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Chapter 2

Cultural Transmission and Religion

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Abstract

Based on population dynamics models, the literature on cultural transmission has studied the formation and diffusion of religious traits through evolutionary and bottom-up forces such as parental socialization. This chapter provides a bird's eye view of this approach and its main extensions. We also emphasize two additional dimensions of the cultural dynamics of religious preferences. The first is cultural blending and religious syncretism, namely the fusion of diverse religious beliefs and practices. The second highlights the importance of

purposeful and centralized authorities, such as religious leaders and institutions, that influence the cultural dynamics of religious beliefs and preferences.

Keywords: Religion, Cultural Transmission, Belief Systems

1. How are Religious Beliefs and Preferences Produced?

The basic building blocks of economic models are beliefs, preferences and constraints (Henrich *et al*., 2005). Given enough information about person *i*'s choices, we could characterize her beliefs and preferences and, under certain consistency conditions, anticipate her choices under a different set of constraints. But that would not tell us why person *j*'s choices would differ from *i*'s, why *i*'s choices at date *t* differ from her own choices at date $t + 1$, or how *i*'s choices are influenced by her identity, political affiliation and exposure to role models and "influencers." Consider for example the curious association between the decision to wear a mask during the COVID-19 pandemic and one's political affiliation. These questions require us to go further and investigate how beliefs and preferences are produced.

Bowles (1998) sets out the ways in which fixed preferences limit economic theory:

> But the scope of economic inquiry is thereby truncated in ways which restrict its explanatory power, policy relevance, and ethical coherence. If preferences are affected by the policies or institutional arrangements we study, we can neither accurately predict nor coherently evaluate the likely consequences of new policies or institutions without taking account of preference endogeneity. [p. 75]

When it comes to religion, many important questions force us to think about the origins, persistence and change in religious belief and preferences. For example, suppose we wish to know why group *A* exhibits higher rates of religious participation than group *B*, or why individual *i* converted to religion *k*, or why religious belief rose in country *C*.

Religion is an ideal training ground for theories of belief and preference formation. Belief in supernatural agents has been found in every

known human society. Religious organizations are among the most ubiquitous and long-lived human institutions. Religious participation also has important consequences for economic decisions (Iannaconne, 1998), including education (Becker and Woessman, 2009; Meyersson, 2014), fertility (McQuillan, 2014) and labor market participation (Berman, 2000; Carvalho, 2013). In addition, there is extensive survey data on religious belief, identity and participation from sources such as the *World Values Survey* and The Association of Religion Data Archives (*ARDA*), as well as a large body of anthropological data on religious belief and participation in small-scale societies gathered in sources such as Murdock's *Ethnographic Atlas*.

This chapter focuses on religious belief and preference formation through *cultural transmission*. Before proceeding, however, let us mention three other approaches (see also Bowles, 1998):

1. *Religious Capital*. Iannaconne (1998) models the accumulation of "religious capital" over an individual's lifetime. This religious "appreciation capital" can be thought of as intensity of religious belief or attachment to a religious group, among other things. In this view, religious belief and preferences are cultivated by participation in religious activities. Religious participation contributes to religious capital. In turn, a larger stock of religious capital means a higher preferred level of religious participation. McBride (2015) explores the implications for religious organizations. Even strict religious groups tolerate free-riding by newcomers in anticipation of future contributions, as newcomers accumulate religious capital. The accumulation of religious capital is consistent with two empirical regularities: (i) the best predictor of an individual's religious affiliation and participation is their parents' religious affiliation and participation and (ii) people who switch religions tend to join groups with similar theology and practices (Iannaconne, 1998).
2. *Motivated Beliefs*. In economics, the term "belief" is typically used to denote a probability distribution over states of the world, which can be updated based on empirical evidence. Such beliefs are instrumental in that forming correct beliefs furthers some other objective. When it comes to religion, however, beliefs are largely non-empirical and often ends in themselves. In this sense, the formation of religious beliefs is motivated.

In his famous study of the Trobriand Islanders, Malinowski (1925) points to the anxiety-reducing benefits of ritual:

> *It is most significant that in the Lagoon fishing, where man can rely completely upon his knowledge and skill, magic does not exist, while in the open-sea fishing, full of danger and uncertainty, there is extensive magical ritual to secure safety and good result.*

Similarly, religious belief can be cultivated for its psychological benefits and be part of the "psychological immune system" (Benabou and Tirole, 2002). Fruehwirth *et al*. (2019) find that religious exposure significantly reduces depression in adolescence. Binzel and Carvalho (2017) present a model in which individuals can respond to unfulfilled aspirations by either working harder to catch up or "dropping out" and immersing themselves in religion. Economic shocks can thus lead to sharp rises in religious participation and these religious awakenings can persist long after the economic shock has subsided. Consistent with this, Bentzen (2019) presents evidence of a sharp rise in religiosity after natural disasters. In addition, Chen (2010) finds that communal Qur'an study and Islamic school attendance in Indonesia rose following the 1997–1998 financial crisis.

3. *Socially Adaptive Beliefs*. Forms of religious belief that solve social dilemmas can evolve through processes such as cultural group selection (Gintis, 2003; Henrich, 2004). Sosis and Ruffle (2003) conduct experimental games in religious and secular kibbutzim and find that religious males are significantly more cooperative than secular males. Levy and Razin (2012) show how belief in supernatural punishment in social dilemmas emerges in equilibrium and boosts cooperation, especially within religious groups. Analyzing a database of nineteenth century utopian communes, Sosis (2000) shows that religious communes are more cooperative and longer lived than secular ones. In addition, Norenzayan (2013) proposes that "Big Gods" who monitor and punish transgressions evolved to support large-scale cooperation as societies scaled up. Skaperdas and Vaidya (2020) argue that the move to Big-God religions was a key factor in the development of the modern state.

Based on population dynamics models, the cultural transmission literature on the formation and diffusion of religious traits has focused on

evolutionary and bottom-up forces driving the persistence or homogenization of religious beliefs in a society. This chapter provides a bird's eye view of this approach and its main extensions. We also emphasize two additional dimensions that bring interesting issues in the cultural dynamics of religious preferences. The first one relates to cultural blending and religious syncretism, namely the fusion of diverse religious beliefs and practices. The second one highlights the importance of purposeful and centralized authorities such as religious leaders and institutions that influence the cultural dynamics of religious beliefs and preferences.

The chapter is structured as follows. In Section 2, we present the baseline model of cultural transmission with two cultural traits and introduce endogenous socialization à la Bisin and Verdier (2001). In Section 3, we outline several extensions of this setup and their connection to the cultural dynamics of religious traits. Further extending the model to multi-trait cultural transmission, Section 4 considers the important issue of cultural blending and its application to syncretism of religious traits. Section 5 focuses on the interaction between institutions and cultural transmission, covering in particular recent works analyzing the influence and impact of religious leaders and organizations in the diffusion and persistence of religious traits, as well as the role of religion in the building up of political legitimacy and state power. Section 6 offers a conclusion briefly discussing avenues for future research.

2. The Bisin–Verdier Model of Cultural Transmission

Mathematical models of cultural transmission were pioneered by evolutionary biologists Cavalli-Sforza and Feldman (1981), anthropologists/biologists Boyd and Richerson (1985) and economists Bisin and Verdier (2000, 2001). Religion is an intensely social phenomenon (Iannaconne, 1998), making cultural transmission — the transmission of traits from person to person — a good fit for modeling the formation of religious beliefs and preferences. This contrasts with religious capital models in which appreciation capital is privately accumulated over an individual's lifetime. Social transmission is important both for the distribution of religious traits in a population and choices such as religious participation and education that govern social transmission.

Let us begin with a baseline model of cultural transmission.

2.1 *Intergenerational cultural transmission*

The population is a continuum of agents. Individuals have either cultural trait a or b, which can be two different religions, or two different levels of religiosity with a types being religious and b types being secular.

The population dynamics are highly simplified. We assume that reproduction is asexual and that each parent has one child. So the population is stationary and normalized to $L = 1$. We consider that cultural transmission is the result of *direct vertical* (parental) socialization and *oblique socialization* in society at large. More precisely, each parent (asexually) produces one child, socializes them and then dies. With probability τ_i, a parent with trait $i \in \{a, b\}$ successfully passes on her trait to her child. For the moment, assume each τ_i is exogenous. With probability $1-\tau_i$ however the child remains "unsocialized". He then becomes subject to a second stage of socialization by the social environment of his parent (i.e., oblique transmission). Specifically, he is matched at random with someone from his parent's generation (i.e., oblique transmission) and acquires their trait.

Let q equal the share of a types in the population. The cultural transmission mechanism is then represented by the following system of equations for P^{ij}, the transition probability that a child from a family with trait i is socialized to trait j:

$$P_{aa} = \tau_a + (1-\tau_a)q,\ P_{ab} = (1-\tau_a)(1-q), \tag{1}$$

$$P_{ba} = (1-\tau_b)q,\ P_{bb} = \tau_b + (1-\tau_b)(1-q). \tag{2}$$

Take for instance trait a. The probability P_{aa} for a child of a family of type a to be socialized to that trait includes two terms: the direct vertical socialization probability τ_a plus the indirect oblique socialization probability $(1 - \tau_a)q$ reflecting the fact when he is not successfully socialized by the family in the first stage (with probability $1 - \tau_a$), he is socialized by a similar type a from the population at large with probability q. Conversely, the probability P_{ab} for a child to acquire the other trait b reflects the fact that the child was not successfully socialized by his parent (with probability $1 - \tau_a$) and was exposed to an oblique role model of type b from society at large (with probability $1 - q$).

Using the law of large numbers and continuous time, this process of cultural socialization results in the following cultural dynamic, describing the diffusion of trait a in the population:

$$\dot{q} = \underbrace{(1-q)P_{ba}}_{\text{inflow}} - \underbrace{qP_{ab}}_{\%\text{outflow}}$$
$$= (1-q)(1-\tau_b)q - q(1-\tau_a)(1-q)$$

or

$$\dot{q} = q(1-q)(\tau_a - \tau_b). \tag{3}$$

Equation (3) is actually a simple version of the replicator dynamics in evolutionary biology for a two-trait population dynamic model.[1] $(\tau_a - \tau_b)$ can be interpreted as the relative "cultural fitness" of trait *a* compared to trait *b*. It is a simple matter to see that:

- *Generically, beginning in any interior state* $q(0) \in (0,1)$, *the cultural dynamic ends up in a monomorphic equilibrium* ($q = 1$ *if* $\tau_a > \tau_b$ *or* $q = 0$ *if* $\tau_a < \tau_b$).[2]

Hence the melting pot result of cultural homogenization.

Given the persistent diversity we observe, how can a polymorphic cultural equilibrium be generated? One way to generate long-run persistence of cultural diversity can be obtained by introducing the possibility of cultural transmission rates which are frequency dependent.

For instance, Boyd and Richerson (1985) consider situations where the commonness or rarity of a trait affects the probability of its transmission more (or less) than proportionally. Typically when individuals are predisposed to adopt the behavior of a larger group, this frequency-dependent bias generates conformity. On the opposite, when individuals tend to adopt more than proportionally traits that are associated with smaller groups, there is an anti-conformist bias.[3] It is then a simple matter

[1] In terms of Cavalli-Sforza and Feldman (1981) and Boyd and Richerson (1985) terminology, this model reflects a cultural transmission process that is equivalent to a linear transmission model characterized by "direct biased" transmission $\tau_a - \tau_b$ between the two cultural variants *a* and *b* of our trait.

[2] If $\tau_a - \tau_b$, $q(t) = q(0)$ for all *t*.

[3] Boyd and Richerson (1985) and Henrich and Gil-White (2001) also identify as "indirect bias" the situation where individuals may also use a cue about one trait (wealth, prestige)

to see that the "melting pot" homogenization result still prevails under conformity bias. On the contrary, an anti-conformist bias preserves cultural diversity in the population.[4]

In the cultural evolution literature, the nature of frequency-dependent bias is assumed to be fixed in the short run and to be subject to genetic evolution in the long run. In particular, it is generally argued that it can be genetically adaptive for individuals to develop frequency-dependent bias such as conformity and indirect bias associated with prestige (Boyd and Richerson, 1985: Henrich and Gil-White, 2001; Panchanathan, 2010). Such co-evolutionary explanations of frequency-dependent transmission rates however necessitate selective forces spanning over long periods of time (many thousands of years). For our purpose of the cultural transmission of religious traits, these genetic processes should probably be best considered as fixed and therefore can hardly explain cultural phenomena of religious resistance/resilience or disappearance due to changes in the social and economic environment under much shorter time scales. As we will see in Section 2.2, bringing a "short/medium" term ~~evolutionary socioeconomic~~ perspective allows cultural transmission rates to be endogenously frequency dependent.

2.2 *An economic model of cultural transmission*

2.2.1 *Endogenous socialization*

Let us now introduce some economics. We bring two important features to the previous framework. The first element is that transmission rates across generations can be the result of costly and purposeful actions by socializing agents. The second feature (related to the first one) is the

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to choose which role model to observe in order to acquire information about another trait.

