Efficient Policy Interventions in an Epidemic

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Abstract

In the context of an epidemic, a society is forced to face a complex system of externalities in consumption and in production. Command economy interventions can support Efficient allocations at the cost of severe information requirements. Competitive markets for *infection rights* (alternatively, Pigouvian taxes) can guarantee instead efficiency without requiring direct policy interventions on the activity of agents and firms. We demonstrate that this is the case also when the infections cannot be associated to the activities which originated them; and moral hazard then ensues. Finally, we extend the analysis to situations where governments have only incomplete information regarding the values of the parameters of the infection process or of firms' production processes.

1 Introduction

A society hit by an epidemic is forced to face a complex system of externalities in consumption and in production. The epidemic diffuses by social contacts between agents, which are an essential by-product of production and consumption-leisure activities. Rational agents and profit maximizing firms, in this society, will take into account the individual costs of the infections generated by their choices but will not internalize the externalities of their actions: hence firms will over-produce, agents will over-consume, and in turn infections will be more widely spread, with respect to the efficient level.

Policy interventions will generally be necessary to design efficient mechanisms to limit infections while allowing for economic activity. In the course of the SARS-Cov-2 epidemic the most frequently adopted (non-pharmaceutical) policy interventions consisted of some form of partial lockdown, that is, command economy interventions directly restricting firms and agents' behavior, selecting which firms produce how much and which agents are allowed to engage in consumption-leisure activities and how much so. In several instances, activities have been ranked in terms of their infectiousness and their opening staggered as the epidemic slowed-down. But command economy interventions are not the only possible mechanism to implement efficient allocations in general, nor are the mechanism with minimal information requirement to be implemented. As for other types of externalities, e.g., pollution, markets for the rights to externality-producing activities, or alternatively, Pigouvian taxes, can be set-up which induce agents and firms to consume and produce efficiently.

In this paper we study the design of these alternative mechanisms markets, say markets for *infection rights* or Pigouvian taxes, in the context of a simple model of a society hit by an epidemic. We determine the conditions under which these mechanisms decentralize efficient allocations in that environment. We characterize then the properties of these mechanisms and show that they allow to identify the subtle net of externalities associated to the effects of firms' and individuals' decisions on infections.

In particular, we characterize the equilibrium prices of infection rights or, equivalently, of optimal Pigouvian taxes on economic activities. We study in detail the information requirements for their implementation showing that they guarantee efficiency without requiring direct policy interventions on the activity of individual agents and firms. We demonstrate that this is the case also when the infections cannot be associated to the activities which originated them, that is, to the production choices of specific firms or to the consumption/leisure activities of specific consumers, and moral hazard then ensues. In contrast with command economy interventions, in the environment we consider it is the design of markets for infection rights or the Pigouvian taxes imposed on economic activities to induce individuals and firms to limit the kinds of activities that are more likely to produce infections. Such activities end up requiring the larger purchase of rights in equilibrium or equivalently, being taxed more heavily.

Finally, we extend the analysis to situations where governments have only incomplete information regarding the values of the parameters of the infection process or of firms' production processes. In this case efficiency cannot be attained, and we identify conditions under which setting the quantity of infection rights to be traded, rather than setting their prices or the tax rate, is superior in terms of social welfare, also to command economy interventions.

While the analysis in the paper is abstract and stylized along many dimensions, its message should be clear: the bag-of-tricks of economic policy institutions does not contain only command economy interventions with severe information requirements - this is true in the context of an epidemic as well, perhaps even especially so.

A few very recent papers have introduced rational, optimizing agents in the framework of epidemiological models, highlighting the importance of individual behavior in response to policy interventions and the trade-offs between health and economic costs; see e.g., Acemoglu et al. (2020), Alvarez et al. (2020), Argente et al. (2020), Atkeson (2020), Bisin and Moro (2020), Eichenbaum et al. (2020), Kaplan et al. (2020), Toxvaerd (2020)). In particular, closer to our focus in this paper, Bethune and Korinek (2020) study the externalities that arise in such framework, by providing a quantitative assessment of the individual and social cost of infections. Farboodi et al. (2020) focus on an environment where, like in our set-up, agents partially internalize the individual but not the social costs of these activities. Our model in contrast is static and clearly more simplified in several aspects. On the other hand, we distinguish infections which take place at work and via social interaction activities and the actions that can be taken to limit each of these channels of infection. Also, a distinguishing feature from all these other approaches to policy interventions is our main emphasis on measures that do not rely on the direct control of some individual choices but rather on the design of additional markets (or taxes) which can induce agents to internalize the social costs of their behavior, in line with the approaches developed for other kinds of externalities, pioneered by Lindahl (1919), Pigou (1920), Coase (1960), Arrow (1969), Baumol (1972).[1]

2 Economy

In this section we first describe a simple abstract society hit by an epidemic. We consider a static environment to highlight in a stark manner the role of various sources of externality in the epidemic. We extend the analysis in Section 4, notably to allow

[1]See, for instance, Dales (1968) for pollution, Bisin and Gottardi (2006) for consumption externalities due to adverse selection.

for heterogeneity of agents and firms and for multiple sectors in Section 4.1 and to allow for productions chains in Section 4.1.1.

Agents. The society is populated by L ex-ante identical (representative) agents and a single (representative) firm. Each agent receives utility from consumption of a private good c and of a consumption/leisure good l which requires social interactions to be enjoyed. Each agent can be infected while interacting socially, with probability I_l, and/or at work, with probability I_h. Let $I = I_l + I_h$.[2] The probability of infection I_l increases with the level of the agent's consumption-leisure activity l; it also increases with the average value of l in the population, $\bar{l}$: $I_l = \delta(l, \bar{l})$, with $\partial\delta/\partial l > 0$, $\partial\delta/\partial\bar{l} > 0$.[3] Agents supply labor inelastically, taking as given the probability of infection at work, I_h, which is determined by the firm's choices. The representative consumer's utility function is

$$u(c, l) - \beta I,$$

with $u(.)$ increasing and concave and β constituting the agent's disutility of becoming infected (for a given level of treatment, as specified below by η).

Firms. Each firm produces the private consumption good with the production function

$$Y = AL_h(1 - I)$$

where Y is output, L_h is the quantity of labor employed in the firm and $L_h I$ is the number[4] of workers in the firm who become infected and are then assumed to be unproductive.

The probability of a worker of being infected at work is given by $I_h = \gamma(1 - a)$, where $\gamma > 0$ and a denotes social distancing and other abatement measures the firm can employ, at costs $C(a, L_h)$ (increasing and convex in a, L_h)[5], to reduce infection at work of all workers employed. Firms' profits are $\pi = Y - wL_h - C(a, L_h)$, where w is the market wage taken as given by the firm.[6]

[2]We assume that whether an agent is infected and whether he/she was infected at work or in the course of his consumption/leisure activities is publicly observed. We shall relax this assumption in Section 4.3, when we discuss how to extend our analysis to allow for various informational asymmetries regarding infections.

[3]We also assume that $\partial^2\delta/\partial l\partial\bar{l} \geq 0$, introducing the possibility of a strategic complementarity in agents' consumption-leisure choices.

[4]Strictly speaking, the expected number.

[5]Except when the analysis is extended to the case of firms with heterogeneous technologies, in Section 4.1, abatement costs can also be considered linear in L_h, so the technology exhibits constant returns to scale.

[6]The consumption good is assumed to be the numeraire.

Public sector. Infections need be treated by the health care sector, which we assume is public and run by the government. Public expenditures in health care are[7]

$$g = \eta(I_h + I_l)L,$$

and are financed by lump-sum taxes T levied on consumers.

Competitive Equilibrium *At a Competitive Equilibrium, i) each agent maximizes his/her utility, by choice of c, l, for given $\pi, I_h, w, T, \bar{l}$:*

$$\max_{c,l} \; u(c,l) - \beta\left(I_l + I_h\right), \quad s.t. \tag{1}$$

$$c = \frac{\pi}{L} + w - T, \tag{2}$$

$$I_l = \delta(l, \bar{l}); \tag{3}$$

ii) each firm maximize profits by choice of a, L_h, for given w, I_l:

$$\max_{Y,a,L_h} \; \pi = Y - wL_h - C(a, L_h), \quad s.t. \tag{4}$$

$$Y = AL_h(1 - I_h - I_l), \tag{5}$$

$$I_h = \gamma(1 - a); \tag{6}$$

iii) the government balances the budget;

$$\eta(I_h + I_l)L = T; \tag{7}$$

iv) markets clear,

$$c = \frac{1}{L_h}\left[AL_h(1 - I_h - I_l) - C(a, L_h) - \eta(I_h + I_l)\right] \tag{8}$$

$$L_h = L \tag{9}$$

v) the externality in social interaction satisfies the consistency condition,

$$l = \bar{l}. \tag{10}$$

Both consumers and firms face a direct disutility from the infections that are generated by their consumption/leisure and their production and abatement decisions. A higher level of consumption/leisure l in fact increases social interactions and

[7] The analysis could be easily extended to the case where the level of treatment, captured by η, here exogenous, is also a choice variable (which in turn affects β).

probability of infection I_l; and this in turn generates a higher disutility for the individual βI_l. Similarly, a firm by lowering its abatement measures saves on costs but also suffers losses because the increase in I_h induced by the reduction in a reduces the fraction of its workers who are productive. But consumption/leisure and abatement decisions also give rise to externalities faced by the society in an epidemic: i) each consumer does not take into account the fact that his/her consumption/leisure activity also increases the probability that other individuals are infected, via the effect of his choice on $\bar{l}$ and hence on $\delta(l, \bar{l})$; ii) he/she also does not consider the fact that this activity negatively affects the firms' productivity, by reducing the fraction of productive workers; iii) furthermore, the consumer ignores that infections entail another cost for the society, given by the health costs η incurred by the government to cure infected agents. Analogously, on the production sector: iv) each firm does not account for the fact that agents infected at work face a utility costs β and v) also produce a societal health cost η.

