T represents the gains from cooperation. R represents instead the cost associated to cooperating, playing *c* when the other player does not; while V represents the gains associated to not cooperating when the other player in the match does. We restrict ourselves to games where cooperation (*c, c*) is efficient from a social surplus point of view: namely, $T > V - R$.

Players with ‘non-cooperative’ preferences (NC players for short) have payoffs as in the payoff matrix above.¹¹ On the other hand, players with ‘cooperative’ preferences (C players) receive $d > 0$ extra units of subjective psychological payoff any time they play cooperatively (choose action *c*) independently of the other player’s actions.¹² Let q_t denote the fraction of C players in the population at time t . Also, let p_t denote the fraction of NC players. Of course $q_t + p_t = 1$.

Young agents are born with no well-defined preferences. Parents with preferences of type $i \in \{C, NC\}$ choose effort τ^i to socialize their children to their own preferences, at cost $H(\tau^i)$.¹³

The socialization mechanism works as follows. Consider a parent with i preferences. His child is first directly exposed to the parent’s preferences (and is socialized with probability τ^i chosen by the parent); if this direct socialization is not successful, with probability $1 - \tau^i$, he is socialized to the preferences of a role model picked at random in the population, that is to preferences C with probability q and to preferences NC with probability $p = 1 - q$.¹⁴

Formally, parents with preferences of type i at time t will then have children with their own same preferences with probability

$$\pi^C = \tau^C + (1 - \tau^C)q \text{ for } C \text{ parents} \tag{1}$$

$$\pi^{NC} = \tau^{NC} + (1 - \tau^{NC})p \text{ for } NC \text{ parents} \tag{2}$$

while they will have children with different preferences with probability

$$1 - \pi^C \text{ for } C \text{ parents} \tag{3}$$

$$1 - \pi^{NC} \text{ for } NC \text{ parents} \tag{4}$$

As a consequence, the dynamics of the fraction of C players in the population are governed by the following equation

$$q_{t+1} = \pi^C q_t + (1 - \pi^{NC})(1 - q_t)$$

or the difference equation:

$$q_{t+1} - q_t = q_t(1 - q_t)(\tau^C - \tau^{NC}) \tag{5}$$

The probability of direct socialization of each population group, τ^i , $i \in \{C, NC\}$ is determined as the optimal choice of each parent with trait i . We assume in fact that parents are altruistic, and altruism motivates the transmission of culture. But we assume parents evaluate their children payoff from playing the game with their own (the parent's) preferences. For instance a parent who is a C player derives utility from his child playing c , even if the child does not.¹⁵

A type i parents' socialization problem is

$$\max_{\tau^i} \pi^i V^{ii}(q_{t+1}^e) + (1 - \pi^i) V^{ij}(q_{t+1}^e) - H(\tau^i)$$

where $V^{ij}(q_{t+1}^e)$ represents the type i parent's evaluation of his child's payoff from playing the prisoner's dilemma in period $t + 1$, if the child has preferences of type $j \in \{C, NC\}$ and the expected fraction of agents in the population with preferences of type C is q_{t+1}^e .

We assume socialization costs are quadratic, $H(\tau^i) = \frac{1}{2}(\tau^i)^2$. This is just simplicity, as it allows closed form solutions, but our qualitative results are preserved for more general convex costs. The solution of the socialization problem is then simply given by:

$$\tau^C = (1 - q_t)\Delta V^C(q_{t+1}^e) \text{ and } \tau^{NC} = q_t\Delta V^{NC}(q_{t+1}^e) \quad (6)$$

where $\Delta V^i(q_{t+1}^e) = V^{ii}(q_{t+1}^e) - V^{ij}(q_{t+1}^e)$.¹⁶

We are looking for dynamic cultural processes such that the time path $\{q_t\}_{t \geq 0}$ satisfies $q_{t+1}^e = q_{t+1}$, so that our analysis does not depend on any systematic mistake in the agent’s prediction of the dynamics of the distribution of the population with respect to preferences. The dynamics of the fraction of the population with trait C , in equation (5), can then be written as:¹⁷

$$q_{t+1} - q_t = q_t(1 - q_t)(\tau^C(q_t, q_{t+1}) - \tau^{NC}(1 - q_t, q_{t+1})) \quad (8)$$

Two explicit remarks discussing some of the general properties of the dynamics of our model of cultural transmission may be useful to interpret our results.

Remark 1. Equation (8) looks very much like what you would get from a standard Replicator Dynamic process if instead of $\tau^C - \tau^{NC}$, one had the difference between normalized expected material payoffs of the two types of individuals. The difference in our formulation is that (i) τ^C and τ^{NC} are direct socialization rates rather than payoffs, and especially that (ii) they are endogenously derived from a purposeful socialization process by parents (material payoffs play only an indirect role in the dynamics through their effects on $V^{ij}(q_{t+1})$).

This difference accounts for the very different implications of our cultural transmission model with respect to a pure evolutionary selection mechanism in which biological fitness coincides with material payoffs, agents are matched randomly to play the Prisoner’s Dilemma, and the dynamics is determined by the Replicator Dynamics. In this case the implied dynamics would be represented by

$$q_{t+1} - q_t = q_t(1 - q_t)(V^C(q_{t+1}) - V^{NC}(q_{t+1}))$$

and hence (it is easy to show) only $q = 0$ is a stable stationary state of the dynamics. Preferences for cooperation are selected out of the population in the long run; see Weibull (1995), example 2.1, p. 39.