[4]Formally, the direct socialization probabilities may be written as

$$\tau_a = f_a(q),\ \tau_b = f_b(1-q),$$

where a conformity bias is captured by the fact that $f_i(.)$ is an increasing function of its argument and $f_i(0) = 0$. Conversely, an anti-conformist bias is reflected by a decreasing function $f_i(.)$, with $f_i(1) = 0$. The cultural relative fitness $\tau^a - \tau^b = \Theta(q)$ is now a function reflecting the "frequency-dependent bias" associated to the cultural variant a.

With a conformity bias, the cultural dynamic converges again monotonically to a monomorphic equilibrium $q = 0$ or $q = 1$, while with an anti-conformity bias, it converges monotonically to the polymorphic state $q^* \in (0,1)$ satisfying the condition $\Theta(q^*) = 0$.

question of the motivation for agents to undertake such costly socialization efforts.

Bisin and Verdier (2001) introduce agency into the previous model of cultural transmission. In particular, they allow a choice of socialization effort, thereby making the transition probabilities endogenous. Parents can choose how intensively to socialize their children, at some cost, through (1) teaching, (2) school choice, (3) residential choice, (4) homogamy, and other costly actions.

To model socialization choice, parents need to have preferences over the traits that their children can acquire. Bisin and Verdier take an approach they call *imperfect empathy*: parents evaluate their children's behavior based on their own preferences. Formally, a parent with trait i gets a payoff of V_{ij} if her child acquires trait j, where $V_{ii} > V_{ij}$ whenever $i \neq j$.

A parent with trait a in state q has payoff function:

$$U^a(q) = [\underbrace{\tau_a + (1-\tau_a)q}_{P_{aa}}]V_{aa} + \underbrace{(1-\tau_a)(1-q)}_{P_{ab}}V_{ab} - c(\tau_a). \tag{4}$$

She chooses socialization effort τ_a at cost $c(\tau_a)$ to maximize this function.[5] A parent with trait b in state q has payoff function:

$$U^b(q) = [\underbrace{\tau_b + (1-\tau_b)(1-q)}_{P_{bb}}]V_{bb} + \underbrace{(1-\tau_b)q}_{P_{ba}}V_{ba} - c(\tau_b). \tag{5}$$

Before deriving optimal socialization efforts let us define the notion of "cultural intolerance." Type i's cultural intolerance is denoted by Δ_i, where

$$\Delta_a = V_{aa} - V_{ab} \text{ and } \Delta_b = V_{bb} - V_{ba}.$$

The first-order conditions for a type and b type are, respectively,

$$(1-q)\Delta_a = c'(\tau_a) \text{ and } q\Delta_b = c'(\tau_b), \tag{6}$$

and population dynamics are given by (3) except that now τ_i is endogenous and given by (6). We have the following proposition.

[5] $c(\tau)$ is supposed to be an increasing convex function with the Inada conditions: c(0) = c′(0) = 0 and $\lim_{\tau \to 1} c(\tau) = \lim_{\tau \to 1} c'(\tau) = +\infty$.

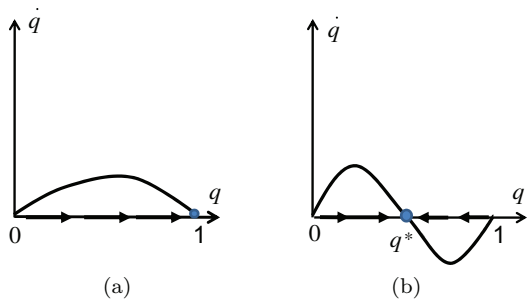

Figure 1: Cultural Transmission Models. (a) Cultural Dynamics, Cavalli Sforza-Feldman (1981) (Case: $\tau_a > \tau_b$). (b) Cultural Dynamics, Bisin and Verdier (2001) (Cultural substitution)

Proposition 1 (Bisin and Verdier, 2001). *Optimal socialization effort varies in the following manner*:

(i) τ_i *is strictly increasing in* "*cultural intolerance*" Δ_i, (ii) τ_a *is strictly decreasing in* q, (iii) τ_b *is strictly increasing in* q, (iv) $\tau_a > \tau_b$ *if and only if* $q < \frac{\Delta_a}{\Delta_a+\Delta_b}$, (v) *From any interior state* $q(0) \in (0,1)$, *the dynamic converges to* $q^* = \frac{\Delta_a}{\Delta_a+\Delta_b}$.

First, higher degrees of cultural intolerance mean more intensive socialization. Second, the smaller a cultural group the more it expends on socialization effort. Third, cultural group i socializes more intensively than group j if its share of the population q is less than its relative cultural intolerance, $\Delta_i/(\Delta_i + \Delta_j)$. If the cultural intolerances are the same across groups, then the minority group will exert greater socialization effort. Otherwise, an intolerant majority can expend more on socialization than a more tolerant minority. Consequently, a *polymorphic* cultural distribution emerges from almost every initial state whenever cultural intolerance is positive for each type.

Introducing endogenous socialization effort in (1) qualitatively changes the population dynamics. It generates an *endogenous* anti-conformist bias that maintains cultural diversity in the population. Figures 1(a) and 1(b) illustrate the starkly different population dynamics in the leading models proposed by Cavalli-Sforza and Feldman (1981) and in the benchmark model of Bisin and Verdier (2001).

As pointed out by Bisin and Verdier (2001), the cultural transmission mechanism described in (1) satisfies the property of *cultural substitution*, by which role models inside the family (direct vertical transmission) act

as *cultural substitutes* to role models outside the family (oblique or horizontal transmission).[6] In such a case, parents have less incentives to socialize their children the more widely dominant are their values in the population. Consequently, as a trait begins to die out in the society at large, parents with that trait socialize more intensively, and this in turn keeps the cultural dynamic away from the boundaries $q = 0$ and $q = 1$. Conversely, when parental role models act as *cultural complements* to other social role models, ~~parents' efforts of socialization are~~ larger the more frequent their trait in the population, providing therefore a transmission force pushing towards cultural homogeneity.[7]

In general, both cultural substitution and complementary effects may be present in the way role models (parental and social) tend to interact to influence children. The relative strength of these effects then shapes the cultural dynamic in society, eventually leading to the existence of multiple long-run possible cultural steady states towards which the society converges, depending on its initial conditions.

Summarizing, the endogenous cultural transmission model in Bisin and Verdier (2000, 2001) allows for population dynamics of the distribution of cultural traits which converge to a heterogeneous distribution. This can explain the observed resilience of ethnic and religious traits.

There is evidence for the endogenous socialization hypothesis. Cohen-Zada (2006) finds that rates of religious schooling for religious minorities in the United States are decreasing in their population share. Bisin and Verdier (2000) present an alternative model of socialization with sexual reproduction in which homogamy increases the likelihood that one's child acquires one's cultural trait. The results are similar. Bisin *et al*. (2004) present evidence consistent with both socialization and homogamy channels from the United States. Using General Social Survey (GSS) data and examining variation across US states from 1972 to 1996, they find that religious minorities socialize more intensively and exhibit higher rates of homogamy than majorities. Calibrating the model and simulating the population dynamics, they show that minority religion shares stabilize at higher levels than predicted by linear extrapolations.

[6]Denote $q_i \in [0, 1]$, the frequency of trait $i \in \{a, b\}$ in the population (i.e., $q_a = q$, $q_b = 1 - q$). Bisin and Verdier (2001) formally define *cultural substitution* as τ_i of a parent of type $i \in \{a, b\}$· (for any $\Delta_i > 0$, $\tau_i(q_i, \Delta_i)$ is a continuous, strictly decreasing function in q_i, and moreover, $\tau_i(1, \Delta_i) = 0$.)

[7]Bisin and Verdier (2001) provide examples of such transmission processes.

While the evidences offered by Bisin *et al*. (2004) and Cohen-Zada (2006) are consistent with a cultural substitution effect between direct and oblique socialization, Patacchini and Zenou (2011) on the other hand present evidence of complementarity in the UK. This suggests that the type of relationship between direct and oblique religious socialization is probably dependent on population conditions and therefore consistent with a multiple steady-state model.

3. Generalizations

One advantage of the economic model of cultural transmission we outline above is the fact that it is versatile enough to allow extensions along several dimensions important for the evolution of religious traits, such as fertility decisions, spatial or social segregation, identity formation and more generally, the possibility of socioeconomic interactions. We turn to these extensions below.

3.1 *Endogenous fertility and cultural transmission*

There is a considerable body of literature in the social sciences that highlights the association between religion and fertility. For instance, it is well known that even controlling for income and education, religious people have more children on average than secular people (Blume, 2009; Frejka and Westhoff, 2008). Historical studies of Western Europe also suggest that fertility declines are often related to differences in religious affiliation and involvement (Anderson, 1986; Derosas and van Poppel, 2006). Studies in the United States highlight as well religious differences in fertility, mainly between Catholics and Protestants (Bouvier and Rao, 1975; Gutmann, 1990; Parkerson and Parkerson, 1988). In developing societies, religious fertility differentials have also been commonly observed, with a particular focus on Muslim-Christian fertility differences (Bailey, 1986; Johnson and Hanks, 2006; Heaton, 2011; Dharmalingam and Morgan, 2004; Jayasree, 1989; Johnson, 1993; Knodel *et al*., 1999; Morgan *et al*., 2002). Regardless of denominational affiliation, people expressing higher religiosity usually tend to have higher fertility and lower contraceptive use (Brewster *et al*., 1998; Goldscheider and Mosher, 1991; Philipov and Berghammer, 2007; Hayford and Morgan, 2008; Zhang, 2008), or tend to

favor formal marriage over cohabitation, which in turn, leads to higher fertility (Berghammer, 2012).

Importantly enough, when it is determined as an endogenous choice of parents, fertility decisions naturally interact with socialization decisions if for no other reason than that socialization costs naturally increase with the number of children to socialize. Consider then our previous model of cultural transmission and extend it to allow for some parental choice of reproductive pattern. Specifically, let $n_i \geq 0$ denote the number of children chosen by parents with trait i, at cost $m \cdot N_i$ (where m is the cost of raising one child). Assuming for simplicity that socialization costs are linear in n_i, and therefore that parents of type i choose $\tau_i \in [0, 1]$ and $n_i \geq 0$ to maximize

$$n_i(P_{ii}V_{ii} + P_{ij}V_{ij}) - n_i c(\tau_i) - m \cdot n_i, \tag{7}$$

where P_{ii} and P_{ij} are as in (1). The dynamics of the distribution of traits in the population is then determined by

$$\dot{q} = q(1-q)\frac{(\tau_a \nu_a - \tau_b \nu_b)}{\nu_a q + (1-\nu_a)(1-q)},$$

where $\nu_i = \frac{n_i}{n_a + n_b}$ and τ_i are determined at equilibrium for $i \in \{a, b\}$.

Bisin and Verdier (2001) point out that the choice of reproduction patterns tends to introduce a cultural complementarity force in the cultural transmission process (i.e., $n_a(q)$ is increasing in q and $n_b(1 - q)$ increasing in $1 - q$). As a matter of fact, parents endowed with a more frequent trait have a higher chance, everything else being equal, to get their children sharing their trait through society's socialization. Such an outcome is perceived by these paternalistic parents as increasing the quality of children, motivating higher fertility rates and consequently more effective cultural transmission of their trait in the population. When fertility interacts with direct socialization more generally, total socialization costs increase with the number of children, and hence parents, when choosing direct children socialization, incur a classic quantity/quality (of children) trade-off. In such a case, as Bisin and Verdier (2001) show, the quantity/quality trade-off is sufficient to re-establish the dynamics associated to cultural substitution, over-riding the cultural complementarity due to endogenous fertility.

Bar-El *et al*. (2013) consider numerical simulations of a version of the previous model to investigate the evolution of secularization in a society. In their framework, the presence of both cultural complementarity and cultural substitution effects eventually leads to multiple steady-state levels of secularization in the population. Also, around a stable steady state, the model predicts a non monotonic parabolic dependence of the current level of secularity on the past level of secularity.[8] The authors provide suggestive evidence of this parabolic relationship using data from the International Social Survey Program (ISSP 1998 National Identity module: Religion II) that collects information on attitudes, religious beliefs and religiosity measures (private prayer habits and Church attendance) in 32 countries.

Bar-Gill and Fershtman (2016) extend the Bisin and Verdier model of cultural transmission with endogenous fertility to the case where paternalistic preferences for transmitting one's own trait are not separable across children. Individuals from different cultures may have different intolerance to having only some of their children adopting different cultural identities. In such a situation, the conversion of one child affects the paternalistic motives associated with another child. They consider two opposite situations: one in which the emphasis is on having at least one child that remains loyal to the parents' religious group (described as a "survivalist type"); the other in which the emphasis is on having all the children maintaining the group's religious trait (described as a "zealot type"). In this context, individuals' fertility decision and their direct socialization effort crucially depend on their degree of zealousness. In this context, they show that integration policies aimed at promoting the cultural conversion of minority groups to the trait of the majority may be ineffective and even result in more resilient and larger minority groups. For instance, whenever individuals are of the "survivalist" type, an integration policy may induce higher fertility rates as individuals may switch from having one child with a high (and costly) direct socialization level to having two children with much lower direct socialization level. This change may result in a larger minority size whenever the effect of higher fertility is stronger than the effect of lower direct socialization.