Due to these externalities, competitive equilibria are not efficient. *Efficient allocations* are those which maximize *social welfare*. In our simple economy social welfare coincides with the representative agent utility and hence the economy admits a unique Efficient allocation.

Efficient allocation. *At the Efficient allocation, c, l, a maximize* social welfare*:*

$$\max_{c,l,a} \quad u(c,l) - \beta(I_l + I_h) \tag{11}$$

$$s.t. \tag{12}$$

$$c = \frac{1}{L}\left[AL(1 - I_h - I_l) - C(a, L) - \eta(I_h + I_l)L\right] \tag{13}$$

$$where \tag{14}$$

$$I_l = \delta(l, l) \tag{15}$$

$$I_h = \gamma(1 - a) \tag{16}$$

For the reasons explained, the Efficient allocation induces an efficient level of infections in the society $(I_l + I_h)\, L$, which is lower than the level of infections at the competitive equilibrium.

3 Market Implementation of the Efficient allocation

In this section we show how the Efficient allocation can be implemented via markets, designed to induce firms to produce - and consumers to choose consumption/leisure activities (with associated social interactions) - efficiently.

We shall discuss several different implementation mechanism, but it is convenient to set *markets for infection rights* as the benchmark.

3.1 Markets for infection rights

Consider more specifically the following institutional market design:

Each agent engaging in consumption/leisure activities is mandated to buy a right per unit of probability of infection I_l induced by his/her activities;

Each firm producing Y units and choosing abatement a is mandated to buy a right per unit of probability of infection I_h induced by its own choices.

Let q_l denote the price of these *infection rights* for consumption/leisure activities and q_h the price of *infection rights* for production.[8]

Competitive equilibrium with infection rights. *A Competitive equilibrium with infection rights is a competitive equilibrium as previously defined but:*

the agent's budget constraint (previously Equation 2) is

$$c + q_l I_l = \frac{\pi}{L} + w - T; \tag{17}$$

the firm's objective (previously Equation 4) is:

$$Y - wL_h - q_h I_h L_h - C(a, L_h); \tag{18}$$

the government chooses the supply of infection rights H_l, H_h and its budget constraint is

$$\eta(I_h + I_l)L = T + q_h H_h + q_l H_l; \tag{19}$$

markets for infection rights also clear,

$$I_h L = H_h, \;\; I_l L = H_l. \tag{20}$$

It is now straightforward to prove that, conditionally on the government supplying tradable infection rights H_l, H_h in an amount equal to the efficient level of infections, while letting prices clear these markets, the Efficient allocation obtains at a competitive equilibrium.

[8]Notice that this institutional design of markets for infection rights requires that both the individual probability of getting infected as well as whether an infection occurs at work or in a consumption/leisure activity are publicly observable in the economy. We will relax both features later.

Proposition (Efficiency of equilibria). *Suppose the government chooses a supply of infection rights H_l, H_h equal to the efficient infections $I_l L, I_h L$. Then the Efficient allocation obtains at the Competitive equilibrium with infection rights.*

The proof of the above decentralization result is by construction. That is, we find prices q_l, q_h, such that the levels of consumption and social interaction c, l chosen by individual consumers, and of production and abatement y, a chosen by firms, and the induced infection rates I_l, I_h, are efficient. The revenue that is generated by the sale of the infection rights is then used to fund the health expenditures incurred by the government, with the difference between the two set equal to a lump sum tax - or transfer if negative - T on consumers. The key step in the argument of the proof is then the characterization of the values of these prices and transfers, stated in the following result.

Proposition (Efficiency of equilibrium - prices). *At the Efficient Competitive equilibrium with infection rights, the price of the rights for production and consumption/leisure, q_h and q_l, respectively, are*

$$q_h = \frac{\beta}{\partial u/\partial c} + \eta. \tag{21}$$

$$q_l = (\eta + A) + (\eta + A)\Delta + \frac{\beta}{\partial u/\partial c}\Delta \tag{22}$$

where $\Delta = \left[\frac{\partial \delta(l,\bar{l})/\partial \bar{l}}{\partial \delta(l,\bar{l})/\partial l}\right]_{\bar{l}=l}$ is the multiplicative effect of each agent's choice l on other agents' infections (via the effect on the average value $\bar{l}$), evaluated at the equilibrium $\bar{l} = l$. Furthermore, the lump-sum tax T is negative:

$$T = -I_l L(q_l - \eta) - I_h L(q_h - \eta) \tag{23}$$

Proof. The first order conditions of the social welfare maximization problem are:

$$[\frac{\partial u}{\partial c}(A + \eta) + \beta]\gamma = \frac{\partial u}{\partial c} \cdot \frac{C_1(a, L)}{L} \tag{24}$$

$$(\frac{\partial u}{\partial c}(\eta + A) + \beta)\delta'(l, l) = \frac{\partial u}{\partial l} \tag{25}$$

where $C_1(a, L)$ denotes the derivative of C with respect to its first argument, a, and - with some abuse of notation - $\delta'(l, l) \equiv \frac{\partial \delta(l,l)}{\partial l} + \frac{\partial \delta(l,l)}{\partial \bar{l}}$, the total derivative of $\delta(l, \bar{l})$ w.r.t. l and $\bar{l}$, evaluated at $\bar{l} = l$. The first order condition for the firm's optimal abatement choice at the Competitive equilibrium with infection rights is instead

$$(A + q_h)\gamma L = C_1(a, L). \tag{26}$$

It is then immediate to verify that conditions (24) and (26) generate the same choice and allocation if q_h is set as in (21).

Consider then the first order condition with respect to l for the agent's maximization problem in the Competitive equilibrium with infection rights,

$$\frac{\partial u}{\partial l} = (\beta + \frac{\partial u}{\partial c} q_l) \frac{\partial \delta(l, \bar{l})}{\partial l}. \tag{27}$$

Conditions (25) and (27) support the same choice if $\left[\frac{\partial u}{\partial c}(\eta + A) + \beta\right] \delta'(l, l) = (\beta + \frac{\partial u}{\partial c} q_l) \frac{\partial \delta(l, \bar{l})}{\partial l}$; that is, if $q_l = (\eta + A) \frac{\delta'(l,l)}{\partial \delta(l,\bar{l})/\partial l} + \frac{\beta}{\partial u/\partial c} \left(\frac{\delta'(l,l)}{\partial \delta(l,\bar{l})/\partial l} - 1 \right)$; which can be rewritten as in (22).

Finally, substituting the expressions obtained for the prices of infection rights into the government budget constraint (19), after some algebra, yields the expression of the lump sum tax T in (23). It is easy to see that both terms in (23) are negative, and hence that T has a negative sign. $\square$

The expressions of the prices of infection rights q_l, q_h allow to clearly see how the various kinds of externalities described in the previous section can be internalized by markets for those rights. The price of infection rights for the firm, q_h, is equal to the marginal utility cost of infection for individuals, $\frac{\beta}{\partial u/\partial c}$ plus the marginal costs for the health care system η. The additional marginal cost of an infection at work, given by the decrease in the productivity of the workforce, does not enter the price of infection rights q_h because it is already internalized in the firm's production and abatement decisions. Turning then to the price of infection rights for the agent, q_l, we see it is composed of three terms. The first, $\eta + A$, represents the marginal costs of an infection for the health care system and for the firms (as a productivity loss). The second and third terms capture the additional marginal costs due to the externality in infections, generated by the effect of each agent's consumption/leisure choice l (which determines the average value $\bar{l}$) on other agents' infections . In particular the second term, $(\eta + A)\Delta$ encodes the component of these additional costs borne by the health care system and the firms, while the third term, $\frac{\beta}{\partial u/\partial c}\Delta$, encodes the component given by utility costs of infected agents.

Finally, the revenue raised by the government at equilibrium by the sale of infection rights at prices q_l, q_h is higher than the cost for the health care system. This is due to the fact that, as explained above, the prices of infection rights at an efficient equilibrium do more than just allow consumers and firms to internalize the health costs of infections, as they also internalize additional utility and production costs. The surplus for the government is then rebated back to consumers through the lump-sum subsidy $-T$ (a negative lump-sum tax in our notation), to support the Efficient allocation.[9]

[9] It is immediate to see, using (22), that the health cost term η still appears in the value of

3.2 Other market implementation mechanisms

Some subtle and important issues arise in the design of markets for infection rights as introduced in the previous section. We discuss here some alternative market mechanisms which implement the Efficient allocation.

Market for infection rights and insurance. In the design of markets for infection rights introduced in the previous section, it is the probability of being infected that is being priced. But this mechanism is equivalent to one where i) agents and firms are required to buy infection right only if they and their workers, respectively, are infected; but ii) markets exists to ex-ante insure this risk. At fair prices $q_l I_l$ and $q_h I_h L_h$, respectively, for agents and firms, the Efficient allocation is implemented.

Markets for infection rights - price-setting. An alternative policy design for the same structure of markets is given by the government setting the prices of these rights at a given level and standing ready to supply the amount requested at these prices by consumers and firms. If prices q_l, q_h are set at the level given by (21), (22), derived in what follows, we show that the Efficient allocation is implemented.

Pigouvian taxes. An alternative interpretation/implementation of markets for infection rights consists in the introduction of a Pigouvian tax scheme on the activities generating infections.[10] The case in which taxes are levied directly on the infections generated by production and consumption-leisure choices is just a simple reformulation of the requirement to acquire infection rights; see the previous paragraph. In this case, the tax rates simply coincide with the prices of rights. But it is similarly straightforward to design a tax scheme whose base is the production of each firm, with rebates based on the firm's abatement choices, and the consumption/leisure activities of each agent.

4 Informational requirements for efficiency

In this section we argue that *markets for infection rights (and hence Pigouvian taxes) are generally superior to command economy interventions*, in that they require less

$q_l - \eta$ and so in the expression of the lump-sum tax T in (23). This is due to the fact that, as we already observed, the adopted specification of the markets for infection rights prescribes that each individual acquires an amount of rights equal to the total infections generated by his/her own social interaction choices, including those due to the externality in infections. But the level of the price q_l that induces agents to internalize the externalities present in their own choices already takes this into account, thus generating a 'double counting'.