Remark 2. Suppose, as a way of illustration, that the gains from socialization, ΔV^i are constant (independent of q_{t+1}) and positive,

for both traits i . Then, from equation (6), τ^C and τ^{NC} only depend on q_t . τ^C is decreasing in q_t while τ^{NC} is increasing. Moreover, in this case τ^C (resp. τ^{NC}) is 0 if $q = 1$ (resp. $q = 0$). In other words, perfect majorities do not socialize their children, because they are freely socialized to the majority trait by society at large. Minorities on the other hand do socialize their children. In fact, the direct probability of socialization of group i , τ^i , is decreasing in the fraction of the population with trait i . In other words, minorities substitute oblique with direct socialization. By equation (8), then, homomorphic steady states, in which q is either 1 or 0, are not stable, and a unique polymorphic steady state, $q \in (0, 1)$ is dynamically stable (with the whole $(0, 1)$ as basin of attraction).

This does not mean that the cultural transmission process we study induces the persistence of any possible trait. When gains from socialization, ΔV^i , depend on the distribution of the population by trait, as is generally the case when agents interact strategically, the persistence of a trait is not implied by cultural transmission. In particular, when those gains from preserving a trait increase with the share of the population with the same trait, minorities might or might not have larger incentives than majorities to socialize their children to their own trait, depending on whether the reduction in socialization rates due to the direct effect of the reduced socialization gains ΔV^i is smaller or larger than the increase in socialization rates due to the substitution of oblique with direct socialization.

This is the case in the strategic environment we study in this paper, in which agents are randomly matched to play the Prisoner's Dilemma. An agent with a preference for cooperation will generally gain more from transmitting this preference to his children when the fraction of cooperative agents in society is large, since in this case his children will more often avoid the costly interactions with non-cooperative agents. Socialization rates will therefore depend in general on the distribution of the population in terms of preferences as well as indirectly on the material payoffs achieved in the strategic interaction game of the next generation.

Moreover, how costly is the interaction with a non-cooperative agent for a cooperative agent will depend on the information structure of the game: if a non-cooperative agent is known to be such in the match, then cooperative agents might adopt conditional strategies, e.g. play c with C agents and nc with NC agents.

In general, homomorphic steady states are unstable as long as

$$(\tau^N(q, q) - \tau^{NC}(1 - q, q)) > 0 \text{ at } q = 0 \text{ or } q = 1$$

Therefore, the crucial aspect of the analysis, which will determine whether homomorphic or polymorphic population will survive in the long run, will consist in characterizing the endogenous gains from socialization ΔV^i for very small minorities, that is populations of agents with common preferences i in the case q is close to 0 or 1. In particular, it will be important to understand how such endogenous gains from socialization are a function of the payoff, the information structure of the game, and the properties of the matching mechanism. This will allow us to derive implications for the dynamics of the distribution of traits in the population.

3. Matching with Complete Information

In this section we study the case in which players can observe the preference type of the opponent after having been matched to play the game. It represents, admittedly in a reduced form, social environments in which the matched agents have previously interacted perhaps in different strategic situations and with different agents. It can be considered a benchmark to be compared to the most interesting case of incomplete information.

A characterization of the Nash equilibria of the one-shot Prisoner's Dilemma, for various configurations of the parameters, is straightforward and is reported in the following table.

Table 2. Nash equilibria (complete information)

Match	<i>(nc, nc)</i>	<i>(nc, c)</i>	<i>(c, c)</i>
Parameters:			
$d > \max\{V, R\}$	(nc, nc)	(nc, c)	(c, c)
$V < d < R$	(nc, nc)	(nc, nc)	(c, c) and (nc, nc)
$R < d < V$	(nc, nc)	(nc, c)	(nc, c) and (c, nc)
$d < \min\{V, R\}$	(nc, nc)	(nc, nc)	(nc, nc)

Playing nc is a dominant strategy for an NC player, independently of which type of player he is matched with. The equilibrium action of C players instead depends on the parameters of the game.

An environment in which the psychological gains from cooperation are small enough, $d < \min\{V, R\}$, is one where nc is a dominant strategy for both NC and C players, in any match. In this case then NC and C players are indistinguishable in terms of equilibrium actions. We exclude this trivial uninteresting case from the analysis and restrict ourselves to the case in which $d > \min\{V, R\}$. We distinguish the three remaining regions of the parameters:

- (a) the environment in which $d > \max\{R, V\}$ is one where the preferences for cooperation of C agents, their psychological gains from cooperating, are very intense. In such a configuration of parameters, C agents are unconditional cooperators (c is a dominant strategy).

If $\min\{R, V\} < d < \max\{R, V\}$, on the other hand, each type C agent's equilibrium action may be contingent on the type of the agent he is matched with. More precisely,

- (b) the environment in which $V < d < R$ is one where non-cooperating in a match in which one agent plays c and the other nc is associated to small gains, but possibly imposes large costs on a cooperating player. C agents are conditional cooperators. First, they do not cooperate with a NC type agent. Second, the game between two C type agents has the structure of an 'assurance' game with two Nash equilibria in pure strategies: (c, c) in where both agents cooperate or (nc, nc) where both do not cooperate.
- (c) the environment in which instead $R < d < V$ is one where non-cooperating in a match in which one agent plays c and the other nc is associated to large gains but imposes small costs. In such a game, C players play c when their opponent plays nc and vice versa. Hence, the game between two C players has a 'chicken game' structure with again two Nash equilibria in pure strategies: (c, nc) and (nc, c) one of the C players cooperates while the other does not.