[8]Up to some level, an increase in secularization in the past increases secularization in the present, while above this level, one finds a negative relation between secularization in the past and the present.

3.2 *Self-segregation, isolation and network homophily*

Sociologists and social psychologists have widely acknowledged the fact that people have significant contact with others like themselves, and that social interactions are characterized by homophily (that is the fact that contact between similar people occurs at a higher rate than among dissimilar people) (McPherson *et al*., 2001). Concerning religious characteristics, this implies that people of different faiths tend to form relatively isolated communities socially or spatially to preserve the persistence of their cultural values. Extreme examples run from the case of the Hutterite and Amish communities who strongly emphasize separation from non-Amish world (including even a reluctance to adopt modern conveniences such as electricity) (Kraybill and Bowman, 2001) to the Haredi (ultra-Orthodox) Jews whose communal self-awareness is found in the clear tendency to avoid the larger society and form sizable enclaves in major cities around the globe, including New York and London (Flint *et al*., 2013; Shilhav, 1993; Valins, 2003). In such examples, expression of religious lifestyles motivates voluntary territorial separation, which allows religiously based lives to remain cut off from external influences and safeguards the younger generation from the perceived threats of secular culture. More generally, religious homophily (often correlated to ethnic homophily) is recognized as significant (Fischer, 1977, 1982; Marsden, 1988; Kalmijn, 1998; Hu *et al*., 2019), especially among individuals with high religiosity (Windzio and Wingens, 2014; Smith *et al*., 2014, 2016; Leszczensky and Pink, 2017).

From a cultural transmission perspective, this dimension implies that socialization at large with society is unlikely to be fully random. In particular the matching process through which children get socialized may be partly controlled by their parents (or other role models) through their choice of schools, neighborhood (where to live), associations or clubs (where to go), and so on. Also, children themselves may choose peer connections in a non-random way, reinforcing or mitigating the bias that parents produced in the first place.

In the benchmark socialization model that we introduced, oblique transmission occurred through random matching with society at large. One may however extend this framework to include the possibility that the cultural composition of society children get exposed to is at least partly under the control of parents. Abstracting from specific details of

the contact process, the transmission probabilities could be generally written as

$$P_{ii} = \tau_i + (1-\tau_i)Q_i,$$
$$P_{ij} = (1-\tau_i)(1-Q_i), \tag{8}$$

where the composition of the social environment of the child, Q_i, is itself a function of the population shares q^i and a costly parental intervention, say, s^i In such a case, the effective "technology" of parental socialization is multi-dimensional, involving various margins of transmission: time, effort and social or spatial segregation decisions. Examples of models along these lines are analyzed in Bisin and Verdier (2001) and Saez-Marti and Sjogren (2008). The marriage segmentation model analyzed in Bisin and Verdier (2000) is also an example of segregation strategies that affect cultural transmission. Other examples consider explicitly a social network structure and analyze how the topology of social connections matters for the cultural dynamics (Buechel *et al*., 2014; Panebianco, 2014; Panebianco and Verdier, 2017; Verdier and Zenou, 2017).

An interesting relevant extension in the context of the cultural transmission of religion is Patacchini and Zenou (2016). It develops a theoretical framework in which parents' involvement in religious activities as well as the peers' influence on the children are the key ingredients in explaining religious outcomes. Contrary to the benchmark model of Bisin and Verdier (2001) where peer effects are conceived as an average intra-group externality that affects identically all the members of a given group, peer effects are only supported by the structure of active bilateral connections or dyads that constitute the social network in which individuals are embedded. In their model, the convexity or concavity of the parental socialization cost function crucially affects whether there is cultural substitutability or cultural complementarity between parental socialization intensity and peers' influence. They test their model using a very detailed dataset of adolescent friendship networks in the United States (the National Longitudinal Survey of Adolescent Health (AddHealth)). Interestingly, they find that, for religious parents, the higher is the fraction of religious peers, the more the effort put in by parents in transmitting their religiosity, indicating cultural complementarity. For non-religious parents, they obtain the reverse result of cultural substitutability,

suggesting that the technology of socialization can be strongly trait dependent (religious versus non-religious).

3.3 *Religious identity and cultural transmission*

The economic model of cultural transmission can also be extended to incorporate insights from the economics of identity formation (Akerlof and Kranton, 2000). This might be particularly relevant for the issue of religious identity. As such, religious identity is anchored in a system of guiding beliefs and symbols and refers to how individuals develop their personal sense of religious meaning and/or spirituality over the course of their lifetimes. Social psychologists and sociologists acknowledge that religion serves a uniquely powerful function in shaping psychological and social processes (Ysseldyk *et al*., 2010; Coyle and Lyons, 2011). Also, it is widely recognized that the formation of religious identity typically occurs within family and community contexts (Goodman and Dyer, 2020), although it is also admitted that some dimensions of its transmission relate both to genetic and cultural factors (White *et al*., 2011).

As for other types of social identity (race, gender or political), the formation of religious identity can be analyzed through two somewhat opposite lenses. A first perspective argues that group identity is driven by a motive for inclusiveness and *cultural conformity*, and therefore, that identity is reduced by assimilation and contact across cultures.[9] The alternative view suggests that cultural minorities are motivated in keeping their own distinctive heritage to generate a sense of positive *distinctiveness* or *cultural distinction*[10] from individuals who are part of that group.[11] When identity formation is characterized by cultural distinction, social interactions across groups might induce the formation of oppositional or radical identities on the part of specific groups. In the context of religious

[9] *Assimilation theories*, in political science and sociology (Gordon, 1964; Moghaddam and Solliday, 1991), *contact theory* in social psychology (Allport, 1953) are the prominent theories of this line of thought.

[10] *Cultural distinction*, as defined here, is a property of individual preferences. It is related but distinct from *cultural substitution* (see Section 2), which is a property of socialization mechanisms.

[11] These ideas have been expressed by the theories of *multiculturalism* (Glazer and Moynihan, 1970). At a broader level, this view is also related to the *social identity theory* in social psychology (Tajfel, 1981; Turner, 1982; Abrams and Hogg, 1988).

identities, this may explain the existence of religious radicalization and its associated social frictions.

For instance, Bisin *et al*. (2011) present a model where an individual's identity is a personal choice that depends on the cultural composition of the neighborhood in which he is raised and his personal negative experiences related to interacting with individuals not sharing his own cultural trait. This dimension is embedded into a cultural transmission model in which parents decide how much to invest in socializing their children to their own trait, anticipating the possible peer effects favoring society's influence and their children's future identity choice. The analysis shows that both cultural substitution and cultural distinction jointly induce resilience and persistence of minoritarian traits, therefore preserving cultural diversity in the long run. Interestingly, the prevalence of an oppositional (radical) culture in a minority group can be sustained when there is enough cultural segmentation in terms of role models, the size the group is significant enough, the degree of radicalization it implies is high enough, and the socioeconomic opportunity cost of the actions it prescribes is small enough.

Using data on ethnic preferences and attitudes provided by the Fourth National Survey of Ethnic Minorities in the UK, Bisin *et al*. (2016) find evidence that might be consistent with intense ethnic and religious identity mostly formed as a cultural distinction mechanism. Consistently, they document that such identities might be more intense in mixed than in segregated neighborhoods.

3.4 *Cultural transmission and social interactions*

In the cultural transmission models we described so far, parental socialization depends on the parents' relative value of having a child with the same cultural trait as theirs, Δ_i, which we referred to as the *cultural intolerance* of trait i. In fact, the Δ_is have been treated as exogenous preference parameters in the frameworks we surveyed up to this point. In many contexts of interest, however, this is too restrictive an assumption. The endogeneity of Δ_i can originate in many different environments. For instance, when individuals interact in markets, their indirect utility may depend on economic variables such as prices and incomes or policy outcomes that depend on the type of society and therefore the distribution of cultural traits that prevails in such society. Similarly, in the contexts of strategic and matching interactions, the payoffs that an individual may obtain is likely to be influenced by

the distribution of cultural traits in the population. In the religious context, it is clear that the degree of religious intolerance of an individual towards other people not sharing his worldview may be influenced (positively or negatively) by the behavior of these individuals and the frequency of their encounters. Also, in a religious community, the Δ_i may be manipulated (opportunistically or not) by community leaders reacting to the social environment in which this community is embedded. In all these situations, it is therefore reasonable to expect cultural intolerance, Δ_i, to be endogenous.

While the implications of the endogeneity of ΔV^i for socialization and population dynamics need to be derived case by case, a reduced form analysis is useful to clarify what to look for in the applications. Suppose, for instance, that each individual (parent or child) chooses $x \in X$ to maximize $u_i(x, q_i)$ for $x_i \in \{a, b\}$, with $x_i = \arg\max_{x \in X} u_i(x, q_i)$ the optimal behavior associated with trait i. Under *paternalism*, direct parental socialization for types i depends on $\Delta_i(q_i) = u_i(x_i, q_i) - u^i(x_j, q_i)$. The first fundamental implication of the endogeneity of Δ_i is the following:

When cultural intolerance Δ_i depends on q_i, paternalism does not necessarily imply that $\Delta_i(q_i) \geq 0$.

In fact, socialization to the parents' trait might put the children at a disadvantage in the child social environment, represented by q_i. While *paternalism or cultural intolerance* is manifested as a preference on the part of parents for sharing their cultural traits with their children, such a preference depends on the economic and social conditions which parents expect for their children. Different economic and social conditions could, in principle, lead parents to socialize their children to a trait different than their own.

Furthermore, when cultural intolerance is endogenous, the dynamic system for the evolution of cultural traits can be written as

$$\dot{q} = q(1-q)[\tau_a(q, \triangle_a(q)) - \tau_b(1-q, \triangle_b(1-q))].$$

While full *cultural substitution* (after integrating the fact that the paternalistic motives $\Delta_i(q_i)$ are frequency dependent) is still sufficient to guarantee population dynamics which converge to cultural heterogeneity, an additional condition on $\Delta_i(q_i)$ is necessary to produce direct socialization maps $\tau_i(q_i) = \tau_i(q_i, \Delta_i(q_i))$ satisfying such property. Bisin and Verdier (2010) denote this assumption as *strategic substitution*.

Strategic substitution: *The social environment is characterized by strategic substitution if*

$$\frac{\partial}{\partial q_i}\Delta_i(q_i) < 0.$$

It is easy to see then that if direct and oblique socialization mechanisms are culturally substitutes then the following holds:

- *In a social environment characterized by strategic substitution, from any interior state* $q \in (0, 1)$, *the cultural dynamic converges to a cultural polymorphism* q^*, *where* $0 < q < 1$.

Strategic substitution guarantees that cultural minorities will face relatively larger gains from socialization, independent of the socialization mechanism. In the case of strategic complementarity, on the contrary, cultural minorities face smaller (even possibly negative) socialization gains. Therefore, depending on the strength of cultural substitution in this case, cultural heterogeneity might or might not be preserved.

In the context of religious traits, $\Delta_i(q_i)$ may depend on the distribution of traits in the population when the nature of the religious trait matters for socioeconomic interactions between individuals in society. This happens, for instance, when the person endowed with that trait is subject to various types of socioeconomic discrimination in areas such as the labor market, access to education or entrepreneurial capital or choices of housing location. An example of this which has recently attracted quite some attention by social scientists concerns the issue of socioeconomic integration of Muslim minorities in western countries (Adida *et al*., 2016). The way $\Delta_i(q_i)$ might vary with q_i depends on the circumstances. For instance, in many social contexts, an increase in q_i may reduce the degree of social and economic discrimination of individuals endowed with that trait. Indeed a larger set of individuals sharing trait i also means more possibilities to trade and interact within that set, reducing therefore the need to interact with others and, in turn, the cost of socioeconomic discrimination by the rest of society.[12] Given such reduced discrimination, the perceived

[12]Likewise, in the context of labor markets when employers' hiring is based on a group-based statistical inference of some non-observable individual characteristic, a higher frequency q_i of individuals' sharing the group trait i may generate additional variation in the

economic cost to transmit the trait intergenerationally is also reduced, increasing therefore the paternalistic motive $\Delta_i(q_i)$ that parents have to transmit their trait. In such a case, there is *strategic complementarity*, which in itself tends to promote a force towards assimilation and a reduction of long-run religious diversity.

Conversely, an increase in the frequency of a religious group q_i may also induce a lower paternalistic motive $\Delta_i(q_i)$ when discrimination has an endogenous taste or identity component. Indeed in such a case, a higher value of q_i may increase the frequency of intergroup contacts. In the context of *cultural distinction*, this triggers negative reactions of other groups to reaffirm their differences. Such reactions may imply in turn increased actions of discrimination and social segmentation against individuals of group i and consequently a lower paternalistic motive $\Delta_i(q_i)$ for the trait to be transmitted. The resulting *strategic substitution* effect leads smaller minority groups to have higher incentives to transmit their traits, driving therefore in itself a force for the preservation of religious diversity.