[10]The Pigouvian qualification on the tax scheme indicates that taxes are assessed against agents and firms for engaging in activities that create negative externalities for society.

information to be implemented. To this end, we study the informational requirements necessary to achieve efficiency in a society hit by an epidemic. We compare the informational requirements necessary for command economy interventions with those necessary to implement markets for infection rights. We also compare the relative informational requirements associated to the different institutional designs of markets for infection rights we discussed.

Command economy interventions require information on the technology of firms, the preferences of agents and the infection process; notably, on the productivity parameter A, the abatement cost function $C(a, L_h)$, the infection at work spread parameter γ, the agents' utility function $u(c, l)$ and preference parameter β, the infection in leisure function $\delta(l, \bar{l})$, and the health cost parameter η. The decentralization result through markets for infection rights requires instead the government to choose the efficient supply of infection rights, that is, "the supply of infection rights H_l, H_h corresponding to the resulting infections $I_h L, I_l L$ at the Efficient allocation". In the simple environment we considered in the previous sections, the informational requirements of determining this level of the supply are not too different from those of implementing command economy interventions. The advocated *superiority of introducing markets for infection rights over command economy interventions* rests mostly on the consideration of richer and more complex economies, where agents and firms are heterogeneous and/or the values of preferences and productivity parameters are only privately known. In the next sections, we extend the analysis to richer environments and discuss the associated informational requirements.

4.1 Heterogeneity and multiple sectors

Consider first adding technological heterogeneity to firm production. In particular, consider different types of firms, indexed by $j = 1, 2, \ldots, J$, and characterized by different technological parameters A_h^j, γ_h^j, for production and infection at work, and different abatement cost functions $C_h^j(a_h^j, L_h^j)$. In this case, command economy interventions can still implement the Efficient allocation, whose definition is aptly and straightforwardly extended. But they require policy makers to set production, labor demand, and abatement y_h^j, L_h^j, a_h^j for each firm j. In other words, the policy maker needs knowledge of each firm's type j and its technological configuration $A_h^j, \gamma_h^j, C_h^j(a_h^j, L_h^j)$. The implementation of markets for infection rights, on the other hand, only requires the knowledge of the distribution in the economy of the technological parameter configurations $\gamma_h^j, A_h^j, C_h^j(a_h^j, L_h^j)$, in order to determine the level of the total supply of rights for infection at work, a much smaller requirement.

The next is a fundamental but easily shown point. At the Competitive equilibrium with infection rights, whose definition is also aptly and straightforwardly extended, all the different firms will be required to trade infection rights, but the price of infection rights q_h will remain determined as in (21). Indeed, while in this economy production, infection, and abatement parameters are heterogeneous across firms, q_h does not depend on these parameters. As a consequence, the design of any implementation mechanism relying on markets for infection rights or Pigouvian taxes does not require information on the technological parameters of each individual firm, but just the the knowledge of their distribution in the economy to calculate the efficient amount of infection rights to supply or their sale prices/taxes. Importantly, however, even though the price of infection rights is the same for all firms, the total cost of infection rights borne by any firm will depend on its own production, infection, and abatement parameters. In particular, firms characterized by a relatively lower productivity A_h^j or a higher marginal cost of abatement $C_{h,1}^j(a_h^j, L_h^j)$, other things equal, will choose lower abatement. In equilibrium these firms are then likely[11] to have a higher probability of infection I_h^j and thus need to buy a larger amount of rights (per worker employed), or the Pigouvian tax revenue levied on - these firms is higher.[12]

Our findings then show that firms and/or sectors whose productivity loss associated to remote work is relatively small, that is, whose abatement costs are low, tend to budget a lower expenditure for the purchase of infection rights (or Pigouvian taxes) at equilibrium. On the other hand, firms and/or sectors relatively concentrated in dense cities whose workers e.g., are likely to use public transportation, will budget a higher expense for infection rights (or Pigouvian taxes).

Consider now extending the analysis to allow for heterogeneity in agents' preferences and infectiveness in consumption/leisure activities. In particular, suppose there are L types of individuals, indexed by $i = 1, 2, \ldots, L$, and characterized by different preferences $\beta^i, u^i(c, l)$, different infectiveness $\delta^i(l, \bar{l})$, and different health care system costs, η_i. With heterogeneous agents distributional issues arise as Efficient allocations are a whole frontier, not a single point. These issues can be addressed in principle by considering lump-sum taxes/subsidies T^i indexed by i but only partly, as the heterogeneity of the disutility β^i of getting infected also affects the extent of the externality which needs to be internalized. In the case of utilitarian welfare, where all agents are equally weighted, the expression for the price of infection rights, once we suitably extend the definition of the Competitive equilibrium with infection rights, becomes:[13]:

[11] The lower level of the productivity and higher marginal cost of abatement of a firm also implies the size L_h^j of its employment will be smaller and this in turn affects the marginal cost of abatement. The overall effect on the (efficient) equilibrium choice of abatement then requires to take this effect also into account.

[12] In contrast, the effect of a more infectious technology (with higher values of γ) is ambiguous, since firms operating such technology choose a higher level of abatement and hence the overall effect on infections could go either way.

[13] See the Appendix for details.

$$q_l^i = A + \eta_i + \sum_{f=1}^{L}(A+\eta_f)\frac{\frac{1}{L}\frac{\partial\delta^f}{\partial\bar{l}}}{\frac{\partial\delta^i}{\partial l_i}} + \sum_{f=1}^{L}\frac{\beta_f}{\frac{\partial u^i}{\partial c}}\frac{\frac{1}{L}\frac{\partial\delta^f}{\partial\bar{l}}}{\frac{\partial\delta^i}{\partial l_i}},\ i=1,..,L \tag{28}$$

In (28) we see that, relative to the previous expression, (22), (i) the multiplicative effect of own infections on infections of other agents (generated by social interaction activities) now varies across individuals: $\Delta^i = \left[\frac{\sum_{f=1}^{L}\frac{\partial\delta^j/\partial\bar{l}}{L}}{\partial\delta^i/\partial l_i}\right]$, (ii) this effect is weighted with the heterogenous utility and health costs across individuals. Furthermore, the term on the right hand side of (28) varies with the agent's type i so that, differently from the case of technological heterogeneity, one single market for infection rights does not suffice to implement an Efficient allocation and personalized (type-indexed) prices (or Pigouvian taxes) q_l^i are required.

These prices/taxes are higher for agents whose relative marginal health care costs η^i are higher and who are relatively more infective, in the sense that they have a larger multiplicative effect Δ^i (or equivalently, for whom the effect of the agent's consumption/leisure on own infection is smaller). The same is true for agents with relatively lower marginal utility for consumption $\partial u^i/\partial c$, as for instance richer agents: a higher price is needed to induce them to internalize their externalities. On the other hand, the individual own cost of getting infected, β^i, does not affect the price of infection rights (or Pigouvian taxes), as its effect is internalized in the agent's choice of consumption/leisure activities.[14]

Nonetheless, and similarly to the case with technological heterogeneity, the total payment/tax required for agents depends not only on the price but also on the amount of rights purchased, that is on the own infection probability I_l^i. This amount will be higher for agents with lower β^i and $\partial u^i/\partial c$, because such agents, other things equal, will choose a higher value of l and hence have a higher infection probability in equilibrium.[15]

Personalized prices of infection rights (or Pigouvian taxes) imply that the informational requirements to design markets for infection rights are somewhat stronger, though not much so if e.g., agents' heterogeneity depends on their demographic characteristics, which are generally observable. Hence the analysis above implies that, for instance, younger agents, with lower health care system costs of infection η^i, should face lower prices/taxes.

[14] The weighted average of the utility cost of all individuals still appears in (28) because it contributes to determine one of the effects of the externality in infections due by agents' consumption/leisure choices.

[15] The effect of the marginal probability of getting infected $\frac{\partial\delta^i}{\partial l_i}$, on the other hand, is ambiguous, for reasons analogous to what we saw for γ^j in the case of the firms. The lower is $\frac{\partial\delta^i}{\partial l_i}$, other things equal, the higher is l_i, so the effect on I_l^i is ambiguous, while q_l^i, as we saw, is also higher.

Similar results obtain if we extend the analysis to allow for a variety of consumption/leisure activities, for which the magnitude of the external effect on the infection of other agents is different. This can be captured by replacing the average level $\bar{l}$ in the expression of $\delta^i(l, \bar{l})$ for all i with a weighted average of the consumption/leisure choices of all individuals, with weights v_i reflecting the intensity of interaction with other agents associated with the kind of consumption/leisure activity that is chosen by an agent. For instance, a high weight v_i may reflect the centrality of the individual in the network of social interactions, or the agent's strong preference for participating in large events like concerts or sport gatherings. The higher v_i, the higher the multiplicative effect Δ^i and the higher the price of rights (or the Pigouvian tax) faced by an individual.

We summarize our findings in the following:

Proposition (Prices of infection rights/Pigouvian taxes - with heterogeneity)**.** *Firms operating technologies with lower productivities or higher (marginal) costs of abatement will face higher infection rights prices/Pigouvioan tax (per worker). The same is true for individuals featuring higher health treatment costs, higher multiplicative effects of own infections, who are richer and have a lower utility cost of getting infected.*

4.1.1 Production chains

The analysis of the informational advantage of markets for infection rights over command economy interventions is even clearer if we allow for production chains, that is, we introduce other goods that are produced (upstream) and are intermediate goods, used as input in the production of the single consumption commodity (downstream). Consider the following reformulation of the firm's production function

$$Y = AD^{\alpha}(L_y(1 - I_y - I_l))^{1-\alpha},$$

where Y is, as before, the output of the single consumption commodity in the economy, while L_y is now the quantity of labor (number of workers, each working one unit) employed in the production of Y, D is the intermediate good used in the production of Y, I_y is the probability that a worker employed in the firm producing Y ends up being infected at work. The quantity D of the intermediate good is produced with labor,

$$D = BL_d(1 - I_d - I_l),$$

where L_d is the quantity of labor in the production of D, I_d is the probability of becoming infected at work for a worker employed in the firm producing D.[16] The

[16] We continue assuming that the total quantity of labor in the economy, $L = L_y + L_d$, is given exogenously - pre-determined at the outbreak of the epidemic.

infection rate of agents at work in the two sectors is, respectively, $I_y = \gamma_y(1 - a_y)$, $I_d = \gamma_d(1 - a_d)$, where a_y and a_d denote social distancing and other abatement measures the firms can employ, at (increasing convex) costs $C_y(a_y, L_y)$, $C_d(a_d, L_d)$, for the two production processes. The total number of agents infected at work is then $I = I_y L_y + I_d L_d$.