Clearly, in configurations (b) and (c) multiple Nash equilibria exist. We adopt therefore the following reasonable selection of

equilibrium and we study the dynamics of the population distribution of traits, represented by q_t^C for such a selection:

- (b) in the parameter region $V < d < R$, we assume that type C plays action c when matched with a C agent. This can be rationalized by noting that the (c, c) equilibrium Pareto dominates the other equilibrium (nc, nc) and therefore that players can reasonably coordinate on it.¹⁸
- (c) in the parameter region where $R < d < V$, we assume that each equilibrium outcome (c, nc) and (nc, c) occurs with equal probability. One way to rationalize this selection device is to assume that when two C players match, there is a probability 1/2 for each agent to play first (e.g. to be a Stackelberg leader) and to implement therefore his best equilibrium outcome (c, nc) or (nc, c) . By symmetry, there is also a probability 1/2 for that agent to play second (e.g. to be a follower) and to have the other outcome implemented.

Proposition 1. There exists a unique stable stationary state of the population dynamics; this state is polymorphic for $d > \max\{V, R\}$ and $R < d < V$; it is monomorphic at $q^ = 1$ for $V < d < R$.*

Consider first the environment in which non-cooperating imposes big costs and has small gains, $V < d < R$. In this case our cultural transmission process gives rise to a stable monomorphism with full cooperation: in the limit, the whole population is composed of agents with preferences for cooperation. How is this possible? When non-cooperating imposes big costs and has small gains players with a preference for cooperation in equilibrium adopt a strategy of conditional cooperation. They cooperate when they match agents with their same preferences and they do not cooperate otherwise. We could call this strategy also *reciprocation*. This strategy is

Table 3. Dynamics (complete information)

Stability of	Heterog. pop.	Coop. pop.	Non-coop. pop.
Parameters:			
$d < \max\{V, R\}$	X		
$V < d < R$		X	
$R < d < V$	X		

obviously quite effective, as it allows agents with cooperative preferences to avoid the cost R of cooperation with non-cooperators.¹⁹ But the reason why such a strategy is so successful in terms of our cultural selection mechanism is that parents with non-cooperative preferences have no incentives to socialize their kids to their own preferences in this case. While parents with non-cooperative preferences, in fact, only care about the ‘objective’ payoffs of the game, conditional cooperation fares very well in terms of objective payoffs, actually better than the strategy of non-cooperation that their children would adopt if socialized. Parents with preferences for cooperation, on the other hand, always socialize their children because they gain psychologically if they cooperate. Consequently, in this environment, parents with preferences for cooperation have higher incentives to socialize their children to their own preferences than parents with non-cooperative preferences, and hence cultural transmission will select the ‘cooperative’ preference trait.

The pure cultural transmission mechanism which, as we have seen, favors the socialization of minorities, is responsible instead for the polymorphic stationary state when $d > \max\{V, R\}$ and $R < d < V$, and players with preferences for cooperation do in fact cooperate in equilibrium when matched with agents with non-cooperative preferences also.

It is interesting to see that this ‘minority favoring’ effect creates a discrepancy between the long run outcomes implied by endogenous cultural evolution and what would be obtained from more standard replicator dynamics. In the standard view where selective forces apply directly to strategies c and nc , given that nc is a dominant strategy in the one-shot PD, the only possible evolutionary stable strategy in this one-shot PD game is obviously nc and cooperation cannot emerge.

In the ‘indirect evolutionary’ approach where preferences evolve according to the fitness (material payoffs) implications of the strategies they induce, again significant differences emerge. It is easy to see, for instance, that for $d > \max\{V, R\}$ and $R < d < V$, the long-run outcome of the ‘indirect evolutionary’ approach generates a long-run monomorphic population at $q = 0$ with no cooperation²⁰ while our setting implies a polymorphic long-run population with some persistent degree of cooperation. For the configuration of parameters $V < d < R$ in which C agents play conditional cooperation, on the contrary, both approaches generate a long-run monomorphic population $q = 1$ with full cooperation.

4. Matching with Incomplete Information

In this section we study the case in which players cannot observe the preference type of the opponent after having been matched to play the game.

A characterization of the Nash equilibria of the one-shot Prisoner’s Dilemma, for various configuration of the parameters, is reported in the following table.

Table 4. Nash equilibria (incomplete information)

<i>Pop. of cooperative agents</i>	<i>Below cutoff</i>	<i>Above cutoff</i>
Parameters:		
$d > \max\{V, R\}$	(c, nc)	(c, nc)
$V < d < R$	(nc, nc)	(c, nc) and (nc, nc)
$R < d < V$	(c, nc)	(mix, nc)
$d < \min\{V, R\}$	(nc, nc)	(nc, nc)

Playing *nc* is a dominant strategy for an *NC* player, for any distribution of preference types in the population. If the psychological gains from cooperation are small enough, $d < \min\{V, R\}$, *nc* is also a dominant strategy for *C* players, for any distribution of types. As in the complete information case, then, for $d < \min\{V, R\}$, *NC* and *C* players are indistinguishable in terms of play; and again we restrict the analysis to the interesting case in which $d > \min\{V, R\}$. If $d > \max\{V, R\}$, *C* agents always play *c*, as in the case of complete information. In this region of the parameters also, then, the dynamics of q_t are as in the case of complete information.