Other channels through which $\Delta_i(q_i)$ may be endogenously determined reflect the idea that cultural transmission does not occur in an institutional vacuum. Indeed, rather than having only families, oblique and peer contacts as decentralized agents of socialization, the cultural dynamic of religious traits often benefits from various institutional organizations (such as community organizations, churches, sects and leaders) that help coordinate and monitor the efforts of a group to maintain over generations the prevalence of its religious characteristics. We turn to some of these dimensions in Section 5.

4. Multiple Religious Traits, Cultural Blending and Syncretism

An important feature of cultural evolution is the phenomenon of cultural blending, namely the fact that, through social contacts and interactions, individuals endowed with different cultural traits create new traits mixing the characteristics of their initial cultures. In linguistics and cultural anthropology, this process is also referred as creolization (Cohen, 2007;

individual characteristic, reducing the relevance of the statistical inference based on the group observable trait i. This in turn may reduce the significance of statistical discrimination.

Stewart, 2016) or hybridization (Nederveen Pieterse, 1994), the mechanism through which creole languages and cultures emerge.

In the context of religions, the analog is religious syncretism, namely the fusion of diverse religious beliefs and practices. Examples of religious syncretism abound in historical times and parts of the world. Historically, the fusion of cultures brought by the conquest of Alexander the Great and then the Roman Empire, tended to bring together a variety of religious and philosophical views that resulted in a strong tendency toward religious syncretism. A well-known example is Gnosticism, a religious dualistic system mixing elements from Eastern mystery religions, Judaism, Christianity and Greek philosophical concepts, and which originated in the late first century AD in non-rabbinical Jewish sects and early Christian sects (Albrile, 2005). In the same vein, emanating from Islam, the Barghawatas in Morocco mixed Sunni, Shi'ite and Kharijite Islamic concepts with elements of astrological and traditional Berber mythology (Le Tourneau, 1986), or the Abangan in Java provided through their Kejawen (Javanism) an amalgam of Islamic beliefs, and other animistic, Buddhist and Hindu aspects (Geertz, 1976). In East Asian societies, notable syncretization of Buddhism with local beliefs includes the Three Teachings, or Triple Religion, that harmonizes Mahayana Buddhism with Confucian philosophy and elements of Taoism, and Shinbutsu-shūgō (Dumoulin, 1976). Similarly various forms of cultural creolization involve religious syncretic dimensions, such as Caribbean Vodou (combining elements of Western African, native Caribbean and Roman Catholic beliefs), Jamaican Rastafari (mixing features from the Bible, Pan-Africanism, Hinduism and Caribbean culture), or Brazilian Candomblé (syncretism between traditional Yoruba religion of West Africa and Roman Catholicism).

How can religious blending and/or emergence of new religious forms be analyzed through the lens of our cultural transmission framework? The natural way to do this is to enrich the space of possible cultural dimensions that can be transmitted.

4.1 *Syncretism and the n-trait Model*

A first step in this direction is to extend the Bisin–Verdier framework to *n* traits (Bisin *et al*., 2009; Montgomery, 2010). Interestingly, this will also confirm its deep relationship to evolutionary game theory. Specifically, based on Montgomery (2010), we will see that when cultural intolerances

are given an appropriate interpretation, the Bisin–Verdier cultural dynamic is the standard replicator dynamic.

Consider again that agents form a continuum. Cultural traits are denoted by $i \in \{1, \ldots n\}$. Each parent (asexually) produces one child, socializes them and then dies. A parent with trait i will have a child with trait $j \neq i$ with probability

$$P_{ij} = (1-\tau_i)q_j \tag{9}$$

and a child with trait i with probability

$$P_{ii} = \tau_i + (1-\tau_i)q_i. \tag{10}$$

In discrete time, the share of trait i is given by

$$q_i(t+1) = \sum_j q_j(t)P_{ji}. \tag{11}$$

Substituting (9) and (10) into (11) and taking the continuous-time limit, we find

$$\dot{q}_i = q_i\left[\tau_i - \sum_j q_j\tau_j\right] \tag{12}$$

for all $i = 1, \ldots n$.

Clearly, when the τs are exogenous, the dynamic converges from every interior state to a monomorphic distribution centered on trait $\arg\max_i \{\tau_i\}_{i=1}^n$. So let us proceed along the lines of Bisin and Verdier (2000) except with n traits and a quadratic socialization cost:

$$\max_{\tau_i} \sum_j P_{ij}V_{ij} - \frac{1}{2}(\tau_i)^2, \tag{13}$$

where V_{ij} is an i type's payoff from having a child with trait j.

The first-order condition is

$$\tau_i^* = \sum_j q_j\Delta_{ij},$$

where Δ_{ij} is an i type's intolerance toward j.

Substituting into the dynamic (12), we have

$$\dot{q}_i = q_i[\sum_j q_j \Delta_{ij} - \sum_j q_j \sum_j q_k \Delta_{jk}] \quad (14)$$

for all $i = 1, \ldots n$.

Interpret Δ_{ij} as the payoff from playing strategy i against j. This links the dynamic system (14) to evolutionary game theory (Smith, 1982; Young, 1998; Sandholm, 2010). More specifically, (14) is the replicator dynamic operating on random matching to play the $n \times n$ game composed of the payoffs Δ_{ij}. The replicator dynamic can arise from natural selection, imitation and reinforcement learning (see the Appendix). Thus, there is a deep connection between the cultural transmission framework and other evolutionary processes. Moreover, we can exploit a large body of results about the replicator dynamic to study cultural evolution.

Specifically, suppose that $\Delta_{ij} = \Delta_i$ for all $j \neq i$ (and $\Delta_{ii} = 0$), that is, each group is intolerant of all other traits to an equal degree. Then, this is a *strictly stable* game. There is a unique Nash equilibrium (distribution of traits), $q_i = \frac{1}{n}$, which is globally asymptotically stable and every trajectory of the replicator dynamic in the interior of the n-dimensional simplex converges to this state. Hence again we have persistent diversity.

AQ01

- **The three-trait example applied to religious syncretism**

Our extended setup may provide some insight into the conditions under which religious syncretism emerges and persists. The simplest way to do so is to consider a three-trait version of this model in which the first two traits (1 and 2) are two specific religious traits and trait 3 is a mixture of them. Denote by q_1, q_2 and q_3 the frequencies of traits 1, 2 and 3, respectively, with $q_1 + q_2 + q_3 = 1$. Following Montgomery (2010), the "cultural payoff" matrix of the evolutionary game associated with the cultural transmission process is

$$\Delta = \begin{Bmatrix} 0 & \Delta_{12} & \Delta_{13} \\ \Delta_{21} & 0 & \Delta_{23} \\ \Delta_{31} & \Delta_{32} & 0 \end{Bmatrix},$$

where Δ_{ij} is the "cultural intolerance" of trait i for trait j.

To be more specific, we are interested in religious syncretism in which two religions 1 and 2 give rise to a mixed trait 3. Assume first that religions 1 and 2 are symmetric in the sense that they have the same degree of intolerance with respect to each other: $\Delta_{21} = \Delta_{12} = \theta > 0$. In the context of syncretism, it appears natural to also assume that these "pure" religions 1 and 2 are less intolerant towards the syncretic mixed trait than towards each other: $\Delta_{13} = \Delta_{23}\ \lambda\theta$, with $\lambda < 1$. Conversely, the syncretic trait is also more tolerant towards each of the "pure" religious traits 1 and 2 than they may be towards the syncretic trait: $\Delta_{31} = \Delta_{32}\ \mu\theta$, with $\mu < \lambda$ to reflect the fact that the syncretic trait is less exclusive than each of the pure religions.

Consider then the situation where religions 1 and 2 coexist in a cultural steady state and syncretism has not yet appeared. In such a case, we are back to the standard two-trait model of Bisin–Verdier, and we have (because of the symmetry) the long-run cultural steady state

$$q_1^* = 1 - q_2^* = \frac{1}{2}$$

and the expected payoff π^* (or expected cultural fitness in the cultural dynamics interpretation) of the two religious traits is

$$\pi^* = \frac{1}{2} \cdot \theta .$$

When is it that syncretism between religions 1 and 2 appear? For this, consider the "cultural fitness of trait 3" at the previous cultural steady state with the population equally divided between the pure traits 1 and 2. Then, the syncretic trait 3 has a chance to diffuse in the population if and only if its expected cultural fitness at this population state is larger than the cultural fitness of any of the two other religious traits, namely

$$q_1^* \Delta_{31} + q_2^* \Delta_{32} > \pi^* .$$

In this simple case, this condition reduces to $\mu > \frac{1}{2}$, that is, the syncretic trait can only diffuse when it embodies enough "exclusivity" towards the two already present traits. Now, after such cultural invasion of the mixed trait, where does the cultural dynamics go? And specifically do we observe in the long run the presence of the three religious traits with $q_i^* > 0$ for $i \in \{1, 2, 3\}$ (that is an interior religious polymorphism)?

Considering general asymmetric situations for the structure of the cultural payoff matrix Δ, Montgomery (2010) defines the following quantities:

$$\Psi_1 = \Delta_{12}\Delta_{23} + \Delta_{13}\Delta_{32} - \Delta_{23}\Delta_{32},$$
$$\Psi_2 = \Delta_{21}\Delta_{13} + \Delta_{23}\Delta_{31} - \Delta_{13}\Delta_{31},$$
$$\Psi_3 = \Delta_{31}\Delta_{12} + \Delta_{32}\Delta_{21} - \Delta_{12}\Delta_{21}$$

and shows that the cultural dynamics converges towards an interior religious polymorphism with $q_i^* > 0$ for $i = 1, 2$ and 3 if and only if $\psi_i > 0$ for $i \in \{1, 2, 3\}$. In such a case, the interior long-run frequencies are given by

$$q_i^* = \frac{\Psi_i}{\Psi_1 + \Psi_2 + \Psi_2} \quad \text{for i} \in \{1,2,3\}. \tag{15}$$

For our simple symmetric context, this collapses to $\psi_1 = \psi_2 = \lambda\theta^2 > 0$ and $\psi_3 = (2\mu - 1)\theta^2 > 0$. The first conditions are automatically satisfied, while the last one is again $\mu > \frac{1}{2}$. Hence, once it is able to diffuse, the syncretic trait stabilizes at a positive frequency but it cannot displace the two pure religious traits. Applying (15) to our simple example provides the long-run cultural polymorphism:

$$q_1^* = q_2^* = \frac{\lambda}{2\lambda + 2\mu - 1} \quad \text{and} \quad q_3^* = \frac{2\mu - 1}{2\lambda + 2\mu - 1}.$$

Interestingly, when $\mu < \lambda$, the long-run fraction of syncretic individuals is always less than 1/3 $\left(\text{as } q_3^* < \frac{2\lambda - 1}{4\lambda - 1} < \frac{1}{3}\right)$. A syncretic trait may survive if it is sufficiently exclusive (i.e., $\mu > 1/2$) but cannot diffuse very ~~largely~~ in the population when it faces alternative strong and exclusive "pure" religions.

4.2 *The continuous-trait model*

While cultural blending and its religious analogy, syncretism can be analyzed in a setup with a discrete number of traits, a more natural way to think about the issue is to allow for some continuous mixing of

traits. In this respect, there is a well-established tradition in evolutionary biology and anthropology that considers continuous-trait models of cultural transmission. These models typically postulate a dynamic of cultural traits, which is driven by exogenous linear mixing (Cavalli-Sforza, 1973; Otto *et al.*, 1994). More specifically, consider a population of N dynasties in which reproduction is asexual and that each parent has one child. Let $R^i(t) \in (0, \infty)$ denote the value of the cultural trait of a representative individual of dynasty i at time t. Assume that transmission from one generation to the next results from cultural blending from two sources: vertical transmission and oblique transmission from the rest of society. Specifically, $R^i(t)$ evolves according to the following process:

$$R_i(t+1) = \tau_i \cdot R_i(t) + (1-\tau_i) O_i(t), \tag{16}$$

where $\tau_i \in (0, 1)$ represents the weight of vertical socialization by parents of type i, and $O_i(t)$ is the pattern of oblique role-model influence to which a child of dynasty i might be exposed. Conveniently, $O_i(t)$ can be defined as a weighted average of the various models in society:

$$O_i(t) = \sum_{j=1}^{j=N} \gamma_{ij} R_j(t),$$

with $\Gamma = [\gamma_{ij}]_{i,j}$ is a row stochastic matrix reflecting the social connectivity of oblique influence across the different dynasties. Denoting $R(t) = (R_j(t))_{i=1,N}$ the N-dimensional vector of the cultural trait, the cultural dynamics can then be written in matrix form:

$$R(t+1) = \mathbf{X} \cdot R(t),$$

where $\mathbf{X} = \mathbf{T} + (\mathbf{I} - \mathbf{T})\mathbf{\Gamma}$ is a row stochastic matrix, $\mathbf{I}$ the identity matrix of dimension N and $\mathbf{T}$ a diagonal matrix of dimension N where the ith diagonal element is τ^i. In some sense, $\mathbf{X}$ reflects the force of the cultural inheritance blending process resulting from the interaction between vertical and oblique transmission. Brueckner and Smirnov (2007, 2008) consider this transmission framework when the vertical influence weights τ^i are exogenous. They show that, when the matrix $\mathbf{X}$ is irreducible and noncyclic, the evolutionary process is fully homogenizing and leads to a

"melting-pot" equilibrium, in which the value of the cultural trait is the same across the population. Cyclicity of the matrix **X**, on the other hand, preserves the possibility of long-term heterogeneity.