Following similar steps to those of the analysis in the previous section, it is straightforward to show that the decentralization of the Efficient allocation requires a single market for rights of infection at work, where firms in all sectors trade rights at the price q_h that has again the same expression as in (21). In this context, the equilibrium price of the intermediate good together with the price of infection rights induce both upstream and downstream firms to coordinate efficiently their production and abatement decisions without any informational requirement on the part of the policy maker.[17] Thus we can say that no additional externality in contagion is generated by the presence of production chains.

It remains the case, however that each firm and each agent must acquire rights for the infections it generates. As a consequence, the observation of the probability of infection for each agents in his/her consumption/leisure activities and in his/her production activity in the firm is necessary to implement and enforce the competitive equilibrium with infection rights. We shall relax this informational requirement in Section 4.3.

4.2 Social preferences/constraints over infections

The decentralization result through markets for infection rights requires the policy maker to set the level of the supply of infection rights H_h, H_l at the efficient level, at which the social welfare function is maximized. The social welfare function we have studied in the previous section coincides with the representative agent's utility, that determines the preferences of society over consumption, leisure as well as infections. Consider instead the case in which social welfare, as far as the public health conditions of society are concerned, is represented by direct preferences over these conditions, in particular over the spread of the infection in the population. Suppose the socially preferred level of infections in the society is represented by I_h^*, I_l^*. In this case, the *Efficient allocation* is the one that maximizes the representative agent's utility subject to (13)-(16) and the additional constraint that the level of infection (I_h, I_l) is equal to (I_h^*, I_l^*). It is clear that the decentralization of this allocation as a competitive equilibrium requires *no information* on the parameters of the economy on the part of the policy maker.

An alternative interpretation of this specification is also possible. So far we assumed that unit health care costs are constant, equal to η, whatever the share of the population that is infected. In many situations however the policy maker faces a

[17] See the Appendix for details.

capacity constraint in the provision of treatment for infected, $(I_h, I_l) \leq (I_h^*, I_l^*)$, e.g., determined by the availability of hospital (or ICU) beds. Whenever the capacity constraint is binding the Efficient allocation described in the previous paragraph also maximizes the representative agent's utility. By setting the supply of infection rights $(H_h, H_l) = (I_h^*, I_l^*)$ the corresponding Competitive equilibrium with infection rights induces the Efficient allocation.

4.3 Moral hazard

As we noticed in Section 3, the argument used in the previous results requires that the individual values of I_h and I_l are observable, that is the probability of being infected for every individual as well as whether he/she is infected at work or via his/her social interaction activities. In this section we show that the decentralization of the Efficient allocation via markets for infection rights does not require this fine degree of observability at the individual level, only the observation of the average infection rate suffices.

Consider a society where only the health status of individuals is observable, but not where infections took place. By an appeal to the law of large numbers, we can say this captures the average value of the total probability of infection I of individuals working in a firm. Such limited observation generates a problem of *moral hazard* in teams, as in Holmstrom (1982), since both the choice of a by the firm employing agents and that of the level l of social interaction by each of these agents contribute to determine the (average) probability that they are infected.

We show in what follows that in this environment it is still possible to decentralize the Efficient allocation with markets for infection rights, provided we allow for lump sum taxes and subsidies not only for consumers but also for firms. Let I denote the average overall probability that an individual working in a firm gets infected, that is, the infection rate of agents working in a firm.[18] The institutional market design introduced in Section 3 is then modified as follows.

Each agent engaging in consumption/leisure activities is mandated to buy - at the price q_h - a right per unit of probability of infection I;

Each firm employing L_h workers is mandated to buy a right per infected worker, IL_h in total, at the unit price q_l.

Not only the agent but also the firm pays a lump sum tax (receives a transfer if negative), given respectively by T and T_h.

[18] The argument and result can also be extended to the case where only the average probability of infection of an individual in the whole society is observed (that is, can be inferred from the available data).

The budget constraint of a representative individual is then now

$$c + q_l I = \pi + w - T \tag{29}$$

and the profits of a representative firm is

$$y - wL_h - q_h I L_h - C(a, L_h) - T_h. \tag{30}$$

Both the individual and the firm take into account how the value of I in the above expressions is affected, though only partly, by their own choices:

$$I = \gamma(1-a) + \sum_{i}^{L_h} \frac{\delta(l_i, \bar{l})}{L_h}. \tag{31}$$

Competitive equilibrium with infection rights and moral hazard. *A competitive equilibrium is as previously defined, except for the following facts:*

the agent's budget constraint is now given by (29);

the firm's objective is now (30)

the government chooses the supply of infection rights H;

and the market clearing condition for infection rights is

$$H = IL. \tag{32}$$

At the Competitive equilibrium with infection rights and moral hazard, total infections IL are equal to the level of the supply of rights H set by the government. It is now straightforward to prove that, when H is set equal to the efficient level of infections, the Efficient allocation is decentralized also in the presence of moral hazard:

Proposition (Efficiency of equilibria with moral hazard)**.** *Suppose the government chooses a supply of infection rights H equal to the Efficient level of infections in the society, IL. Then the Competitive equilibrium with infection rights and moral hazard induces the Efficient allocation.*

The proof of the decentralization result with moral hazard develops along the same lines as the proof of Proposition (Efficiency of equilibrium). In fact the same value of the firm's price q_h as in (21), and L times the value of the worker's price q_l obtained in (22) ensure that the first order conditions of the social welfare maximization problem coincide with those of the worker's and firm's maximization problems in the Competitive equilibrium with infection rights and moral hazard. On the other hand, the value of the lump sum transfers both to agents and firms have to be suitably increased, to reflect the presence of moral hazard for the reasons explained in what follows.

Proof. Consider the first order condition for the firm's choice of abatement at a competitive equilibrium in the current environment. Substituting the market clearing conditions for infection rights, (32), as well as for labor and the consumption good yields:

$$(A + q_h)\gamma L = C_1(a, L),$$

the same expression as the one obtained in the previous section, where I_h is observable (with no moral hazard). It then follows that the value of q_h inducing the optimal choice of a is unchanged. However each firm needs to acquire a greater amount of infection rights, IL_h rather than $I_h L_h$. To keep its net payments the same a lump sum rebate is thus needed.

The first order condition for the worker's optimal choice of leisure/social interaction at a competitive equilibrium, after substituting the market clearing conditions, is instead:

$$\left(\beta + \frac{\partial u}{\partial c} q_l \frac{1}{L}\right) \frac{\partial \delta}{\partial l} = \frac{\partial u}{\partial l},$$

It differs from the one obtained in the previous section for the fact that q_l is now multiplied by $1/L$. To be able to still match the FOC's for a Pareto optimum, q_l must then be L times its value in the absence of moral hazard. This feature, together with the fact that in equilibrium each agent must also acquire a greater amount of infection rights, equal to $I_l + I_h$ instead of I_l, requires a higher value of the lump sum rebate received by agents, to offset the extra payment made by them. □

Hence the price of infection rights for firms supporting the efficient allocation is the same with and without moral hazard. The only difference is that now a lump sum transfer to firms is required. This is due to the fact that each firm needs to acquire a greater amount of infection rights, IL_h rather than $I_h L_h$ and so ends up paying also for the infections caused by its workers' decisions (that is, $\sum_i^{L_h} \delta(l_i, \bar{l})$). This additional payment operates as lump sum tax paid by the firm via its purchase of infection rights, which must be offset with a lump sum rebate for the same amount to keep the level of its profits unchanged.

In contrast, the price of infection rights faced by agents in the moral hazard society is L times the one with no moral hazard. This is due to the moral hazard problem induced by the fact that each worker must acquire now an amount of rights equal to the average infection rate among workers in the firm $\sum_i^L \frac{\delta(l_i, \bar{l})}{L}$, rather than to his/her individual infection rate $\delta(l_i, \bar{l})$. The agent's utility is only directly affected by the latter, but the agent ends up paying only a fraction $1/L$ of the infections generated by his/her chosen level of interaction. To preserve his/her incentives, the price of infection rights is then multiplied by L. Furthermore, the other amount of rights the agent must purchase depend on the interaction choices of other individuals working in the firm, $\sum_{j\neq i}^{L} \frac{\delta(l_j,\bar{l})}{L}$, as well as on the abatement decisions by the firm, I_h. The payment for these other amounts is then a constant, independent of the agent's own decisions, analogous to a lump sum tax, which must be rebated back to the individual by suitably increasing the value of $-T$ above the level obtained in Proposition (Efficiency of equilibrium), in the case without moral hazard.

To sum up, in the situation considered moral hazard can be fully overcome and the incentives of firms and agents sustained simply by increasing the amount of infection rights they are required to purchase and possibly by suitably increasing the price of rights. Budget balance is then preserved with suitable lump sum rebates.

Finally, the Efficiency of equilibria with moral hazard proposition extends to an economy which accounts for technological heterogeneity and multiple production as well as consumption/leisure sectors; that is, competitive equilibria with infection rights are efficient even if it is not observable whether agents are infected in a production or consumption/leisure activity nor, a fortiori, in which production of consumption/leisure activity.