The most interesting cases are in the region of the parameters in which $\min\{R, V\} < d < \max\{V, R\}$. In this case in fact, with complete information, *C* agents choose actions contingent on the type of the match, while this is not possible with incomplete information. With incomplete information *C* agents of type *c* can only condition their action in equilibrium on the fraction of agents of type *C* in the population, q_t .²¹

The interesting question we address in this environment is whether incomplete information helps or hinders the evolution of cooperation in the population. Consider the two cases in turn: $V < d < R$ first, and then $R < d < V$.

Proposition 2. If $V < d < R$, there exists a cutoff population distribution $\bar{q} = (R - d)/(R - V)$ such that:

- any initial distribution $q_0 < \bar{q}$ is a stable stationary state;
- \bar{q} is a dynamically stable stationary state, with basin of attraction $[\bar{q}, 1]$.

With complete information, in this case, agents with preferences for cooperation adopt the conditional strategy of playing c only when facing a C agent in equilibrium; as a consequence, in the limit all of the population is composed of agents with a preference of cooperation (e.g. $q_t \rightarrow 1$). With incomplete information, instead, agents of type C can only condition their action in equilibrium on the fraction of agents of type C in the population, q_t . In equilibrium they in fact play c only when the fraction of agents with preferences in the population is large enough (above the cutoff). As a consequence, when C agents are a majority in the population, and they play c in all matches, agents with non-cooperative preferences have some incentive to socialize their children to their own preferences. This prevents them from bearing the cost R in any match with NC agents. This aspect contrasts with the complete information case in which non-cooperative parents have no incentive to socialize their children when C parents are a majority. The dynamics of the population reflect this reduced advantage of C agents with incomplete information when cooperation is associated with big costs and small gains, that is when $V < d < R$. We conclude that in this case incomplete information unambiguously hinders evolution of cooperation.

One may also note that under that configuration of parameters, the endogenous cultural transmission process and the 'indirect evolutionary' approach predict similar outcomes. Indeed, when $V < d < R$, as long as $q < \bar{q}$, the two preferences C and NC are observationally identical and induce therefore the same material payoffs. Hence any initial distribution $q_0 < \bar{q}$ is a stable stationary state; when $q > \bar{q}$, it is easy to see that the expected material payoff of the C preference's player is $qT - (1 - q)R$ less than $q(T + V)$ the expected payoff of the NC player. Therefore, according to the 'indirect evolutionary' approach, the fraction of C preferences should decrease and, as in proposition 2, \bar{q} is a dynamically stable stationary state, with a basin of attraction $[\bar{q}, 1]$.

Consider now instead the case in which $R < d < V$.

Proposition 3. *If $R < d < V$, there exists a unique stable stationary state of the population dynamics, $q^C = (d - R)/(d - R + V)$ whose basin of attraction is $(0, 1)$.*

One can again analyze the role of information for that configuration of parameters. With complete information, in this case, there also exists a unique stable stationary state of the population dynamics whose basin of attraction is $(0, 1)$. The composition of the population at the stationary state is different in the complete and incomplete information cases: with complete information $q^* = (d - R)/\frac{1}{2}(T + d - R + V)$, while with incomplete information $q^C = (d - R)/(d - R + V)$.

In this region of the parameters it is therefore possible that incomplete information favors rather than hinders cooperation. When $T > d - R + V$, incomplete information is associated with a higher fraction of agents with preferences for cooperation at the stationary state. The intuition of this result is rather subtle. The disadvantage of agents with a preference for cooperation when information is complete follows from the fact that in this case *NC* agents have strong socialization incentives even if they represent a relatively large share of the population. This is because in this case *C* agents actually play the conditional strategy of cooperating in matches with *NC* agents, and randomize in matches with other cooperative agents. When instead information is incomplete, such conditional strategies are not possible, and *C* agents randomize when they represent a large enough fraction of the population. This strategy is not as costly from the point of view of *NC* agents relying on objective payoffs only.

Note, finally, that when $R < d < V$, as in the case of complete information, some differences will arise between the two adaptive processes: endogenous cultural transmission and ‘indirect evolutionary’ approach. Under the latter, when $q < \bar{q}$ the expected material payoff of the *C* preference’s player is $qT - (1 - q)R$ less than $q(T + V)$ the expected payoff of the *NC* player. Therefore, the long-run monomorphic population $q = 0$ obtains with basin of attraction $[0, \bar{q}]$. On the other hand, for $q > \bar{q}$ the two preferences *C* and *NC* are now observationally identical and induce the same material payoffs. Hence any initial distribution $q_0 \in [\bar{q}, 1]$ is a stable stationary state. This is in contrast to the result of proposition 3,

which states that with endogenous cultural transmission a globally stable polymorphic population prevails in the long-run at $q^C = (d - R)/(d - R + V)$.

5. Conclusions

In this paper, we propose and study an endogenous cultural selection mechanism for preference traits. We focus on cooperative behavior, and we ask how agents with a taste for cooperation can survive in the long-run when they interact with agents who do not cooperate. In our model, agents are randomly matched in a one-shot interaction to play the standard Prisoner's Dilemma, and may or may not have complete information about the opponent's preferences (to mimick various possible social settings).

Our main contributions are threefold. First, we provide explicit microfoundations for a cultural selection mechanism to study the population dynamics of traits over time. In particular, we focus on an endogenous socialization mechanism in which parents spend costly effort to transmit their trait to their offspring. In our mechanism, direct (parental) and oblique (societal) cultural transmission interact endogenously as parents take into account societal pressures to choose their desired socialization effort.