Another mechanism through which cultural diversity may be preserved, even in the presence of linear cultural blending, is the presence of learning errors in the process of cultural transmission. To see this, consider for instance that the continuous trait $R^i(t)$ in dynasty i evolves according to the following stochastic process:

$$R_i(t+1) = \tau \cdot R_i(t) + (1-\tau)\bar{R} + \varepsilon_i(t),$$

where $\varepsilon_i(t)$ reflects a learning error term reflected by an independently and identically distributed random shock with zero mean and constant variance σ^2, and $\tau \in (0, 1)$ is now a common weight of vertical socialization.[13] It is then a simple matter to see that the average trait in the population E_i $(R_i(t))$ converges towards $\bar{R}$, while the variance of the trait $V(t) = E_i[(R_i(t) - E_i(R_i(t)))^2]$ converges towards $\sigma^2/(1-\tau)$. With no learning noise (i.e., $\sigma^2 = 0$), as usual, cultural blending reduces variation in each generation until it is all gone. With positive learning mistakes, the population comes to rest at a positive amount of cultural variation, reflecting the balance between two forces: the homogenization effect of blending inheritance and the maintenance of variation due to error in learning.

In the religious context, the previous models suggest that syncretism (and full-trait blending) is prevented to emerge only because of specific features of the social interaction context (cyclicity of the social influence matrix **X**) or because of fundamental learning errors. From the simple three discrete trait example provided in Section 4.1, one may wonder how these conclusions are robust to endogenous cultural transmission efforts.

In the spirit of the Bisin–Verdier framework, some related models consider this possibility and allow for endogenous economic choices of the τ^i values (Vaughan, 2010; Büchel *et al*., 2014; Panebianco, 2014). In order to ensure long-run cultural convergence, they impose however additional structure on the interacting matrix **Γ**. In particular, when a child's trait is a weighted average of his parent's trait and the mean value of the

[13] More complex and interesting models along these lines are discussed in Boyd and Richerson (1985).

trait in the society, cultural blending prevents the long-run cultural heterogeneity result of Bisin and Verdier (2001). Indeed, in such linear weighting models, direct vertical socialization and oblique socialization interact in such a way that there is a standard mean reverting linear process, leading naturally to cultural homogeneity in the long run. From the point of view of religious traits, this blending view of cultural transmission should favor syncretism rather than polarization.

An interesting model investigating the issue of cultural hybridization and syncretism in a continuous trait setup is Kuran and Sandholm (2008). The paper considers two interrelated mechanisms of cultural influences: behavioral adaptations driven by payoffs to coordination, and preference changes shaped by socialization and self-persuasion. Preferences and behaviors vary continuously. Importantly, the setup posits a two-speed formulation whereby gradual changes in preferences are accompanied by immediate behavioral adjustments that maintain equilibrium play. Using techniques from evolutionary game theory, the model analyzes the set of conditions under which cross-cultural contacts across cultural groups promote cultural hybridization and homogenization, and characterizes both the ultimate composition of the hybrid culture and the speed of cultural change. The model suggests some interesting political economy implications of cultural blending and religious syncretism. Because cultural groups and religious communities are not homogeneous with respect to their relevant cultural trait, the costs and benefits of cultural adjustment are not shared equally. Significant cultural conflicts may arise therefore within and across cultural groups. Also, communities benefit from having other communities adjust their behaviors.

With a proper probabilistic structure of cultural transmission however, cultural diversity may still depend on the notion of cultural substitution between vertical role models and other influences. Cheung and Wu (2018) provide, in this respect, an elegant extension of the Bisin and Verdier (2001) to this kind of setting.

Specifically, they consider a population of unit mass, where each agent in the population has a trait from set $T = [0, 1]$. The population state is a distribution of traits over T and thus is described by a probability measure over T. Denote by $\Delta_{zy} := V_{zz} - V_{zy}$ the cultural intolerance a z-parent has towards trait $y \in T$. Assume that V_{zy} is continuous in z and y, and hence Δ_{zy} is continuous in z and y. $\Delta_{zy} \in [0, 1]$ for any z $y \in T$, and $\Delta_{zy} = 0$ only if $y = z$. Describing a population state over T by its probability

distribution μ,[14] one may define the aggregate cultural intolerance of a z-parent at population state μ by

$$\Delta_z(\mu) = \int_{y \in T} \Delta_{zy} \mu(dy).$$

Denoting by $\tau_z(\mu)$ the socialization rate exerted by a z-parent at population state μ, the cultural evolutionary dynamic is then characterized by the following differential equation for all (integrable) subset of traits A:

$$\dot{\mu}(A) = \underbrace{\int_{y \in A} \int_{z \in T \setminus A} (1 - \tau_z(\mu)) \mu(dz) \mu(dy)}_{\text{inflows}} - \underbrace{\int_{y \in A} \int_{z \in T \setminus A} (1 - \tau_y(\mu)) \mu(dy) \mu(dz)}_{\text{outflows}}. \tag{17}$$

The rate of change $\dot{\mu}(A)$ in the mass of agents with traits in set A is equal to the "inflow" of children whose parents' traits are not in set A but who themselves adopt traits in set A, minus the "outflow" of children whose parents' traits are in set A but who themselves adopt traits not in set A. It is a simple matter to see that Equation (17) rewrites as

$$\dot{\mu}(A) = \int_{y \in A} \int_{z \in T} [\tau_y(\mu) - \tau_z(\mu)] \mu(dy) \mu(dz).$$

Now, at state μ, a z-parent solves the following maximization problem:

$$\max \tau_z V_{zz} + (1 - \tau_z) \int_{y \in T} V_{zy} \mu(dy) - c(\tau_z),$$

which in the standard quadratic cost function case $c(\tau_z) = \tau_z^2 / 2$, yields

$$\tau_z(\mu) = V_{zz} - \int_{y \in T} V_{zy} \mu(dy) = \int_{y \in T} \Delta_{zy} \mu(dy) = \Delta_z(\mu).$$

[14] More rigorously, let B be the Borel σ-algebra on T. Denote by $M^+(T)$ the space of probability measures on (T, B). A population state is a distribution of traits over T and is described by a probability measure $\mu \in M^+(T)$.

From this, the cultural dynamic with continuous traits takes the following form:

$$\dot{\mu}(A) = \int_{y\in A}\int_{z\in T}[\Delta_y(\mu) - \Delta_z(\mu)]\mu(dy)\mu(dz)$$
$$= \int_{y\in A}\Delta_y(\mu)\mu(dy) - \mu(A)\int_{z\in T}\Delta_z(\mu)\mu(dz).$$

As Montgomery (2010) for discrete traits, Cheung and Wu note again the important connection with evolutionary game dynamic on continuous sets of strategies (Oechssler and Riedel, 2001). Indeed let the population game where a unit mass of agents are randomly matched in pairs to play a two-player symmetric game with continuous strategy set T and payoff function Δ_{yz} (i.e., the single match payoff of an agent playing strategy y against an opponent playing strategy z). Defining then $F_y(\mu)$ the expected payoff of an agent playing pure strategy $y \in T$ at population state μ as

$$F_y(\mu) = \int_{z\in T}\Delta_{yz}\mu(dz) = \Delta_y(\mu)$$

and noting that $\mu(T) = 1$, the cultural dynamic can be rewritten as

$$\dot{\mu}(A) = \mu(A)\left[\frac{\int_{y\in A}F_y(\mu)\mu(dy)}{\mu(A)} - \frac{\int_{z\in T}F_z(\mu)\mu(dz)}{\mu(T)}\right].$$

The cultural dynamic of continuous traits is then equivalent to an imitative dynamic with continuous strategies (Oechssler and Riedel, 2001). Using sophisticated measure theory tools for such dynamic systems, Cheung and Wu show that cultural substitutability is again essential for the preservation of long-run cultural heterogeneity. Furthermore, when one parameterizes an agent's cultural intolerance towards another agent Δ_{zy} as an increasing function of their cultural distance $|z - y|$, they highlight that the curvature of the cultural intolerance function plays an important role in determining the long-run cultural phenomena. In particular, when the cultural intolerance function is convex, only the most extremely polarized state distribution with mass points at the extreme traits $z = 0$ and $z = 1$ is a stable limit point of the cultural dynamics.

In the context of transmission of religious traits, this result suggests that radicalization in religious attributes rather than syncretism tends to occur when the perceived distance between these associated attributes

increasingly generates some degree of intolerance and exclusivity in the transmission process across individuals.

5. Institutions and Cultural Transmission

So far, we presented cultural transmission models that take a strong evolutionary and bottom-up perspective on the diffusion and evolution of religious beliefs and preferences. A central issue neglected by this approach is the fact that some important entities (religious organizations and clubs, churches, community leaders, states) participate actively in the process of religious socialization and, as such, are able to internalize some of the dynamic implications of the cultural transmission of religious traits in the society. Hence, on top of the evolutionary and purely decentralized dimensions of cultural transmission as reflected so far by vertical and oblique socialization, there also exist coordinated and forward looking aspects related to purposeful and centralized authorities and organizations. In this section, we cover a recent strand of the literature that incorporates these features. First, we examine the effect of religious leaders on the previously decentralized process of cultural transmission. Second, we analyze religious clubs in conjunction with other socializing institutions such as the media and education system. Finally, we review work on the co-evolution of culture and institutions, with a focus on the political economy of religious organizations.

5.1 *Religious leaders*

Religious leaders have always played a tremendously important role in the transmission and diffusion of religious beliefs and religious attributes across the world. Some regarded as prophets, such as Abraham and Moses in Judaism, Muhammad in Islam or Guru Nanak in Sikhism, diffused the world of God because of their privileged relationship with him. Others, like Siddhārtha Gautama in Buddhism, Peter and Paul in Christianity, Gurus of Sikhism, or Foundation members of Islam and the early Caliphs, founded or helped found a faith community or spread a religion or a belief-system. Some religious leaders, such as Martin Luther King Jr, Saint Theresa, Mahatma Gandhi or the Dalai Lama, became role models from the way their lifestyle exemplified the values of the faith community. More commonly, religious leaders, like clergy, imams, gurus, priests,

rabbis and lamas, are individuals who play an active role in the contemporary faith community.

An emerging economic literature has started to analyze more closely how including religious leaders matters for the cultural dynamics of religious traits (Verdier and Zenou, 2015, 2018; Hauk and Mueller, 2015; Carvalho and Koyama, 2016; Carvalho *et al*., 2017; Prummer and Siedlarek, 2017; Chen *et al*., 2019; Carvalho and Sacks, 2020).[15]

From the perspective of our framework for cultural transmission, religious leaders bring three new analytical features. First, cultural leaders have their own motivations and objectives related to the diffusion of their religion. Because of this, they can take actions that promote or discourage the transmission of their religious trait in the society. In particular, because of the legitimacy they draw in their community, religious leaders can inculturate and manipulate directly or indirectly the paternalistic motives Δ_i of their believers, affecting therefore the dynamic sustainability of their religion in the population.

Second, also because of their privileged position, cultural leaders have the capacity to internalize group-related effects of cultural transmission for their community. In particular, they may incentivize and coordinate collective action by their followers, changing again endogenously the payoffs of acquiring or maintaining the religious trait they promote in the population. Their forward-looking perspective means they internalize the dynamic externalities associated with the diffusion of cultural and religious attributes.

Finally, cultural leaders tend to compete across communities, or within their own community, to acquire and maintain their privileged positions. From a conceptual point of view, the process of cultural transmission of religion is therefore not only determined by decentralized parental and oblique transmission motives (i.e., the "demand side" of cultural transmission) but also by the competitive context in which cultural leaders "offer" incentives and motivations for cultural diffusion (i.e., the market structure of the "supply side" of cultural transmission).