4.4 Private information

In the environment considered so far the Efficient allocation is decentralized at the Competitive equilibrium if the government supplies the efficient amount of infection rights and lets prices clear the markets for such rights. The welfare result - in Proposition (Efficiency of equilibrium) - shows that this is equivalent to the government mandating the Efficient allocation of a, l and the associated production and consumption levels to firms and agents. The different informational requirements in the two cases have been discussed in the previous subsections. Notably, a variant of the design of the market for infection rights, that also decentralizes the Efficient allocation, has the government fixing the prices of infection rights q_l, q_h at the Competitive equilibrium value and supplying all the rights demanded at these prices by agents and firms. Our previous analysis of the informational requirements for the market for infection rights applies also to this alternative specification. There is however another dimension in which these two specifications and the command economy should be assessed; that is how they fare in the presence of shocks, observed by agents and firms but not observed by the policymaker.

In what follows we consider a society as described in Section 2 but where the firm's productivity parameters and/or the parameters capturing the infectiousness of production and consumption/leisure activities are subject to *unanticipated* shocks, whose realization is known to the agents but not to the policy maker; that is, subject to *private information.* In this society, firms and agents can adjust their choices to the observed value of the shock, while the intervention of the government cannot be modified in the light of the shock. We will compare how command economy interventions and the two variants of the markets for infection rights fare in this situation, when the parameters which define these interventions are set at the level which allow to attain the Efficient allocation prior to the realization of the shock.

In the case of the command economy intervention the values of a, l - and hence the allocation - are determined and cannot respond in any way to the realization of the shock. When a shock occurs we have so a welfare loss, unlike in the situations considered in the previous sections. With a market for infection rights, when the policy maker fixes the quantity supplied of these rights, this remains unchanged but consumers' and firms' demand may vary with the shock and so prices adjust to clear. Alternatively, when the policy maker sets the prices of the rights, quantities - the supply of infection rights - adjust in response to the changes in demand. In both cases, with markets for infection rights, even though the policymaker does not observe the realization of the shock and so the specification of the policy (the supply, or the price of the rights) remains unchanged after the shock, the allocation that is obtained in equilibrium may still indirectly responds to the shock because of the change in firms' and agents' behavior. The magnitude and sign of this response may be different for the two design specifications, where either quantities or prices are fixed. It also depends on the parameters which are subject to the shock. Hence in this context, with privately observed shocks to the parameters determining the technology and the infection process, the command economy intervention and the institutional designs where the policy maker fixes quantities or prices of infection rights are no longer necessarily equivalent in terms of welfare.

It is then important to determine whether a design of markets for infection rights is apt to reduce the welfare losses associated to the command economy, for the different kinds of shocks; and if so which design of the market for infection rights is preferable in terms of social welfare. This is the objective of the analysis that follows.

More specifically, we study the welfare losses associated to the following institutional designs of the policy interventions:

Command economy The government mandates a and l at the Efficient level prior to the parameter shock; the levels of a, l remain then unchanged after the shock.

Quantity setting of infection rights The government sets the level of the supply H_h, H_l of infection rights equal to the Efficient level of infections prior to the parameter shock, that is, the infections induced by the Efficient allocation; after the shock, in a Competitive equilibrium with infection rights firms and individuals choose a, l facing prices q_h, q_l such that markets clear when the supply of rights remains fixed.

Price setting of infection rights The government sets the prices of infection rights at the level q_h, q_l corresponding to the value at the Competitive equilibrium with infection rights prior to the parameter shock; after the shock, firms and individuals choose a, l facing an unchanged value of these prices, while the government supplies all requested infection rights so as to clear the market.

A pathbreaking paper by Martin Weitzman (1974) tackles these issues with great clarity. In the following analysis we follow closely the approach in that paper, extending the analysis to our society.[19] As in Weitzman (1974), we take a quadratic approximation to the agents' utility function as well as to the cost of abatement function in order to evaluate welfare losses. Since the social welfare maximization problem in our society is two-dimensional, entailing the choice of l and a, we simplify further the analysis by considering in particular the case where the agents' utility is linear in consumption. Therefore, we have $u(c, l) = \theta c + l - \frac{1}{2}\lambda l^2$ and $C(a, 1) = \frac{1}{2}ca^2$. Furthermore, we assume for simplicity (without any substantial loss of generality) that $L = 1$ and $\delta(l, \bar{l}) = \delta_1 l + \delta_2 \bar{l}$. Under these assumptions it is possible to evaluate the welfare losses at the allocations obtained with the three distinct designs of policy interventions relative to the Efficient allocation after the shock realization.

Specifically, we concentrate on shocks to parameters affecting the generation of infections, $\gamma, \delta_1, \delta_2$, and to parameters affecting the costs of infections, in terms of output and utility losses, η, A, c, β. Under the above assumptions, both the Efficient and the equilibrium levels of a and l are determined independently by a separate equation and hence the effects of shocks can be analyzed separately for each of the two variables. Results can be illustrated using diagrams analogous to the ones in Karp and Traeger (2018); see Figure 1 and 2.

Firms' abatement choice - Figure 1. Under the above assumptions, the first order condition of the social welfare maximization problem with respect to firms' abatement choice a, (equation 24), simplifies to:

$$ca\theta = \beta\gamma + \theta\gamma(A + \eta), \tag{33}$$

Equation (33) determines the efficient level of the firm's abatement choice a. The term on the left-hand-side of this equation describes the marginal cost of abatement, while the term on the right-hand-side describes the marginal benefit, in terms of direct utility gains and of the utility of the output gains, both due to the reduction of infections at work induced by a marginal increase in a.

Consider first the command economy intervention. In this design, the policy maker mandates a at the efficient level prior to any shock, that is, the one which solves (33) for the values of the parameters before a shock occurs. A shock to any of

[19] An interesting survey of a subsequent literature on the application of the ideas in Weitzman (1974) to environmental control can be found in Karp and Traeger (2018).

the parameters $\gamma, \eta, A, c, \beta$ however changes the efficient level of a. If for instance the value of A after the shock increases to $A' > A$, as in panel (a) of Figure 1, then the efficient abatement level also increases to the value a' solving $ca'\theta = \beta\gamma + \theta\gamma(A' + \eta)$.[20] We have so a positive welfare loss in the case of a command economy intervention, represented by the red shaded area in the figure. The same is true in the case of shocks to any of other parameters γ, η, c, β. Panel (b) of Figure 1 describes the case of an increase to c' of the abatement cost (with the welfare loss described by the blue shaded area).

Consider in turn the design given by quantity setting of infection rights. The policy maker sets the supply of rights H_h at the efficient level of infections at work prior to the shock, equal to $I_h L = (1-\gamma)aL$, obtained from equations (6) and (20), with a determined by the solution of (33). The level of a obtained in equilibrium with this design, therefore, remains unchanged in response to shocks, as in the case of the command economy intervention, except when a shock occurs to the parameter γ governing the infection process. When γ changes to γ', the equilibrium level of a changes to a' satisfying the new market clearing equation $(1-\gamma)aL = H_h = (1-\gamma')a'L$. It is easily shown that the indirect response of a induced in equilibrium in the quantity setting design is in the same direction as the change in the Efficient level of a in response to the shock but, for some values of the parameters the magnitude of the change is greater than at the optimum, that is, we have overshooting in the response. It then follows that, except when the overshooting is sufficiently large[21], the quantity setting design is socially preferable to the command economy intervention.

Consider finally the price setting of infection rights design. The policy maker sets the price of infection rights for the firm at the level $q_h = \eta + \beta/\theta$, that is, so as to satisfy equation (21),[22] the expression for the equilibrium price of these rights derived in Proposition (Efficiency of equilibrium - prices), which supports the Efficient allocation prior to any shock. We then see from the condition for the firms' optimal choice of a in a competitive equilibrium, (26), that in response to any shock to the parameters γ, A, c, when q_h is kept constant the equilibrium value of a changes to the level that is efficient after the shock realization.[23] We see in fact from (21) that the level of the price supporting the Efficient allocation after shocks to these parameters is unchanged. Hence there is no welfare loss associated to this design.

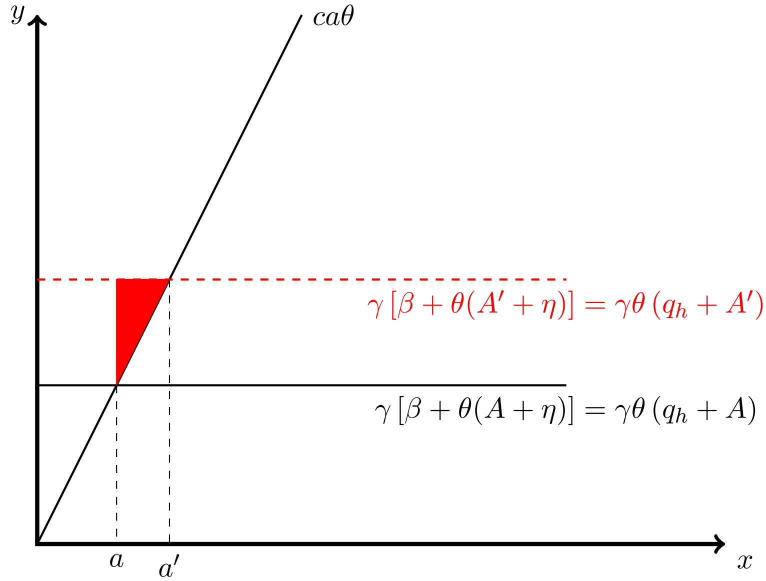
[20] In the figure, a' is obtained at the intersection between the new horizontal dashed line (in red), describing the new level of the marginal benefits of abatement, and the unchanged positively sloped line, describing the marginal cost of abatement.

[21] Explicit conditions on the parameters under which this and the previous property occur are derived and reported in the Appendix.