Second, we contrast the implications of our transmission and selection mechanism for the long-run population dynamics of traits with those induced by other mechanisms, such as replicator dynamics or the *indirect evolutionary* approach, that are both based on fitness criteria. For various ranges of the parameter values, our mechanism generates a polymorphic population with a long-run presence of cooperative agents, even where replicator and indirect evolutionary mechanisms would bring about a monomorphic population in which only non-cooperation survives. This is mostly due to the endogenous nature of our mechanism that produces a *minority favoring* effect.

Third, we further analyze the implications of our mechanism for long-run dynamics under different assumptions on the completeness of information about the type of the opponent with whom one is being matched. Interestingly, we find that while, in general, incomplete information hinders the survival of cooperation in the long-run, a region of the parameters exists in which the long-run fraction of cooperative agents is larger under incomplete than complete

information. This is due to the interplay between the kind of strategies available to agents under the two informational regimes, and the endogenous socialization choices of NC agents while in a majority.

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NOTES

1. A recent survey of this literature, centered around the original contribution by Axelrod (1984), is contained in Axelrod and D'Ambrosio (1996). Behavioral evidence on the related phenomenon of reciprocity in various strategic environments is surveyed in Gintis (2003a).
2. See Hechter and Opp (2001) for recent surveys within a multi-disciplinary perspective.
3. These results have been extended in many directions; see Fudenberg and Tirole (1991).
4. This body of literature extends from the social sciences to biology. Without any hope of being exhaustive, one can cite recent work in Economics: Bowles (2001), Bowles and Gintis (1998), Gintis (2000, 2003a), Guth and Yaari (1992), Guth and Kliemt (1994), Bester and Guth (1998), Guttman (2000, 2001a, b, 2003), Hirshleifer and Martinez Coll (1988), Hirshleifer (1999), Kockesen et al. (2000), Stark (1999); in Political Science: Axelrod (1984), Cohen et al. (2001); in Sociology: Bendor and Swistak (2001); in Biology: Dawkins (1976), Smith (1982), Trivers (1971); in Evolutionary Anthropology: Boyd and Richerson (1985, 1988, 1992), Rogers (1988).
5. See also Bowles and Gintis (2002), Gintis (2003b).
6. In a similar spirit, Becker and Madrigal (1995) study cooperation that is induced by habitual behavior: agents gain psychological payoffs by playing cooperatively repeatedly.
7. The relative importance of the cultural and genetic component in the cognitive and behavioral characteristics of children is a hotly debated issue, the *nature/nurture* issue. While recent popular books by Herrnstein and Murray (1994), Harris (1998), and Pinker (2002) have notably favored the nature position, the scientific debate is more balanced; see, e.g., Ceci and Williams (1999), Otto et al. (1994).
8. More recently, the idea that socialization can help reduce conflicts has been investigated by Guttman et al. (1992), Raub (1990), and Guttman (2001a, b).
9. We do not study the children's own decisions regarding preference adoption and/or internalization. The analysis of oblique transmission, though, is designed to indirectly capture in part these aspects of preference formation; see Gintis (2003a) and Becker and Mulligan (1997) for a more explicit analysis of internalization.

10. Becker (1993) discusses a related model of socialization of preferences for altruism and of guilt, in which also the socialization choices of parents are purposeful, but under the assumption that parents are endowed with perfect empathy towards their children (and varying degrees of altruism).
11. We use the wordings *NC* player, *NC* agent, agent with *NC* preferences, interchangeably; the same for *C* player, etc.
12. Our analysis is unchanged if we model *C* preferences with a psychological cost to play non-cooperatively.
13. Parents have no technology to socialize their children to preferences different to their own, even if they might want to do so. The assumption can be relaxed at some cost without changing qualitative results.
14. To simplify notations, we omit time subscripts whenever there is no risk of confusion.
15. Some evidence for this form of 'paternalistic' altruism (or 'imperfect empathy') is discussed in Bisin and Verdier (2000), while a justification in terms of natural selection in sufficiently rapidly changing environments is provided by Bisin and Verdier (2001).
16. We implicitly normalize the parameters of the model, i.e. the payoffs R, V, d , so that $\tau^i \leq 1$, for any q_t, q_{t+1}^e . This is necessary, since τ^i is a probability.
17. We will look for simplicity at the continuous time limit of the dynamics of q_t by assuming that socialization is instantaneous. In other words, we will consider a discrete time model with periods of length h letting then $h \rightarrow 0$. The dynamics reduces to the following form (see the proofs of the Propositions):

$$\dot{q}_t = q_t(1 - q_t)(\tau^C(q_t, q_{t+h}) - \tau^{NC}(1 - q_t, q_{t+h})) \quad (7)$$

and we can derive explicit closed form solutions in terms of parameters R, d, V .

18. This is also the most interesting selection to consider, since for the other Nash equilibrium (*nc, nc*), this parameter's region would be otherwise indistinguishable from the region in which $d/\min\{V, R\}$.
19. Evolutionary biologists have noted such selective advantages of these class of strategies; see Trivers (1971).
20. When $d > \max\{V, R\}$, the expected material payoff of a *C* player is $qT - (1 - q)R$, which is less than $q(T + V)$, the expected material payoff of a *NC* player. Hence any monotonic material payoff selection mechanism will imply a monomorphic population at $q = 0$. A similar conclusion holds for the case $R < d < V$, where the expected material payoff of a *C* player is $q\frac{1}{2}(T + V - R) - (1 - q)R$, which is again less than $q(T + V)$.
21. As in the previous analysis of complete information, multiple equilibria arise and equilibrium selection is necessary; we adopt the same selection mechanism with incomplete information.
22. If we allowed agents of type *C* to socialize their children to trait *NC*, they would for q_{t+h}^C close to 1. Qualitative results are unchanged though.