To illustrate, consider for instance the framework of Hauk and Mueller (2015), building upon Bisin and Verdier (2001), ~~and~~ which introduces the possibility for leaders to manipulate the paternalistic motives Δ_i of parents to transmit their trait. They assume that cultural leaders can be either

[15] See also Prummer (2019) for an insightful survey focusing on cultural leaders.

intrinsically motivated by proselytism (i.e., maximize the number of people their religion is successfully distributed to), or alternatively they can enjoy rents (pecuniary or not) associated with the overall level of religious transmission exerted in their community. Assume that a cultural leader ~~is promoting~~ religious trait a. Focusing on steady states of the cultural dynamics, this means that a proselytist leader is interested in maximizing

$$q^* = \frac{\Delta_a}{\Delta_a + \Delta_b},$$

while a rent-seeker leader would like to maximize (assuming quadratic costs of socialization) something proportional to

$$q^* \tau_a^* = q^*(1 - q^*)\Delta_a = \left(\frac{\Delta_a}{\Delta_a + \Delta_b}\right)^2 \Delta_b.$$

As is easily noted, the cultural leader always has an incentive to raise as much as possible the perception of cultural differences Δ_a of ingroup members. Indeed, this motivates parents to socialize their children to that trait. An increase in this difference can be achieved in two ways: through the provision of cultural values (raising V_{aa}) or through claims of "religious superiority" making the alternative trait (other religion or secularism) appear inferior (lowering V_{ab}). Both strategies generate symmetric outcomes from the point of view of the religious leader, regardless of his objective function (proselytism or rent seeker). For ingroup parents of type a, things however are different. Parents indeed have to live with a chance that their children change religion. High values of V_{aa} mean that high perceived benefits drive parental socialization efforts, while low values of V_{ab} imply that parents inculturate their children mostly out of fear.

While it seems natural to think that religious leaders have some influence on their ingroup believers, one may also think that they may undertake actions to affect the perception that outgroup members have about their ingroup. This would be done in our setting by changing Δ_b through some manipulation of V_{ba}. In the case of a proselytist leader from group a, clearly his interest is in reducing Δ_b through an increase in V_{ba} that makes his religious trait a look more attractive to individuals from group b. Surprisingly enough, when the leader is of a rent-seeking type, he may not necessarily be interested in making his religious trait look good. Actually, he might want to increase Δ_b by lowering V_{ba}, the cultural perception of

the outgroup towards the ingroup, something Hauk and Mueller describe as *cultural alienation*. The reason is based on the *cultural substitution* effect already mentioned by Bisin and Verdier. Indeed, as Δ_b increases, the outgroup members socialize more intensively their children to keep their trait (i.e., trait b). This in turn leads the population of ingroup believers to become more minoritarian in the population (i.e., q^* goes down). By the cultural substitution effect, they consequently intensify in turn their own socialization effort τ_a^*. This effect on parental socialization may countervail the reduced size of the ingroup, so that the rents that the leader receives $q^*\tau_a^*$ may actually increase. In such a case, the religious leader prefers to have a small ingroup of believers who are intensively active in the religious education of their children, something which is the source of the rents he enjoys. Overall the analysis indicates that religious leaders have incentives to amplify disagreement about their religious values, and that the population does not always benefit from such actions.

Religious leaders are certainly motivated not only to spread their beliefs, but also to care about the economic well-being of their community members. Prummer and Siedlarek (2017) incorporate both features in the leader's objective function. In this setup, leaders influence the beliefs and attitudes of community members by establishing rules and religious norms. Departing from the Bisin and Verdier approach, they consider a continuous value framework, implying that religious leaders care about the *intensive margin* with which their religious trait is displayed. Importantly, the leader can only indirectly affect the economic well-being of his community members, which depends on the economic outcomes that are induced in the market by the norms the believers follow. The leader then eventually faces the following trade-off. On the one hand, he would want his community to identify faithfully with the values of the religious norms he supports. On the other hand, such norms are not necessarily well adapted to market behavior, and he would nevertheless like his community to be economically integrated and wealthy. Prummer and Siedlarek (2017) show that three possible outcomes arise out of this trade-off. The leader may select the most extreme level of religiosity, or he may support some intermediate level, or finally, he may completely refrain from encouraging any display of religious behavior. Which outcome emerges, depends on the economic environment and the interplay between earnings and religious beliefs in the leader's objective function.

The previous analyzes studies the optimal manipulation by cultural leaders at the cultural steady state but do not consider the dynamic

transition path to that steady state. The latter exercise requires developing a dynamic framework where forward looking leaders fully take into account the effects of their manipulation of religious values on the cultural diffusion of their religious trait. Verdier and Zenou (2018) provide such a framework. More specifically, they consider a situation where leaders provide community public goods that affect positively the paternalistic motive of their believers to transmit their trait to their children.

Formally, consider a cultural leader for group a who provides an amount G^a of a public good specific to trait a. If, for example, we consider traits a and b as "religious" and "secular," then G^a will be a religious public good (a church, religious center or a mosque). Assume now that the provision of G^a increases the paternalistic motive of a parent of type a to transmit his trait to his children such that $\Delta_a = \Delta_a^0 + vG^a$ with $v > 0$ a positive constant. Following the same lines as in our benchmark model in Section 2.3, the cultural dynamic now rewrites as

$$\dot{q}_t = q_t(1 - q_t)[\tau_a - \tau_b],$$

where

$$\tau_a = (1 - q_t)(\Delta_a^0 + vG_t^a) \text{ and } \tau_b = q_t\Delta_b,$$

where we allow *a priori* the provision of G_t^a to be time varying. Consider also that the technology to produce the public good G^a has constant marginal cost c up to a capacity constraint $\bar{G}$, so that $G^a \in [0, \bar{G}]$. While capturing the essential features of a convex production technology, this assumption is convenient to fully characterize the transitional dynamics of the socialization mechanism under cultural leadership.

Note that when the leader is never active ($G^a = 0$), the cultural dynamic converges towards the steady state

$$q(0) = \frac{\Delta_a^0}{\Delta_a^0 + \Delta_b},$$

while when the leader is providing constantly the maximum public good provision $G^a = \bar{G}$, the cultural dynamic converge towards

$$q(\bar{G}) = \frac{\Delta_a^0 + v\bar{G}}{\Delta_a^0 + v\bar{G} + \Delta_b},$$

which obviously is the largest possible long-run diffusion of trait a in society.

To analyze the role of a perfect forward-looking religious leader, Verdier and Zenou (2018) assumes that the utility of a leader for group a is given by

$$\int_0^{\infty} e^{-\rho t}(W^a q_t - cG_t^a)\, dt, \tag{18}$$

where ρ is the discount rate and where the religious leader's rents $W^a q^a$ increase with the size q of group a. Also, to highlight in the purest way the role of the leader on cultural diffusion, they assume that the initial state of the population is exactly the cultural steady state $q(0)$ when there is no leader intervention.

The equilibrium pattern of public good provision G_t^{a*} of the religious leader is then the solution of the following optimal control problem:

$$\begin{aligned} &\max_{0\le G_t^a \le \bar{G}^a} \int_0^{\infty} e^{-\rho t}(W^a q_t - cG_t^a)dt \\ s.t.\, \dot{q}_t &= q_t(1-q_t)[(1-q_t)(\Delta_a^0 + vG_t^a) - q_t\Delta_b] \\ s.t.\, q_0 &= q(0) \text{ given.} \end{aligned} \tag{19}$$

Because of the linear dependence of the problem on the control G_t^a, this optimization problem is of a bang-bang nature. Applying a characterization method based on a "Most Rapid Approach Path" formulation of the problem,[16] Verdier and Zenou (2018) show that the optimal cultural trajectory of this problem has the property that it approaches as rapidly as possible some point q^* and stays there forever, given the constraint that q^* can be reached using the control $G^a \in [0, \bar{G}]$. The characterization of the state point q^* (and the associated control function $G^{a*}(t)$) clearly depends on the shape of the objective function of the leader and the initial state $q(0)$ of the population.

Specifically, Verdier and Zenou (2018) show that the cultural leader can only be active when the marginal religious rent W^a is above a certain threshold $\overline{W}$. Second, even when such circumstances are satisfied (i.e., when $W^a > \overline{W}$), they show that the degree of activity of the religious leader varies in a nonmonotonic way with the initial size $q(0)$ of his

[16]The associated Hamiltonian is not concave in G_t^a, qt, and therefore the first-order approach is generally not sufficient to characterize optimal trajectories.

ingroup a. Specifically, there is a range (q_L, q_H) such that the leader is active and does influence the cultural diffusion of his religious trait only when $q(0)$ falls within that range. Typically, when the initial size $q(0)$ of group a is less than the first threshold q_L, it is not profitable for the cultural leader to promote more socialization than what parents of the ingroup already do. The system stays therefore at its initial steady state $q(0)$. Similarly, when vertical socialization by parents is strong enough to generate a cultural steady state with a steady-state size group $q(0)$ larger than the second threshold q_H, then it does not either pay to the leader to promote more socialization to his ingroup members. Families indeed already do enough of a good job that the leader need not spend additional resources to stimulate more cultural transmission.

When however the group size is intermediate (i.e., $q(0) \in (q_L, q_H)$), the cultural leader is active to push forward cultural dynamics in the direction of a higher steady state than would prevail without his intervention (i.e., $q(0)$). When the steady-state value at permanent full capacity $q(\bar{G})$ is less than the threshold q_H (above which the leader stops being active), then the dynamic system converges asymptotically towards $q(\bar{G})$ with a religious leader permanently active at this full capacity. When conversely, the cultural dynamic with full capacity reaches in finite time the threshold q_H, then the leader reduces endogenously his provision of public good just to ensure that cultural evolution remains critically at this threshold long-run steady state q_H.[17]

The model allows for discussion of some interesting comparative dynamics along the transition path of cultural evolution. Importantly, the analysis indicates that a shift in a parameter determining the context in which the religious group evolves has different short run versus long-run effects in terms of the socialization activity of the group. Typically, there will always be some over-reactions or under-reactions compared to the long-run effect that can be expected. These transitory dynamics have important policy implications in terms of the reaction of minority religious community associations to changes in their environment.

Finally, Verdier and Zenou (2018) extend the framework presented above to the case of competition between two perfectly forward-looking

[17] In other words, the optimal trajectory for the leader is to be active to full capacity (i.e., $G^a(t) = \bar{G}^a$) up to the moment where the ingroup size reaches the level min $[q_H, q(\bar{G})]$. Whenever that occurs in finite time T (i.e., when $q_H < q(\bar{G})$), the leader ensures that the cultural steady state stays at q_H by choosing a "singular" interior policy $G^a(t) = G^F < \bar{G}$.

religious leaders with their respective ingroups *a* and *b*.[18] The leaders may differ in terms of their discount factor ρ. When both evaluate the future in the same way, then there is a symmetric dynamic equilibrium where both select the same levels of public goods and their effort at influencing the cultural dynamics simply cancel each other out: the long-run cultural steady state is the same as if no leaders were present. If, however, one leader is more patient than the other, then the less patient leader does not provide any public good and remains inactive. Interestingly, differences in time-discount factors across leaders can be interpreted as differences in institutional stability. In such a case, a leader part of an organization that has a strong base, can naturally be more forward looking. This higher institutional stability ~~magnifies~~ into a higher capacity to influence the cultural diffusion of his religious trait, at the expense of other outgroups with less institutional stability. A plausible application of the setting concerns the case where the more patient leader has ingroups with a strong and stable religious commitment, whereas the less patient leader is a secular leader, facing the political uncertainty of elections and support. The model suggests that the secular leader may then refrain from opposing the more patient, religious leader. This may be indicative of why extremist leaders may face too little opposition in democratic societies.

Using a similar "Most Rapid Approach Path" approach, Almagro and Andrés-Cerezo (2020) also explore, in the context of nation-building, how a forward-looking leader (i.e., a central state) may promote the diffusion of a cultural trait (national identity) on its territory. The key control variable in this context is the share of a fixed resource that is allocated to the provision of a public good specifically attached to the national identity trait. Homogenization of the population towards such trait is constrained by political unrest, electoral competition and the intergenerational transmission of local identities within the family. Different from Verdier and Zenou (2018), the zero-sum character of the conflict over resources pushes the cultural dynamic toward homogeneous steady states and extreme levels of allocations of the public good. A common feature is the fact that the long-run distribution of cultural traits in the society is highly dependent on initial conditions.

[18]The issue of religious competition between leaders is also tackled by Verdier and Zenou (2015, 2018) as well as Hauk and Mueller (2015). Verdier and Zenou (2015) consider two leaders who myopically invest in their own cultural trait, while Hauk and Mueller (2015) focus on competition once the cultural dynamics are already at their steady states.

Carvalho and Sacks (2017) is another study considering explicitly the transition dynamics of religious traits' diffusion. Specifically, they discuss the conditions under which a forward-looking religious leader is willing and able to radicalize a community, transitioning it from an inclusive and liberal community to an exclusive and strict club. They identify two mechanisms which are critical to radicalization, prestige-biased cultural transmission and niche construction. Both are important in cultural evolutionary theory (Henrich and Gil-White, 2001; Odling-Smee *et al*., 2003) but largely ignored in economics. Prestige bias occurs when actively religious members of the community have greater visibility and prestige, giving them disproportionate influence over cultural transmission. Niche construction occurs when a leader can induce blanket discrimination against community members, and thereby shield the religious club from outside pressures. In both cases, the religious leader begins by forming a small but extreme club, using it to radicalize the community over time through cultural transmission and niche construction. Religious competition, however, rules out these dynamic radicalization strategies.

5.2 *Religious clubs and other socializing institutions*

There are potentially severe *free rider* and *externality* problems associated with cultural transmission. It is natural that organizations emerge to deal with these problems. Carvalho (2016) analyzes how religious clubs regulate the process of cultural transmission. In addition, social transmission in the Bisin–Verdier framework is replaced by institutional transmission of a "mainstream trait," for example through the education system or mainstream media. See also Carvalho and Koyama (2014) for a model of education choices when the education system transmits a mainstream cultural trait.