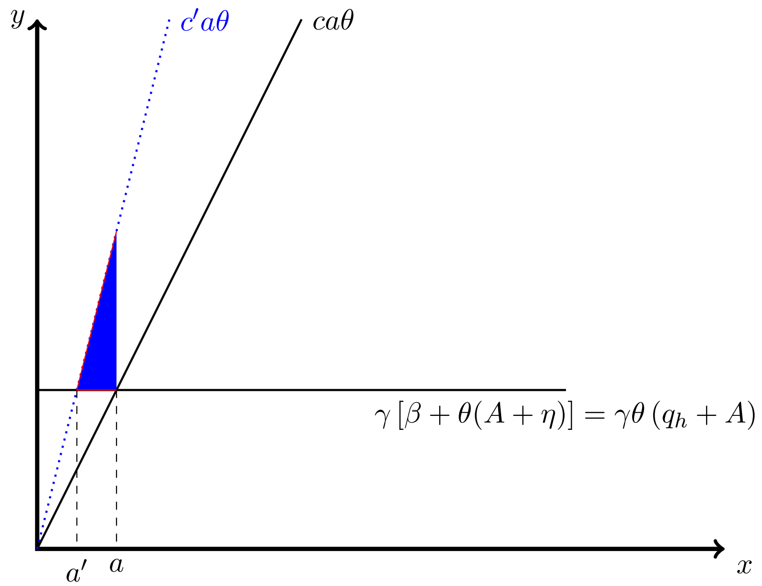
[22] When the marginal utility for consumption is constant and equal to θ.

[23] For instance, in Figure 1 the equilibrium value of the abatement choice after the shock equals the new efficient level a' both in panels (a) and (b).

Figure 1: **Firms' Abatement Choice**



(a) Shock: un-anticipated increase in A.



(b) Shock: un-anticipated increase in c.

Note: Color shaded areas represent welfare losses for command economy and quantity setting designs.

On the other hand, we readily see from (26) that the firms' choice of a in equilibrium remains unchanged after shocks to η, β,[24] so that in this case the welfare loss is positive and equal to the loss associated to the command economy and the quantity setting designs. We conclude summarizing the results as follows,

Proposition (Welfare losses - abatement choice)**.** *The price setting design of infection rights induces, in the face of* private information *over the shocks to any of the parameters $\gamma, \eta, A, c, \beta$, an abatement choice which is preferable in terms of social welfare, at least weakly, to both to the command economy and the quantity setting designs.*

Agents' consumption/leisure choice - Figure 2. The efficient level of the agents' consumption/leisure choice l is obtained as a solution of the first order condition of the social welfare maximization problem with respect to l, (equation (25)), which under the above assumptions simplifies to:

$$1 - \lambda l = \beta\left(\delta_1 + \delta_2\right) + \theta(A + \eta)\left(\delta_1 + \delta_2\right). \tag{34}$$

The analysis of the welfare losses associated to the value of l obtained for the three designs for policy interventions follows the logic of the analysis we carried out for firms' abatement choices.

In the command economy design the policy maker sets l to solve (34) prior to the shock. Hence, for shocks to any of the variables $\delta_1, \delta_2, \eta, A, \beta$ we have a positive welfare loss. Figure 2 describes the cases of a decrease in β (panel (a)) and in δ_2 (panel (b)), and the welfare loss due to the fact that agents' leisure choice remains unchanged in response to these shocks is represented by the green shaded areas in the two panels.

In the quantity setting design the policy maker sets the supply of rights H_l at the level $I_l L = (\delta_1 + \delta_2) l L$ (from equations (10) and (20)), with l determined by (34) before the shock occurs. It is immediate to verify that, as we saw for the abatement choice, when we have a shock to the production or preference parameters (η, A, β) the equilibrium value of l remains unchanged and so the welfare losses are the same as for the command economy. When instead a shock to the parameters of the infection technology δ_1, δ_2 occurs, with quantity setting the equilibrium value of l varies; again the change is in the same direction as the change of the efficient level[25] and, for some parameter values, we have an over-reaction, that is the magnitude of the equilibrium

[24] Recall that we do not consider preference shocks to the marginal utility of consumption, θ, as they do not refer to the direct effects of infections. It can be shown however that for such shocks the equilibrium value of a in the price setting design remains unchanged, as in the command economy and the quantity setting designs. Hence the welfare loss is the same in all these cases.

[25] If, say δ_1 increases to δ_1', the new equilibrium value of l'' satisfies $(\delta_1' + \delta_2)l'' = (\delta_1 + \delta_2)l$; hence it decreases, as the efficient value, obtained from (34), does.

variation is greater than the one required for efficiency. Figure 2, panel (b) describes a situation in which we have under-reaction in response to a decrease in δ_2: the new equilibrium level with quantity setting is[26] l'' and the welfare loss is the (smaller) grid-patterned area.

Finally, in the price setting design the policy maker sets q_l at the level $\left[(\eta + A)\frac{\delta_1+\delta_2}{\delta_1} + \frac{\beta}{\theta}\frac{\delta_2}{\delta_1}\right]$, which satisfies equation (22) derived in Proposition (Efficiency of equilibrium - prices). The change in the equilibrium level of l in response to shocks is then obtained from the first order condition for an individual optimum (27) when q_l is kept constant at the set level. In Figure 2 this is given by l' in panel (a) while it is unchanged at l in panel (b). It is immediate to verify that the welfare losses associated to this design (for shocks to $\delta_1, \delta_2, \eta, A, \beta$) are now also always positive. They are smaller than the losses associated to the command economy and the quantity setting designs for shocks to β (as we see in panel (a) of Figure 2, where these losses are given by the smaller, grid-patterned area); while they are equal for shocks to η, A. In the first case, in fact, the equilibrium value of l varies in the same direction, in response to a shock, though by a smaller amount, than the efficient value. In the second case, instead, the equilibrium value l does not vary. On the other hand, in the case of shocks to δ_2 we have no change in l in the price setting design, as for the command economy, while we do have a change in l in the quantity setting design. It can be shown that, for a large set of parameter values[27], the welfare loss is smaller in the latter case (this is the case in the situation described in panel (b) of Figure 2). Finally, the ranking of the welfare losses with respect to shocks to δ_1 complex. This is because the equilibrium change in l in the price setting design is in the same direction but larger than required by efficiency ex-post, that is we have always an over-reaction; while in the quantity setting design, as noticed in the previous paragraph, it can be either larger or smaller. We can show that again for a large set of parameter values the welfare loss is smaller in the quantity setting design.[28] We conclude summarizing the results as follows,

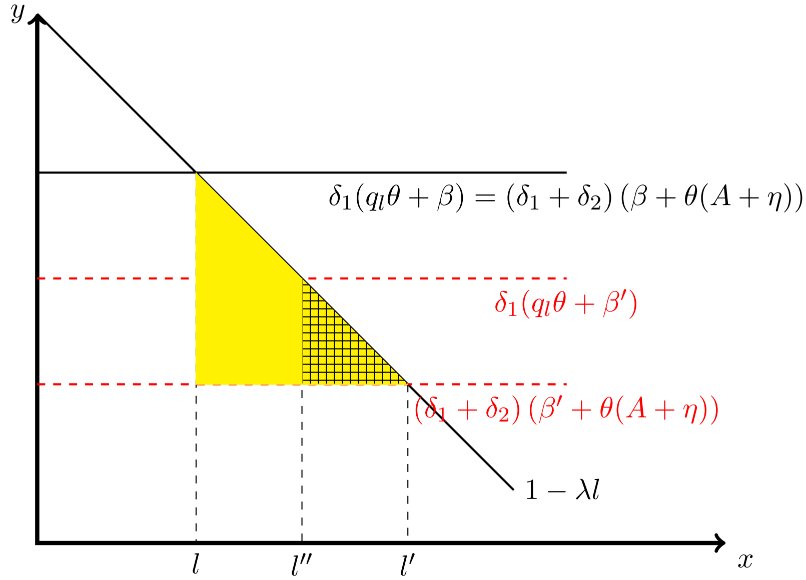
Proposition (Welfare losses - consumption/leisure choice). *In the face of* privately observed *shocks to infection process δ_1, δ_2, the quantity setting of infection rights design proves superior in terms of social welfare, for most parameter values, in the sense of inducing lower welfare losses associated to the consumption/leisure choice, to both to the command economy and the price setting designs. In contrast, in the face of shocks to the preference parameter β, price setting is preferable; and for all other shocks the three designs are equivalent since l does not respond to shocks.*

[26] The value of l'' is obtained as a solution of the equation: $(\delta_1 + \delta_2)l = (\delta_1 + \delta_2')l''$.

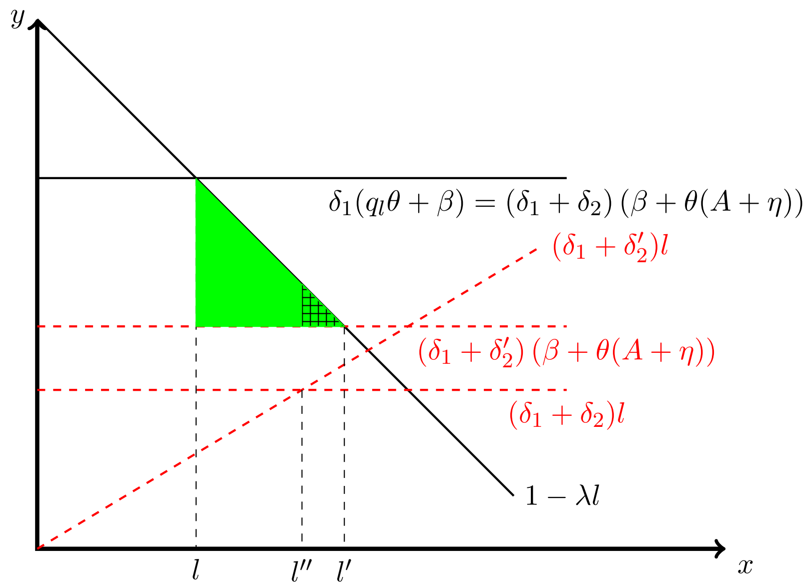
[27] See again the Appendix for details.