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Appendix

Proposition 1: Proof. We compute the parent's expected evaluation of the future payoff of his child and analyze the dynamics of q_t^C for any parameter configuration of the game.

Case 1: $d > \max\{V, R\}$. In this case, the subjective flow gains per unit of time can be computed as:

$$V^{C,C}(q_{t+h}) = q_{t+h}(T + d) + (1 - q_{t+h})(-R + d)$$

$$V^{C,NC}(q_{t+h}) = q_{t+h}(T + V)$$

$$V^{NC,NC}(q_{t+h}) = q_{t+h}(T + V)$$

$$V^{NC,C}(q_{t+h}) = q_{t+h}T + (1 - q_{t+h})(-R)$$

Let $\Delta V^C(q_{t+h}) = V^{C,C}(q_{t+h}) - V^{C,NC}(q_{t+h})$, and $\Delta V^{NC}(q_{t+h}) = V^{NC,NC}(q_{t+h}) - V^{NC,C}(q_{t+h})$. Then, we have

$$\Delta V^C(q_{t+h}) = q_{t+h}(d - V) + (1 - q_{t+h})(d - R) > 0 \tag{9}$$

$$\Delta V^{NC}(q_{t+h}) = q_{t+h}V + (1 - q_{t+h})R > 0 \tag{10}$$

From this and 6, it follows that τ^C and τ^{NC} are always strictly positive for $(q_t; q_{t+h}) \in (0, 1) \times [0, 1]$. Substituting 6 into 5 the difference equation becomes

$$q_{t+h} - q_t = hq_t(1 - q_t) \frac{[d - R - q_t(d + V - R)]}{1 - hq_t(1 - q_t)[R - V]}$$

Taking the limit $h \rightarrow 0$, one gets the differential equation for q_t as

$$\dot{q}_t = q_t(1 - q_t)[d - R - q_t(d + V - R)]$$

As a consequence, the homomorphic steady states $q = 0$ and $q = 1$ are dynamically unstable and the unique polymorphic steady state $q^* = (d - R)/(d - R + V)$ is dynamically stable (its basin of attraction is $(0, 1)$).

Case 2: $V < d < R$. As type C agents play C when faced with a type C agent (and NC when faced with a NC agent). The ‘subjective’ flow gains per unit of time are:

$$V^{C,C}(q_{t+h}) = q_{t+h}(T + d)$$

$$V^{C,NC}(q_{t+h}) = 0$$

$$V^{NC,NC}(q_{t+h}) = 0$$

$$V^{NC,C}(q_{t+h}) = q_{t+h}T$$

and

$$\Delta V^C(q_{t+h}) = q_{t+h}(T + d) > 0 \quad (11)$$

$$\Delta V^{NC}(q_{t+h}) = -q_{t+h}T < 0 \quad (12)$$

From this and 6 it follows that τ^C is positive and that $\tau^{NC} = 0$ for $(q_t; q_{t+h}) \in (0, 1) \times [0, 1]$. Substituting 6 into 5 and reorganizing, one gets

$$q_{t+h} - q_t = h \frac{(q_t)^2(1 - q_t)^2(T + d)}{1 - hq_t(1 - q_t)^2[T + d]}$$

and taking the continuous time limit gives:

$$\dot{q}_t = (q_t)^2(1 - q_t)^2(T + d) > 0$$

As a consequence, the homomorphic steady state $q = 1$ is dynamically stable with a basin of attraction $(0, 1)$.

Case 3: $R < d < V$. We compute the ‘subjective’ flow gains per unit of time as:

$$\begin{aligned}
 V^{C,C}(q_{t+h}) &= q_{t+h} \cdot \frac{1}{2}(T + V + d - R) + (1 - q_{t+h})(d - R) \\
 V^{C,NC}(q_{t+h}) &= q_{t+h}(T + V) \\
 V^{NC,NC}(q_{t+h}) &= q_{t+h}(T + V) \\
 V^{NC,C}(q_{t+h}) &= q_{t+h} \cdot \frac{1}{2}(T + V - R) - (1 - q_{t+h})R
 \end{aligned}$$

and

$$\Delta V^C(q_{t+h}) = -\frac{1}{2}q_{t+h}(T + V + d - R) + (d - R) \tag{13}$$

$$\Delta V^{NC}(q_{t+h}) = q_{t+h} \frac{1}{2}(T + V + R) + (1 - q_{t+h})R > 0 \tag{14}$$

Note that for q_{t+h} close to 1, $\Delta V^C(q_{t+h}) < 0$. Agents of type C do not want to socialize children as C players.²² Formally then

$$\tau^C = (1 - q_t^C) \max\{0, \Delta V^C(q_{t+h})\}$$

and $\tau^C = 0$ for $q_{t+h} > (d - R)/\frac{1}{2}(T + V + d - R)$.