Organizations cultivate cultural traits through (i) rules of participation in cultural activities and (ii) excluding nonmembers from social interactions. For example, regarding (i), communal prayer, scriptural study and religious sacrifice can convert someone into a believer in an organization's doctrine. The exclusivity condition (ii) means that a cultural trait can be viewed as a *club good*, which is a central subject of the economics of religion (Iannaccone, 1992; Berman, 2000; McBride, 2008).

Consider a society consisting of a finite set of risk-neutral individuals I, partitioned into two (nonempty) communities I_a and I_b (e.g., secular and religious).

There are two cultural traits $k \in \{a, b\}$. Let a be the "mainstream" trait and b be the alternative trait. For example, a could be the official religion of a society, or it could be a secular belief system. There are two risk-neutral organizations (or groups), with organization A (B) cultivating trait a (b).

Each $i \in I_\theta$ receives a payoff of $V_{\theta k}$ from acquiring trait k, where $V_{aa} - V_{ab} \equiv \Delta_a > 0$ and $V_{bb} - V_{ba} \equiv \Delta_b > 0$. Hence, we refer to members of I_a as mainstream types and members of I_b as alternative types.

Let c be an individual's (privately known) cost of joining an organization, which is determined by an independent draw from the distribution F. It is assumed that $F(0) = 0$ and F is twice differentiable and strictly log-concave on $(0, \infty)$.

The timing of the game is as follows:

- Date 0 (*Strictness*). Each organization $\ell \in \{A, B\}$ announces its strictness, s_ℓ, which is the minimum level of participation required of its members.
- Date 1 (*Membership*). Each individual i can choose to become a member of an organization, $m_i = \ell \in \{A, B\}$, or be unaffiliated, $m_i = n$. M_ℓ is the set of organization ℓ members and $N = I - M_A \cup M_B$ is the set of unaffiliated agents.
- Date 2 (*Participation*). Each member $i \in M_\ell$ chooses participation level $x_i \geq s_\ell$ in group ℓ's activities, at cost x_i^2. Unaffiliated agents are excluded: x_i 0 for all $i \in N$.
- Date 3 (*Cultural Transmission*). Group and institutional transmission occurs and the final distribution of traits is determined.

The likelihood $i \in M_\ell$ acquires the trait cultivated by ℓ through group transmission is the average level of participation among ℓ members:

$$\overline{x}_\ell \equiv \frac{1}{|M_\ell|} \sum_{i \in M_\ell} x_i.$$

If group transmission fails, institutional transmission occurs and i acquires the mainstream trait a with a probability of one.

The expected payoff to a (θ, c)-type agent i who joins organization ℓ cultivating trait k is

$$u_i = V_{\theta k}\overline{x}_\ell + V_{\theta a}(1 - \overline{x}_\ell) - x_i^2 - c.$$

The probability that i ∈ N acquires the mainstream trait a is one, so the payoff when unaffiliated is $V_{\theta a}$.

Each organization ℓ maximizes aggregate participation in its activities:

$$X_\ell = \sum_{i \in M_l} x_i.$$

For example, it could be that organizations are paternalistic, maximizing the welfare of their members but not internalizing their cost of participation. Alternatively, religious organizations might have financial and political interests in the tangible products of participation, including cultural power, public good provision and political opposition.

This leads to the following result:

(Carvalho, 2016) *Recall the cultural intolerance of I_b members is $\Delta_b \equiv V_{bb} - V_{ba}$. There exists a unique SPE of this game. In this equilibrium,*

Intolerance Δ_b can be reinterpreted as "cultural tension," which is an important concept in the sociology of religion. According to Stark and Finke (2000, p. 143), "All religious groups can be located along an axis of tension between the group and its sociocultural environment," where tension is defined in terms of "distinctiveness, separation and antagonism." Here, tension dictates strictness and total participation:

(Carvalho, 2016) *In the subgame, perfect equilibrium of the game, organization B's strictness s_B^* and total participation X_B^* are strictly increasing in Δ_b.*

The lessons from introducing clubs and other socializing institutions are as follows:

- Organizations that form will tend to be ones cultivating oppositional culture.
- Stricter organizations will have doctrines far from the mainstream worldview.
- Groups cultivating oppositional cultural traits will have an advantage in collective action.

Now, suppose we extend this game to an infinite horizon with discrete time, letting the share of mainstream types $|I_a^t|/|I|$ equal the share of agents who acquired trait a in the previous period q^{t-1}. This is consistent with

imperfect empathy and the desire of parents to have children acquire their own cultural trait. The asymptotic behavior of such a dynamic would be $\lim_{t\to\infty} q^t = 1$. That is, the alternative trait would be driven to extinction through the force of institutional transmission.

From the Carvalho (2016) setup that connects religious doctrine and participation, the implications for religion are as follows. Religious groups that are marginalized in mainstream society should generate the most intensive participation. If, for example, we rank all Protestant denominations by their doctrinal tension with the mainstream secular worldview, denominations with higher doctrinal tension should have more intensive participation. This is roughly what we see. However, the long-run prediction is that such groups die out due to institutional transmission of the secular worldview from the state, media and public education system. This can be avoided if alternative religions could obtain some control over institutional transmission, possibly through the collective action which they are able to generate, or insulate themselves from it through socialization within the family.

5.3 *Political economy, cultural transmission and religious legitimacy*

The connections between religion and political economy have been widely acknowledged by social scientists and more recently by economists in different contexts (Cosgel and Miceli, 2009; Platteau, 2011; Cosgel *et al*., 2012; Chaney, 2013; Auriol and Platteau, 2017; Rubin, 2017; Barro and McCleary, 2019; Carvalho *et al*., 2020). Economic and political developments affect religiosity and the diffusion of beliefs in a society. In turn, the extent of religious participation and beliefs influence economic performance and political institutions. Typically, a religion by providing (or not) legitimacy to state powers reduces (or increases) the transactions costs of law enforcement and authority, while conversely, state powers try to regulate positively (through the establishment of state religions) or negatively (through secularism and laicity) the extent of religious influence on public matters. Mediating this connection between the political and the spiritual spheres is the prevalence of religious beliefs in the population, itself the endogenous result of inculturation processes (transmission and conversion), and public policies favoring (or not) their diffusion in society.

To highlight these connections between political economy and religion, one may take our cultural transmission perspective and develop a framework analyzing the dynamic interaction of political institutions and religious culture. A starting point for this is Bisin and Verdier (2017) who formalize the evolution of institutions and culture and study their joint dynamics. We then see how this framework can be fruitfully applied to the issue of religious legitimacy in some specific historical and political economy contexts (Bisin *et al.*, 2019, 2020).

5.3.1 *The joint evolution of culture and institutions*

Bisin and Verdier (2017) formalize the evolution of institutions and culture and study their joint dynamics. There are two building blocks. The first block describes the mechanism of institutional change. Here in line with Acemoglu and Robinson (2000, 2006), institutions are conceptualized as mechanisms through which social choices are delineated and implemented. More specifically, institutional change represents an effective commitment mechanism on the part of the political elites to imperfectly and indirectly internalize the lack of commitment and the externalities which plague social choice problems. The second block is just the Bisin–Verdier cultural transmission framework outlined in Section 2.

To illustrate, consider a simple society constituted by two groups $i \in \{E, S\}$ (E for *elite* and S for the rest of *society*), with distinct cultural traits, objectives and technologies. At each period t, let us describe in a relatively abstract way a societal policy game which is played between private individuals and a hierarchical public authority (the state) controlling socioeconomic policies. Individuals in each group $i \in \{E, S\}$ are characterized by an objective function $V^i = U^i(a^i, \mathbf{p}, \mathbf{A})$ that depends on the "cultural" type i of the individual, private actions a^i by that individual, a policy vector $\mathbf{p}$ implemented by the state during the period and some aggregator measure of socioeconomic outcomes $\mathbf{A} = \mathbf{A}(\mathbf{a}, \mathbf{p}, \mathbf{q})$ that captures the interactions between the private agents and the public authorities. $\mathbf{A}$ naturally depends on the aggregate vector $\mathbf{a}$ of actions by individuals of the two groups, the public policy vector $\mathbf{p}$ and the distribution of cultural types in the population (captured by the frequency distribution $\mathbf{q}$ of types in society).

Collective decisions on socioeconomic policies are made in accordance with the distribution of political power between the two groups encoded and represented by institutions. Specifically, the institutional system is characterized by weights β^E and $\beta^S = 1 - \beta^E$ associated with the two

groups E and S in the decision making problem of the state with respect to the policy vector $\mathbf{p}$. A given institutional setup $\beta_t = (\beta_t^E, \beta_t^S)$ in period t induces a set of policies $\mathbf{p}^*(\beta_t, \mathbf{q}_t)$ and actions $\mathbf{a}^*(\beta_t, \mathbf{q}_t)$, as the equilibrium of the societal policy game between individuals and the public authority. Importantly, the equilibrium outcomes $\mathbf{p}^*$ and $\mathbf{a}^*$ also depend on the distribution of cultural traits $\mathbf{q}_t$ prevailing in the population during period t.

With respect to institutional change, Bisin and Verdier (2017) note that the key issue about societies is the fact that they are characterized by economic and political externalities that are not fully accounted for by private and public decisions. Externalities typically arise because of socioeconomic or political imperfections associated with the existence of various frictions going from asymmetric/incomplete information, matching problems, limited rationality and cognitive biases, strategic behaviors associated with market power, private opportunism and lack of political commitment. In any of these situations, the equilibrium outcomes $\mathbf{a}^*(\beta_t, \mathbf{q}_t)$ and $\mathbf{p}^*(\beta_t, \mathbf{q}_t)$ of the societal policy game do not fully internalize their impacts on aggregate social outcomes $\mathbf{A} = \mathbf{A}(\mathbf{a}, \mathbf{p}, \mathbf{q})$, and as a result, inefficient policies and social allocations are implemented.

Taking a simple mechanism design approach, the institutional structure corresponding to a power structure β_t at any point in time might then have an incentive to change the distribution of political power in the future to internalize the externalities responsible for the inefficiencies at equilibrium. This is a fundamental driver of institutional change in society. Similar to the governance theory of organizations (Coase, 1937; Williamson, 1996), it induces as a general principle that the political group most likely to internalize the externality is the group receiving more residual decision rights along the institutional dynamics, i.e., the group having a higher political weight in the state policy choice problem.

As schematically illustrated in Figure 2, for any cultural population profile at a given time t, $\mathbf{q}_t$, this mechanism provides a mapping from the institutional system at t, β_t, into the one at $t+1$, β_{t+1}.

As mentioned, the second part of the framework is the cultural transmission block. In this respect the Bisin–Verdier cultural transmission setup is augmented by the fact that the degree of paternalistic motivations for cultural transmission now depend on the equilibrium outcome of the societal policy game $\Delta_i = \Delta_i(\mathbf{a}^*, \mathbf{p}^*)$. Under cultural substitution, parental socialization is generally stronger for cultural minorities and, *ceteris paribus*, for the group which is relatively favored at the equilibrium outcome

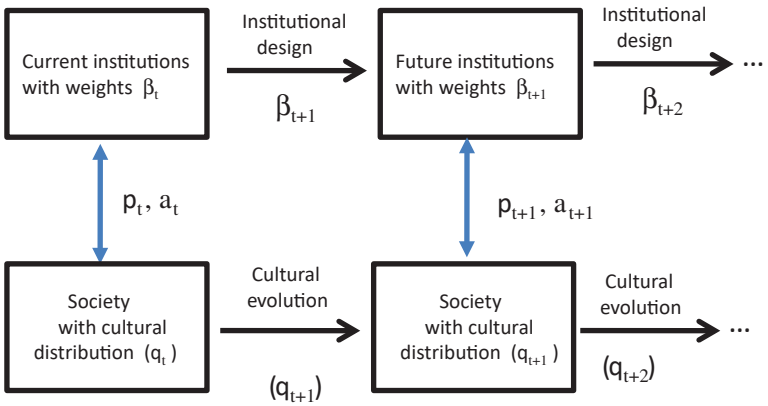

Figure 2: Joint Dynamics of Culture and Institutions

(**a***, **p***) of the policy game. These considerations determine the cultural dynamic of the profile of society $\mathbf{q}_t$ over time. Given that (**a***, **p***) are themselves function of the institutional weights β_t, the diffusion of cultural traits across generations are consequently also influenced by the status of the institutional system, β_t. This mechanism induces a mapping from $\mathbf{q}_t$ into $\mathbf{q}_{t+1}$, which represents the cultural dynamic.