[28] See the Appendix for details. [This happens when $[\beta + \theta(A + \eta)](\delta_1 + \delta_2) > 1/3$, hence it is always true when the equilibrium change in l in the quantity setting design is smaller than required by efficiency ex-post.]

Figure 2: **Agents' Consumption/Leisure Choice**



(a) Shock: un-anticipated decrease in β.



(b) Shock: un-anticipated decrease in δ_2

.

Note: Color shaded areas represent welfare losses for command economy and quantity setting (panel a) and for command economy and price setting (panel b). The dashed area in panel (a) (resp. b) represented the smaller welfare losses with price setting (resp. quantity setting) when the equilibrium outcome is give by l'' (instead of l').

To sum up, we have shown that the design of a market for infection rights allows to reduce, at least weakly, the welfare losses that we have with a command economy intervention due to shocks to parameter values of the economy that are only privately observed by individuals and firms. Furthermore, while price setting for such market proves better in the case of firms' abatement choices, this is not the case for agents' consumption/leisure choices. In the face of shocks to the infectiousness of consumption/leisure activities, quantity setting proves more effective at reducing welfare losses.

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5 Appendix

5.1 Heterogenous agents

We focus our attention in this case on the Efficient allocation that maximizes utilitarian welfare, where all types of agents have the same welfare weight[29]. This is obtained as solution of the following programme:

$$\max_{\{c_i,l_i,a\}} \sum_i^L \left[u^i(c_i,l_i) - \beta_i(I_l^i + I_h)\right]$$

$$s.t.$$

$$\begin{aligned}
\sum_i^L c_i &= \left[AL(1 - \frac{\sum_i^L (I_h + I_l^i)}{L}) - C(a,L) - \eta(\frac{\sum_i^L (I_h + I_l^i)}{L})L\right] \\
I_l^i &= \delta^i(l_i, \frac{\sum_f l_f}{L}) \text{ for } i = 1,..L \\
I_h &= \gamma(1-a)
\end{aligned}$$

The first order condition with respect to consumption/leisure for type i is then:[30]

$$\frac{\partial u^i}{\partial l_i} = \left[\beta_i + (A+\eta_i)\frac{\partial u^i}{\partial c}\right]\frac{\partial \delta^i}{\partial l_i} + \left[\sum_f \left(\beta_f + (A+\eta_f)\frac{\partial u^i}{\partial c})\frac{1}{L}\frac{\partial \delta^f}{\partial \bar{l}}\right)\right]$$

Hence the effect of the externality in infections must be weighted with the heterogeneous utility and health care costs across individuals who get infected. The first order condition with respect to the choice of the same variable at a competitive equilibrium with infection rights is instead essentially the same as the one obtained with homogeneous agents, (27):

$$\frac{\partial u^i}{\partial l_i} = (\beta_i + \frac{\partial u^i}{\partial c}q_l^i)\frac{\partial \delta^i}{\partial l_i}$$

Hence the expression of the price supporting the efficient allocation is:

$$q_l^i = A + \eta_i + \frac{\sum_f (A+\eta_f)\frac{1}{L}\frac{\partial \delta^f}{\partial \bar{l}}}{\frac{\partial \delta^i}{\partial l_i}} + \frac{\sum_f \beta_f \frac{1}{L}\frac{\partial \delta^f}{\partial \bar{l}}}{\frac{\partial u^i}{\partial c}\frac{\partial \delta^i}{\partial l_i}}$$

[29] To focus on the effects of agents' heterogeneity on their consumption/leisure choices, we assume all firms are identical and so is the probability of getting infected at work for all agents.

[30] Note that at an abitrary point on the welfare frontier, characterized by individual welfare weights ξ^i, the first order condition is

$$\xi^i\frac{\partial u^i}{\partial l_i} = \left[\xi^i\beta_i + (A+\eta_i)\frac{\partial u^i}{\partial c}\xi^i\right]\frac{\partial \delta^i}{\partial l_i} + \left[\sum_f \left(\xi^f\beta_f + (A+\eta_f)\frac{\partial u^i}{\partial c}\xi^i)\frac{1}{L}\frac{\partial \delta^f}{\partial \bar{l}}\right)\right]$$

hence we see that, because of the utility costs of getting infected, welfare weights matter.

5.2 Production Chains

Following a similar procedure as in the proof of Proposition (Efficiency of Equilibrium), we compare the first order conditions of the firms at the equilibrium with those of the social optimum. In fact, to show that efficiency is obtained at an equilibrium with infection rights, it suffices to study the conditions determining the level of abatement in the consumption good (downstream) and intermediate good (upstream) production. The first order condition of the social welfare problem with respect to a_d is:

$$\begin{aligned}\left(\frac{\partial u}{\partial c}\eta+\beta\right)\gamma_d\frac{L_d}{L}+\rho B\gamma_d\frac{L_d}{L} &= \frac{\partial u}{\partial c}\frac{C_{d,1}(a_d,L_d)}{L}, \\ &\Leftrightarrow \left(\frac{\partial u}{\partial c}\eta+\beta+\rho B\right)\gamma_d=\frac{\partial u}{\partial c}\frac{C_{d,1}(a_d,L_d)}{L_d}\end{aligned}$$

where ρ is the Lagrange multiplier of the constraint

$$\frac{D}{L}=B(\frac{L_d}{L}-\gamma_d(1-a_d)\frac{L_d}{L}-\frac{L_d}{L}\delta l);$$

that is, the shadow price of the intermediate good. We then see this condition is satisfied at a competitive equilibrium, where the firms' optimality condition is

$$(q_h+pB)\,\gamma_d=\frac{C_{d,1}(a_d,L_d)}{L_d}$$

when

$$p=\frac{\rho}{\frac{\partial u}{\partial c}},\quad q_h=\eta+\frac{\beta}{\frac{\partial u}{\partial c}};$$

as in the one sector model.

Turning then to the upstream firms, the first order condition of the social welfare problem with respect to a_y is:

$$\left(\frac{\partial u}{\partial c}\eta+\beta\right)\gamma_y\frac{L_y}{L}+\frac{\partial u}{\partial c}\gamma_y\frac{L_y}{L}A(1-\alpha)D^{\alpha}(L_y-I_y-\frac{L_y}{L}I_lL)^{-\alpha}=\frac{\partial u}{\partial c}\frac{C_{y,1}(a_y,L_y)}{L}$$

while the analogous condition for a firm's optimum at a competitive equilibrium is:

$$q_h\gamma_y+\gamma_yA(1-\alpha)D^{\alpha}(L_y-I_y-\frac{L_y}{L}I_lL)^{-\alpha}=\frac{\partial u}{\partial c}\frac{C_{y,1}(a_y,L_y)}{L_y}$$

We see the solution of the two equations is the same when q_h takes the same value as above.

5.3 Welfare losses wih unobserved shocks

5.3.1 Firms' abatement choice

Under the assumptions stated in Section 4.4, the component of the representative consumer's utility that is affected by the abatement choice is given by

$$\theta a\gamma(A+\eta)+\beta\gamma a-\theta\frac{c}{2}a^2$$

Substituting into the above expression the socially efficient value of a, solving (33), yields

$$\theta\frac{c}{2}\left(\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}\right)^2,$$

which describes then the welfare gain due to the efficient choice of a.

Command Economy In the case of a command economy intervention, a is kept constant - in the face of parameter shocks - at the value $\left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right)$, given by the solution of (33) prior to any parameter shock. When any of γ,c, η, A, β changes, there is a positive welfare loss (relative to the efficient value of the welfare gain computed above, captured by

$$\begin{aligned}
& \theta\frac{c}{2}\left[\left(\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}\right)^2\right]-(\theta\gamma(A+\eta)+\beta\gamma)\left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right)+\theta\frac{c}{2}\left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right)^2 \\
= & \ \theta\frac{c}{2}\left[\left(\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}\right)^2-2\left(\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}\right)\left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right)+\left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right)^2\right] \\
= & \ \theta\frac{c}{2}\left[\left(\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}\right)-\left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right)\right]^2
\end{aligned}$$

described by the color shaded areas in Figure 1.

Quantity setting of infection rights In the case of quantity setting, that is when H_h is set at the value $\bar{H}_h = LI_h = \gamma\left[1-\left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right)\right]$, the welfare loss is the same as above, except when we have a shock to γ, say $\gamma' > \gamma$. When that happens, the constant level $\bar{H}_h$ of the supply of rights induces a change in the value of a, to a' such that:

$$\gamma'(1-a')=\gamma[1-\left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right)].$$

Hence

$$a' = \left(1 - \frac{\gamma}{\gamma'}\right) + \frac{\gamma}{\gamma'}\left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right) > \left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right),$$

that is the equilibrium value of a increases in response to the increase in γ, as does the Efficient value. Furthermore, the magnitude of the increase of the equilibrium value of a may be greater than that of the efficient response:

$$\left(1 - \frac{\gamma}{\gamma'}\right) + \frac{\gamma}{\gamma'}\left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right) > \frac{\gamma'}{\gamma}\left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right)$$

satisfied if and only if $\frac{\gamma}{\gamma'+\gamma} > \left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right)$. Recalling that an interior optimum $a < 1$, we see that this inequality holds for some parameter values. Hence we may have overshooting in equilibrium.