Let us note for convenience,

$$g_0(q, h) = \frac{q + hq(1 - q)[d - R - dq]}{1 + \frac{1}{2}hq(1 - q)[T + V + d - R - dq]} \tag{15}$$

$$g_1(q, h) = \frac{q - hq^2(1 - q)R}{1 + \frac{1}{2}h[q^2(1 - q)](T + V - R)} \tag{16}$$

$g_0(q)$ (resp. $g_1(q)$) characterizes the evolution of the time path of q_t^C when $\tau^C > 0$ (resp. $\tau^C = 0$). More precisely, denoting $q_\gamma = (d - R)/\frac{1}{2}(T + V + d - R)$:

$$q_{t+h} = g_0(q_t, h) \text{ when } q_{t+h} \leq q_\gamma \tag{17}$$

$$q_{t+h} = g_1(q_t, h) \text{ when } q_{t+h} > q_\gamma \tag{18}$$

Lemma 1. There exists a $\bar{q}^C(h) \in [q_\gamma, 1)$ such that: (i) the dynamics of the system is described by: $q_{t+h}^C = g_0(q_t^C, h)$ when $q_t^C \leq \bar{q}^C(h)$ and $q_{t+h}^C = g_1(q_t^C, h)$ when $q_t^C > \bar{q}^C(h)$ with $\lim_{h \rightarrow 0} \bar{q}^C(h) = q_\gamma$.

Proof. (i) Consider first that we have $q_{t+h} > q_\gamma = (d - R)/\frac{1}{2}(T + V + d - R)$, then the equilibrium path is given by $q_{t+h} = g_1(q_t, h)$ and the domain of validity of this dynamic equation should satisfy $g_1(q_t, h) > q_\gamma$. Substitution of the expression of $g_1(q_t, h)$ provides the following inequality:

$$\Psi(q, h) = -hd(T + V) \frac{q^2(1 - q)}{2} + \frac{1}{2}(T + V + d - R)q - (d - R) \geq 0$$

One can see that $\Psi'(q, h) = -dh(T + V)[q(1 - q) - (q^2/2)] + \frac{1}{2}(T + V + d - R)$ and $\Psi''(q, h) = -dh(T + V)[1 - 3q]$. Hence $\Psi'(q, h)$ has a minimum at $q = \frac{1}{3}$ and $\Psi'(\frac{1}{3}, h) = \frac{1}{2}[(d - R) + (T + V)(\frac{1}{3} - dh)] > 0$ when $dh < \frac{1}{3}$. Hence for h small enough $\Psi'(q, h)$ is positive for all $q \in [0, 1]$ and $\Psi(q, h)$ is increasing with $\Psi(0, h) = -(d - R) < 0$ and $\Psi(1, h) = \frac{1}{2}(T + V + d - R) > 0$ (as $V > d$). Hence there exists a unique $\bar{q}(h)$ such that $\Psi(\bar{q}(h), h) = 0$ and that $\Psi(q, h) \geq 0$ if and only if $q \geq \bar{q}(h)$. Moreover, as $\Psi(q_\gamma) = -[dh(T + V)[q^2(1 - q)/2]]_{q=q_\gamma} \leq 0$ it follows that $\bar{q}(h) \geq q_\gamma = (d - R)/\frac{1}{2}(T + V + d - R)$ and that $\lim_{h \rightarrow 0} \bar{q}(h) = q_\gamma$.

Consider now that $q_{t+h} < q_\gamma$, then the equilibrium path is given by $q_{t+h} = g_0(q_t, h)$ and the domain of validity of this dynamic equation should satisfy $g_0(q_t, h) > q_\gamma$. Substitution of the expression of $g_0(q_t, h)$ provides the inequality:

$$\Psi(q, h) = -dhK(T + V) \frac{q^2(1 - q)}{2} + \frac{1}{2}(T + V + d - R)q - (d - R) < 0$$

Hence $q < \bar{q}(h)$. Thus the characterization of the dynamics $q_{t+h} = g_0(q_t, h)$ when $q_t \leq \bar{q}(h)$ and $q_{t+h} = g_1(q_t, h)$ when $q_t > \bar{q}(h)$.

When $q_t \leq \bar{q}(h)$, this can be rewritten as:

$$q_{t+h} - q_t = \frac{hq_t(1 - q_t)[d - R - q_t d - \frac{1}{2}q_t(T + V + d - R) + \frac{1}{2}(q_t)^2]}{1 + \frac{1}{2}hq_t(1 - q_t)[T + V - R + d - dq_t]}$$

which gives at the limit $h \rightarrow 0$:

$$\dot{q}_t = q_t(1 - q_t) \left[d \frac{(q_t)^2}{2} - dq_t - q_t \frac{T + V + d - R}{2} + (d - R) \right]$$

$$\text{when } q_t \leq \lim_{h \rightarrow 0} \bar{q}(h) = q_\gamma$$

In order to characterize the dynamics in this case we need to study the sign of the second order polynomial $P(q, q)$ given by:

$$P(q, q) = \left[d - R - dq - \frac{1}{2}q(T + V + d - R) + \frac{1}{2}d(q)^2 \right]$$

With some algebra we re-write

$$P(q, q) = \left[q \left[\frac{1}{2}qd - \frac{1}{2}(T + V + d - R) \right] + (d - R - qd) \right]$$

Note that $P(q, q) > 0$ (resp. < 0) iff

$$qd \left(\frac{1}{2}q - 1 \right) > \frac{1}{2}q(T + V + d - R) - (d - R) \quad (\text{resp. } < 0) \quad (19)$$

Consider the function $\Theta(q) = qd(\frac{1}{2}q - 1) - \frac{1}{2}q(T + V + d - R) + (d - R)$. This function is decreasing in q for $q \in [0, 1]$ and $\Theta(0) = d - R > 0$ and $\Theta(1) = -(T + V - R)/2 - R < 0$. Hence, there is unique polymorphic solution $q^* \in (0, 1)$ such that $\Theta(q^*) = 0$. Moreover, $\Theta(d - R)/\frac{1}{2}(T + V + d - R) = -[2d(d - R)(T + V)/(T + V + d - R)^2] < 0$. Hence $q^* < q_\gamma$. It is also easy to see that $\dot{q}_t > 0$ (resp. < 0) when $q_t < q^*$ (resp. $q^* < q_t < q_\gamma$).