The politico-economic and cultural structures of society are then characterized by the joint dynamics of institutions and culture (β_t, $\mathbf{q}_t$) as described in Figure 2. This system eventually reaches a long-run steady state (β*, q*). Importantly, the joint dynamics may reinforce or hinder each other in response to shocks, depending on whether culture and institutions are dynamic complements or substitutes. To get an intuition on this, consider for instance the case of complementarity and take an exogenous shock to the system that makes more salient the existence of an externality or a political commitment issue. Such a shock triggers an institutional response aimed at internalizing the externality and/or committing policy choices. This institutional response implies augmenting the political weight to the group who gains relatively more from a policy change that helps correct the externality and/or the commitment issue. When the strength of this institutional response is positively related to the frequency of the cultural traits carried by that group, and that such more empowered group has in turn a higher success at diffusing those specific traits, then complementarity between institutions and culture prevails. Over time, institutional and cultural dynamics reinforce each other and therefore act as dynamic complements.

5.3.2 *Religious legitimacy*

The previous logic can be fruitfully applied to the study of religious legitimacy and its implications in terms of institutional and cultural change in a society. For this, define an elite as legitimate when the people believe in its right to rule. Such a belief is ingrained into a set of values and normative statements describing how society should be organized. While some legitimacy principles can be derived from rational premises, legitimacy often takes its roots in the existence of internalized values and worldviews provided by specific organizations or individuals. Given the nature of the beliefs and values that they promote, religious institutions and their members (priests, clerics, etc.) are important agents contributing to the construction of legitimate orders which elites can leverage for their authority and policy making.

In the context of religious legitimacy, Bisin *et al*. (2019) highlight three basic principles driving the joint evolution between institutions and culture:

1. Legitimacy helps (secular) elites to affirm their authority and reduce the transactions costs associated with the implementation of their policy choices.
2. The capacity of the religious clerics to supply legitimacy to the elite relies fundamentally on how religious values promoted by the clerics are disseminated in society.
3. The diffusion of religious values is in turn facilitated by institutions that entrust more political power to the clerics.

The first item is at the source of specific institutional changes which determine an evolution of the distribution of political power between the elite and those supplying the legitimacy: the religious clerics. The second item implies that institutional changes associated with legitimacy depend on the cultural profile of society in terms of religious beliefs and values. Finally, the last item indicates that the institutional system reflecting the structure of power between elite and religious clerics impacts strongly on the dynamics of cultural diffusion of religious values in the population.

In a recent work, Bisin *et al*. (2020) discuss the implications of these different elements for the emergence of religious legitimacy and the associated joint evolution of culture and institutions. For this, they consider a society composed of a political elite, clerics and civil society (merchants,

workers, popular masses, etc.). The political power of the religious clerics reflects their relative control of policies and reforms. It relates, for instance, to their control of the judicial administration, the local police and taxation. It is also linked to their presence in key institutions providing social services to the population. Clerics care about the provision of a religious good, e.g., places of worship or of religious study, which they control and they extract (material or immaterial) rents from.

The interaction between rulers, clerics and civil society involves a number of externalities and commitment issues not internalized by individual decisions. For instance, the religious good constitutes a public good for individuals in society, in that it facilitates individual participation in religious activities. In turn, participating in religious activities results in psychological and emotional benefits, as a consequence of a (perceived) closer relationship with the divine. Favoring a more intense participation in religious practices, the religious good provided for by the clerics legitimizes the political control of the elites, e.g., reducing at the margin the psychological cost associated with taxation and other forms of extraction. Less directly, participating in religious activities may also increase the scope of social interactions between religious individuals. This can have positive effects on productivity, since it potentially improves prosocial attitudes, e.g., coordination and cooperation; also, it promotes informal information networks in principle very useful in trading and other economic activities. At the same time though, religious beliefs and restrictions may also increase the costs of economic activities and transactions. Religious regulations on occupational choices in labor markets, restrictions on credit markets at a positive interest rate, or prohibitions of adoption of new technologies are typical examples distorting the allocation of resources away from efficiency.

In this environment, the public provision of the religious good controlled by the clerics will generally not internalize these social externalities, and depending on the state of society and its organizational features, can be inefficiently low or high. Because of commitment issues, the elites controlling political power may not internalize the public good aspect of the religious good. Also, they may not internalize fully how such religious good and the legitimacy it provides allows them to extract more (and more efficiently) resources from society. Consequently, when the legitimacy effect is sufficiently strong, institutional change pushes for a shift in the structure of power towards religious clerics. In turn, clerics exercise this power by providing the religious good in larger quantities, which in turn favors

religious practices and activities, propagating beliefs within the population that justify the ruling and extractive capacity of the political elite.

Bisin *et al*. (2020) show that this mechanism of political empowerment of religious clerics has important consequences in terms of institutional trajectory and cultural change. In particular, since clerics care about the provision of religious services, their prevalence in society should increase religious parents' direct inculturation efforts, that is, vertical socialization. Religious legitimacy then makes culture and institutions complements: institutional change devolving political power to the clerics reinforces the incentives of the religious members of society to transmit their values, while in turn, a higher fraction of religious individuals augments the political incentive to credibly change the institutional structure so as to empower the clerics.

Interestingly, the dynamic complementarity between institutional change and diffusion of religious values gives rise to multiple stationary states in the joint dynamics. On the one hand, the system may converge towards a *strong religious state* characterized by the wide diffusion of religious norms and influential clerics imposing religious restrictions that facilitate the extractive power of political rulers (eventually at some cost in terms of economic efficiency). On the other hand, one may have a *secular state* where religious norms do not diffuse, clerics become steadily less influential on economic and political aspects of social life, and civil society (merchants, workers or popular masses) eventually gain control over production and redistribution. As can be expected in such situations, history matters in the sense that the joint evolution of religious values and institutions crucially depends on the initial conditions. Following the analytical implications of this setup, Bisin *et al*. (2020) discuss how the historical divergence between the Christian West and the Muslim East can be understood in terms of the political economy relationships between political elites and religious elites, and the process of building up political legitimacy in the two respective regions.

In a similar vein, but focusing this time on the role of the Reformation in the secularization of the West, Cantoni *et al*. (2018) propose a conceptual framework in which the introduction of religious competition shifts the way religious authorities provide legitimacy to rulers in exchange for control over resources, and consequently the balance of power between secular and religious elites. Using original microdata, they document an important, unintended consequence of the Protestant Reformation: a reallocation of resources from religious to secular purposes. Indeed as secular

authorities acquired enormous amounts of wealth from monasteries closed during the Reformation, this transfer of resources had significant consequences. In Protestant regions it shifted the allocation of upper-tail human capital, with graduates of Protestant universities increasingly studying secular subjects and taking thereafter secular, especially administrative, occupations rather than church-sector-specific positions. They also show that this process affected the sectoral composition of fixed investment. Particularly in Protestant regions, new construction shifted from religious toward secular purposes, especially the building of palaces and administrative buildings, which reflected the increased wealth and power of secular lords.

6. Conclusion

This chapter has surveyed the literature on cultural transmission and its application to religion. The study of religion makes clear that preferences are endogenous and must be brought within the scope of economic analysis. The central theoretical framework of Bisin and Verdier (2000, 2001) connects the bottom-up evolutionary dynamic approach to the diffusion of cultural traits to the standard microeconomic choice approach of socialization decisions on the part of families. The model is flexible enough to incorporate various features important for the formation and transmission of religious traits such as endogenous fertility, homophily, religious identity, spatial and social segregation. We highlighted how generalizations of this setup to n discrete traits and continuous traits can also bring interesting and new insights on the genesis of important religious phenomena, such as syncretism.

Importantly, we emphasize that this setup is versatile enough to allow the inclusion of more centralized forces of religious change. These include churches, religious clubs, community leaders, the media, the education system, or other state and private institutions. Introducing these top–down entities qualitatively changes the cultural dynamics of religious beliefs. Further, it opens up a set of new and interesting issues. In this chapter, we touched upon the role of coordination and competition between these centralized entities, and how that may matter for the diffusion or radicalization of religious beliefs. We also outlined some of the political economy and institutional implications associated to the provision of religious legitimacy.

Other dimensions may certainly be worth exploring. One of them for instance relates to demography and how religious leaders and organizations may strategically influence marriage and fertility norms to promote the persistence and diffusion of their religious beliefs. This strategy also connects to important political economy implications, as illustrated for instance by "revenge of the cradle" and other pronatalist policies promoted for example by Roman Catholics in Northern Ireland, or Haredi (ultra-Orthodox) groups in Israel.

Future empirical work is also likely to present new avenues for extending the model. In this respect, one potentially fruitful line of research could be to design cultural transmission models of religious beliefs with and without leaders and analyze how they give differentially interesting empirical implications across religions and denominations.

In the end, we hope this survey encourages further theoretical and empirical work on cultural transmission and its application to the many open questions in the study of religion.

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Appendix

The Replicator Dynamic

In this Appendix, we derive the replicator dynamic taking the biological approach of interpreting payoffs as *reproduction rates* of strategies. Because payoffs are frequency dependent, this extends the notion of survival of the fittest from an exogenous environment to an interactive setting.

Evolutionary game theory is the study of bounded rational populations of agents who may (or may not) evolve or learn their way into equilibrium by gradually revising simple, myopic rules of behavior. Strategies that do better, given what everyone else is doing, proliferate. Following this approach, we could also show that the replicator dynamic emerges from a variety of learning protocols including imitation and reinforcement learning (Sandholm, 2010).

Once again the population consists of a continuum of agents. Consider a population game with a set of (pure) strategies $S = \{1, \ldots, n\}$, with typical members i, j and s. The mass of agents programmed with strategy i is m_i, where $\sum_{i=1}^{n} m_i = m$.

Players do not choose strategies through deliberation. Rather, they are *programmed* with a strategy, and strategies with higher payoffs proliferate.

Let $q_i = \frac{m_i}{m}$ denote the share of players programmed with strategy $i \in S$ and $q = (q_i)\, i \in S$ be the full distribution.

The set of population states (or strategy distributions) is $Q = \{q \in [0,1]^n : \sum_{i \in S} q_i = 1\}$. That is, Q is the unit simplex in $\mathbb{R}^n$.

The set of vertices of Q are the pure population states — those in which all agents choose the same strategy. These are the standard basis vectors in $\mathbb{R}^n$:

$$e_1 = (1,0,0, \ldots), e_2 = (0,1,0, \ldots), e_3 = (0,0,1, \ldots), \ldots$$

A *continuous* payoff function F: $Q \to \mathbb{R}^n$ assigns to each population state a vector of payoffs, consisting of a real number for each strategy. F_i: $Q \to \mathbb{R}$ denotes the payoff function for strategy i.

Consider the expected payoff to strategy i if i is matched with another strategy drawn uniformly at random from the population to play the following *two-player* game:

	1	2	…	n
i	$u(i, 1)$	$u(i, 2)$	…	$u(i, n)$

The expected payoff to strategy i in state q is

$$\begin{aligned} F_i(q) &= q_1 u(i,1) + q_2 u(i,2) \ldots + q_n u(i,n) \\ &= \sum_{j=1}^{n} q_j u(i,j) \\ &= \sum_{j=1}^{n} q_j F_i(e_j). \end{aligned}$$

The average payoff in the population is

$$\begin{aligned} \bar{F}(q) &= q_1 F_1(q) + q_2 F_2(q) \ldots + q_n F_n(q) \\ &= \sum_{i=1}^{n} q_i F_i(q). \end{aligned}$$

Note that this is the same as the payoff from playing the mixed strategy q against itself.

To derive the replicator dynamic, suppose that payoffs represent *fitness* (rates of reproduction) and reproduction takes place in continuous time. This yields a continuous-time evolutionary dynamic called the *replicator dynamic* (Taylor and Jonker, 1978). The replicators here are pure strategies that are copied without error from parent to child. As the population state q changes, so do the payoffs and thereby the fitness of each strategy.

Let the rate of growth of strategy i be

$$\frac{\dot{m}_i}{m_i} = [\beta - \delta + F_i(q)],$$

where β and δ are "background" birth and death rates (which are independent of payoffs). This is the interpretation of payoffs as fitness (reproduction rates) in biological models of evolution.

What is the rate of growth of q_i, the *population share* of strategy i? By definition, $q_i = \frac{m_i}{m}$. Hence $\ln(q_i) = \ln(m_i) - \ln(m)$, which means

$$\begin{aligned}\frac{\dot{q}_i}{q_i} &= \frac{\dot{m}_i}{m_i} - \frac{\dot{m}}{m} \\ &= [\beta - \delta + F_i(q)] - \sum_{j=1}^{n} q_j [\beta - \delta + F_j(q)] \\ &= F_i(q) - \bar{F}(q).\end{aligned}$$

That is, the growth rate of a strategy equals the excess of its payoff over the average payoff.

The following results are immediate:

- Those sub-populations that are associated with better than average payoffs grow and *vice versa*.
- The sub-populations associated with pure best replies to the current population state $q \in Q$ have the highest growth rate.
- Support invariance: $\dot{q}_i = q_i[F_i(q) - \bar{F}(q)]$, so that if $m_i = 0$ at T, then $m_i = 0$ for all $t > T$.

Again, though we have derived the replicator dynamic based on a biological interpretation of payoffs as reproduction rates, the replicator dynamic also emerges at the population level from a variety of learning protocols including imitation and reinforcement learning (Sandholm, 2010).