Hence with quantity setting of infection rights we still have a welfare loss. Since we may have overshooting, it is possible that the welfare loss is greater than with the command economy intervention. This happens iff:

$$\theta\frac{c}{2}\left[\left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right) - \frac{\gamma'}{\gamma}\left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right)\right]^2 < \theta\frac{c}{2}\left[\left(\left(1-\frac{\gamma}{\gamma'}\right) + \frac{\gamma}{\gamma'}\left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right)\right) - \frac{\gamma'}{\gamma}\left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right)\right]^2$$

$$\Leftrightarrow \left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right)^2 - 2\frac{\gamma'}{\gamma}\left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right)^2 < .$$
$$< \left(\left(1-\frac{\gamma}{\gamma'}\right) + \frac{\gamma}{\gamma'}\left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right)\right)^2 - 2\frac{\gamma'}{\gamma}\left(\left(1-\frac{\gamma}{\gamma'}\right) + \frac{\gamma}{\gamma'}\left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right)\right)\left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right)$$

$$\Leftrightarrow 2\frac{\gamma'}{\gamma}\left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right)\left(\left(1-\frac{\gamma}{\gamma'}\right) + \frac{\gamma}{\gamma'}\left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right) - \left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right)\right) <$$
$$< \left(\left(1-\frac{\gamma}{\gamma'}\right) + \frac{\gamma}{\gamma'}\left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right)\right)^2 - \left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right)^2$$

$$\Leftrightarrow 2\frac{\gamma'}{\gamma}\left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right) < \left(\left(1-\frac{\gamma}{\gamma'}\right) + \frac{\gamma}{\gamma'}\left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right)\right) + \left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right)$$

$$\Leftrightarrow \frac{\gamma'}{\gamma}\left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right) - \left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right) < \left(1-\frac{\gamma}{\gamma'}\right) + \frac{\gamma}{\gamma'}\left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right) - \frac{\gamma'}{\gamma}\left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right)$$

$$\Leftrightarrow \left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right)\left(2\frac{\gamma'}{\gamma} - 1 - \frac{\gamma}{\gamma'}\right) < \left(1-\frac{\gamma}{\gamma'}\right)$$

$$\Leftrightarrow \left(\overline{\frac{\theta\gamma(A+\eta)+\beta\gamma}{c\theta}}\right) < \frac{\gamma(\gamma'-\gamma)}{2\gamma'^2-\gamma\gamma'-\gamma^2} = \frac{\gamma(\gamma'-\gamma)}{\gamma'(\gamma'-\gamma)+(\gamma'-\gamma)(\gamma+\gamma')} = \frac{\gamma}{\gamma'+\gamma+\gamma'}$$

that is when we have a sufficiently large overreaction with quantity setting.

5.3.2 Individuals' consumption leisure choice

Quantity setting of infection rights The supply of infection rights H_l is fixed at the level $I_l L = (\delta_1 + \delta_2)\bar{l}L$, with $\bar{l}$ satisfying $1 - \lambda\bar{l} = \beta(\delta_1 + \delta_2) + \theta(A + \eta)(\delta_1 + \delta_2)$. When $\delta_1 + \delta_2$ increases, say by an infinitesimal amount ε, the equilibrium value of l, which we obtain from the equation $(\delta_1 + \delta_2 + \varepsilon)(\bar{l} + \Delta l)L = \bar{H}_l$, must decrease (by $(\delta_1 + \delta_2)\Delta l = -\varepsilon\bar{l}$). To find the change in the efficient level of l in response to the shock, observe that he right hand side of the FOC for an optimum (34) increases by $[\beta + \theta(A + \eta)]\,\varepsilon$, which means that $1 - \lambda l$, the term on the left-hand-side, should also increase by this amount, or $\Delta l = -[\beta + \theta(A + \eta)]\,\varepsilon/\lambda$. Comparing the two we see that the magnitude of the change in equilibrium is smaller than at the optimum iff

$$\begin{aligned} \frac{\varepsilon\bar{l}}{\delta_1 + \delta_2} &< \varepsilon\frac{\beta + \theta(A + \eta)}{\lambda} \\ \bar{l}\lambda &< (\beta + \theta(A + \eta))(\delta_1 + \delta_2) \\ 1 - \beta(\delta_1 + \delta_2) + \theta(A + \eta)(\delta_1 + \delta_2) &< (\beta + \theta(A + \eta))(\delta_1 + \delta_2) \\ \bar{l}\lambda &< 1/2 \end{aligned} \tag{35}$$

At an interior solution we have $\bar{l}\lambda = 1 - (\beta + \theta(A + \eta))(\delta_1 + \delta_2) < 1$ so the above inequality may or may not be satisfied. We have so an over reaction when $1 - (\beta + \theta(A + \eta))(\delta_1 + \delta_2) > 1/2$, or $1/2 > (\beta + \theta(A + \eta))(\delta_1 + \delta_2)$.

Price setting of infection rights To find the effect of fixing prices at the level $\bar{q}_l = \left[(\eta + A)\frac{\delta_1 + \delta_2}{\delta_1} + \frac{\beta}{\theta}\frac{\delta_2}{\delta_1}\right]$ we need to consider the first order condition for an individual optimum (27)

$$1 - \lambda l = (\beta + \theta\bar{q}_l)\delta_1$$

with the price set at the level $\bar{q}_l$. We then see that in the case of shocks of η, A, δ_2 the equilibrium value of l is unchanged, hence the welfare loss is the same as in the command economy. On the other hand, when δ_2 changes we saw above that with the quantity setting design l changes. By a similar argument as in the case of the abatement choice, we can say that the welfare loss is smaller in the quantity setting design than in the command economy or price setting designs (with no change in l) if

$$\begin{aligned} \left(\frac{\bar{l}}{\delta_1 + \delta_2} - \frac{\beta + \theta(A + \eta)}{\lambda}\right)^2 &< \left(\frac{\beta + \theta(A + \eta)}{\lambda}\right)^2 \\ \left(\frac{\bar{l}}{\delta_1 + \delta_2}\right)^2 &< 2\frac{\bar{l}}{\delta_1 + \delta_2}\frac{\beta + \theta(A + \eta)}{\lambda} \\ 1 - (\beta + \theta(A + \eta))(\delta_1 + \delta_2) &< 2[\beta + \theta(A + \eta)](\delta_1 + \delta_2) \\ 1 &< 3(1 - \bar{l}\lambda) = 3[\beta + \theta(A + \eta)](\delta_1 + \delta_2) \end{aligned} \tag{36}$$

Note that if (35) holds, (36) holds as well, which means that if the magnitude of the change when the supply of infection rights is fixed is smaller than at the optimum (under reaction, though in the 'right' direction), then the welfare loss is smaller in that case. This means that in order to have a smaller welfare loss with no change in l than with the one induced by fixed supply, we must an over -reaction with fixed supply, and a sufficiently big one.

When instead we have an (infinitesimal) change in δ_1, the equilibrium value of l in the price setting design varies, by the amount $-\left(\theta\bar{q}_l+\beta\right)/\lambda=-\left(\beta\left(\delta_1+\delta_2\right)+\theta(A+\eta)\left(\delta_1+\delta_2\right)\right)/\delta_1\lambda$. In this case we have that the magnitude of the change of l in equilibrium is always greater than at the optimum (over-reaction) since

$$\begin{gathered}\frac{(\beta(\delta_1+\delta_2)+\theta(A+\eta)(\delta_1+\delta_2))}{\delta_1\lambda}>\frac{\beta+\theta(A+\eta)}{\lambda}\\ \Leftrightarrow\beta\left(\delta_1+\delta_2\right)+\theta(A+\eta)\left(\delta_1+\delta_2\right)>\delta_1\left(\beta+\theta(A+\eta)\right)\\ \Leftrightarrow\beta\left(\delta_2\right)+\theta(A+\eta)\left(\delta_2\right)>0\end{gathered}$$

always holds. Comparing then the welfare losses in the quantity setting and the price setting designs, they are smaller in the first case if

$$\left(\frac{\bar{l}}{\delta_1+\delta_2}-\frac{\beta+\theta(A+\eta)}{\lambda}\right)^2<\left(\frac{(\beta(\delta_1+\delta_2)+\theta(A+\eta)(\delta_1+\delta_2))}{\delta_1\lambda}-\frac{\beta+\theta(A+\eta)}{\lambda}\right)^2$$

$$\left(\frac{\lambda\bar{l}-[\beta+\theta(A+\eta)](\delta_1+\delta_2)}{(\delta_1+\delta_2)\lambda}\right)^2<\left(\frac{(\beta(\delta_1+\delta_2)+\theta(A+\eta)(\delta_1+\delta_2))-\delta_1(\beta+\theta(A+\eta))}{\delta_1\lambda}\right)^2$$

$$\left(\frac{1-2[\beta+\theta(A+\eta)](\delta_1+\delta_2)}{(\delta_1+\delta_2)\lambda}\right)^2<\left(\frac{\delta_2(\beta+\theta(A+\eta))}{\delta_1\lambda}\right)^2$$

$$\left(1-2\left[\beta+\theta(A+\eta)\right]\left(\delta_1+\delta_2\right)\right)^2\left(\delta_1\right)^2<\left(\delta_2\left(\beta+\theta(A+\eta)\right)\right)^2\left(\delta_1+\delta_2\right)^2$$

$$\begin{aligned}&\left(\delta_1\right)^2+4\left[\beta+\theta(A+\eta)\right]^2\left(\delta_1+\delta_2\right)^2\left(\delta_1\right)^2-4\left(\delta_1\right)^2\left[\beta+\theta(A+\eta)\right]\left(\delta_1+\delta_2\right)<\\&<\left(\delta_2\right)^2\left(\beta+\theta(A+\eta)\right)^2\left(\delta_1+\delta_2\right)^2\end{aligned}$$

$$\left(\delta_1\right)^2+\left[\beta+\theta(A+\eta)\right]^2\left(\delta_1+\delta_2\right)^2\left[4\left(\delta_1\right)^2-\left(\delta_2\right)^2\right]<4\left(\beta+\theta(A+\eta)\right)^2\left(\delta_1+\delta_2\right)^2$$

$$\left(\delta_1\right)^2<\left(\beta+\theta(A+\eta)\right)^2\left(\delta_1+\delta_2\right)^2\left[4(1-\left(\delta_1\right)^2)+\left(\delta_2\right)^2\right]$$

When the quantity setting design is superior to command economy (where there is no change), that is when $\left(\beta+\theta(A+\eta)\right)\left(\delta_1+\delta_2\right)>1/3$, the above condition can be written as

$$13\left(\delta_1\right)^2<\left[4+\left(\delta_2\right)^2\right]$$

always satisfied when $\delta_1,\delta_2<1/2$. Hence under these conditions we conclude that welfare losses are minimal in the quantity setting design.