One can finally check that, at the limit $h \rightarrow 0$, when $q_t > \bar{q}(h)$,

$$\dot{q}_t = -q_t(1 - q_t) \left[q_t R + \frac{1}{2}(q_t)^2(T + V - R) \right] < 0$$

$$\text{for } q_t > \lim_{h \rightarrow 0} \bar{q}(h) = q_\gamma$$

Therefore the qualitative dynamics has unstable corner solutions $q = 0$ and $q = 1$ and that $q^* < q_\gamma$ is a stable interior stationary state with a basin of attraction $(0, 1)$.

Taking the limit $h \rightarrow 0$, the dynamic system becomes

$$\dot{q}_t = q_t(1 - q_t) \left[d \frac{(q_t)^2}{2} - dq_t - q_t \frac{T + V + d - R}{2} + (d - R) \right]$$

when $q_t \leq q_\gamma$ (20)

$$= -q_t(1 - q_t) \left[q_t R + \frac{1}{2}(q_t)^2(T + V - R) \right] < 0 \text{ when } q_t > q_\gamma \quad (21)$$

There is then a unique dynamically stable polymorphic steady state $q^* \in (0, q_\gamma)$ whose basin of attraction is $(0, 1)$ and the homomorphic states $q = 0$ and $q = 1$ are dynamically unstable.

Case 4: $d < \min\{V, R\}$ In this case trivially

$$\Delta V^C(q_{t+h}) = \tau^C(q_t, q_{t+h}) = \Delta V^{NC}(q_{t+h}) = \tau^{NC}(1 - q_t, q_{t+h}) = 0$$

As a consequence, any initial condition for q_t is maintained over time (as a stable stationary point). ■

Proposition 2: Proof. Agents of type C play c iff

$$q_{t+h}(T + d) + (1 - q_{t+h})(d - R) > q_{t+h}(T + V)$$

i.e. iff

$$q_{t+h} > \frac{R - d}{R - V} < 1 \quad (22)$$

If (22) is satisfied, then

$$\tau^C(.) = (1 - q_t)(d - R + q_{t+h}(R - V))$$

which is positive for $q_{t+h} > (R - d)/(R - V)$, is = 0 for $q_{t+h} = (R - d)/(R - V)$ and $q_t = 1$.

Also, for $q_{t+h} > (R - d)/(R - V)$,

$$\tau^{NC}(.) = q_t(R - q_{t+h}(R - V)),$$

and hence is strictly positive and decreasing in q_{t+h} , for $q_{t+h} > (R - d)/(R - V)$. For $q_{t+h} \leq (R - d)/(R - V)$, instead, $\tau^{NC}(.) = 0$. From this, the dynamics of q_t can be described as:

$$q_{t+h} = \frac{q_t + hq_t(1 - q_t)[-R + (1 - q_t)d]}{1 - hq_t(1 - q_t)(R - V)} \text{ for } q_{t+h} > \frac{R - d}{R - V} \quad (23)$$

$$= q_t \quad \text{otherwise} \quad (24)$$

which gives

$$q_{t+h} - q_t = \frac{hq_t(1 - q_t)[-(R - d) + q_t(R - V - d)]}{1 - hq_t(1 - q_t)(R - V)} \text{ for } q_{t+h} > \frac{R - d}{R - V} \quad (25)$$

$$= 0 \quad \text{otherwise} \quad (26)$$

As $-(R - d) + q_t(R - V - d) \leq 0$ for all $q_t \in [0, 1]$ and taking the limit $h \rightarrow 0$, the characterization of the dynamics then follows immediately. ■

Proposition 3: Proof. Agents of type C play c iff

$$q_{t+h}(T + d) + (1 - q_{t+h})(d - R) > q_{t+h}(T + V)$$

i.e. iff

$$q_{t+h} < \frac{d - R}{V - R} < 1 \quad (27)$$

If (27) is satisfied, then

$$\tau^C(\cdot) = (1 - q_t)(d - R - q_{t+h}(V - R))$$

which is decreasing in q_t and q_{t+h} and is $= 0$ at the cutoff $q_{t+h} = \bar{q} = (d - R)/(V - R)$. If (27) is not satisfied, then $\tau^C(\cdot) = 0$.

Also, for any $(q_t, q_{t+h}) \in (0, 1) \times [0, 1]$, $\tau^{NC} > 0$. Also,

$$\tau^{NC}(\cdot) = q_t(q_{t+h}(V - R) + R)$$

if (27) is satisfied, and hence is increasing in q_t and q_{t+h} , is > 0 for $q_t > 0$, and is $= 0$ for $q_t = 0$. From this, we get the following dynamics

$$q_{t+h} - q_t = \frac{hq_t(1 - q_t)[d - R - q_t(d - R + V)]}{1 - hq_t(1 - q_t)(R - V)} \text{ for } q_{t+h} < \frac{d - R}{V - R} \quad (28)$$

$$< 0 \quad \text{otherwise} \quad (29)$$

Taking the limit $h \rightarrow 0$, the characterization of the dynamics is then

$$\dot{q}_t = q_t(1 - q_t)[d - R - q_t(d - R + V)] \text{ for } q_t < \frac{d - R}{V - R} \quad (30)$$

$$< 0 \quad \text{otherwise} \quad (31)$$

From this, it is straightforward to get the characterization of the dynamics as given in the proposition. ■

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