The Shifting Bottleneck Heuristic

- successful heuristic to solve make-span minimization for job shop
- iterative heuristic
- determines in each iteration the schedule for one additional machine
- uses reoptimization to change already scheduled machines
- can be adapted to more general job shop problems
  - other objective functions
  - workcenters instead of machines
  - set-up times on machines
  - ...

Basic Idea

Notation: M set of all machines

Given:
fixed schedules for a subset $M_0 \subset M$ of machines
(i.e. a selection of disjunctive arcs for cliques corresponding to these machines)

Actions in one iteration:

- select a machine k which has not been fixed (i.e. a machine from $M \setminus M_0$)
- determine a schedule (selection) for machine k on the base of the fixed schedules for the machines in M_0
- reschedule the machines from M_0 based on the other fixed schedules

Selection of a machine

Idea: Chose unscheduled machine which causes the most problems (bottleneck machine)

Realization:

- Calculate for each operation on an unscheduled machine the earliest possible starting time and the minimal delay between the end of the operation and the end of the complete schedule based on the fixed schedules on the machines in M_0 and the job orders
- calculate for each unscheduled machine a schedule respecting these earliest release times and delays
- chose a machine with maximal completion time and fix the schedule on this machine

Technical realization

- Define graph $G' = (N, A')$:
  - N same node set as for the disjunctive graph
  - A' contains all conjunctive arcs and the disjunctive arcs corresponding to the selections on the machines in M_0
- $C_{max}(M_0)$ is the length of a critical path in G'

Comments:

- with respect to G' operations on machines from $M \backslash M_0$ may be processed in parallel
- $C_{max}(M_0)$ is the makespan of a corresponding schedule

Technical realization - cont. 1

for an operation (i, j); $i \in M \setminus M_0$ let

- r_{ij} be the length of the longest path from U to (i, j) in G'
- Δ_{ij} be the length of the longest path from (i, j) to V in G'
- $d_{ij} = C_{max}(M_0) - \Delta_{ij} + p_{ij}$

Comments:

- r_{ij} is the release time of (i, j) w.r.t. G'
- d_{ij} is the due date of (i, j) for a schedule with makespan $\leq C_{max}(M_0)$

Technical realization - cont. 2

For each machine from $M \setminus M_0$ solve the nonpreemptive one-machine problem with release times and due dates and objective to minimize the maximum lateness

Result: values $L_{max}(i)$ for all $i \in M \setminus M_0$

Action:

- Chose machine k as the machine with the largest maximum lateness
- schedule machine k according to the optimal schedule of the one-machine problem
- add k to M_0 and the corresponding disj. arcs to G'

Remark:

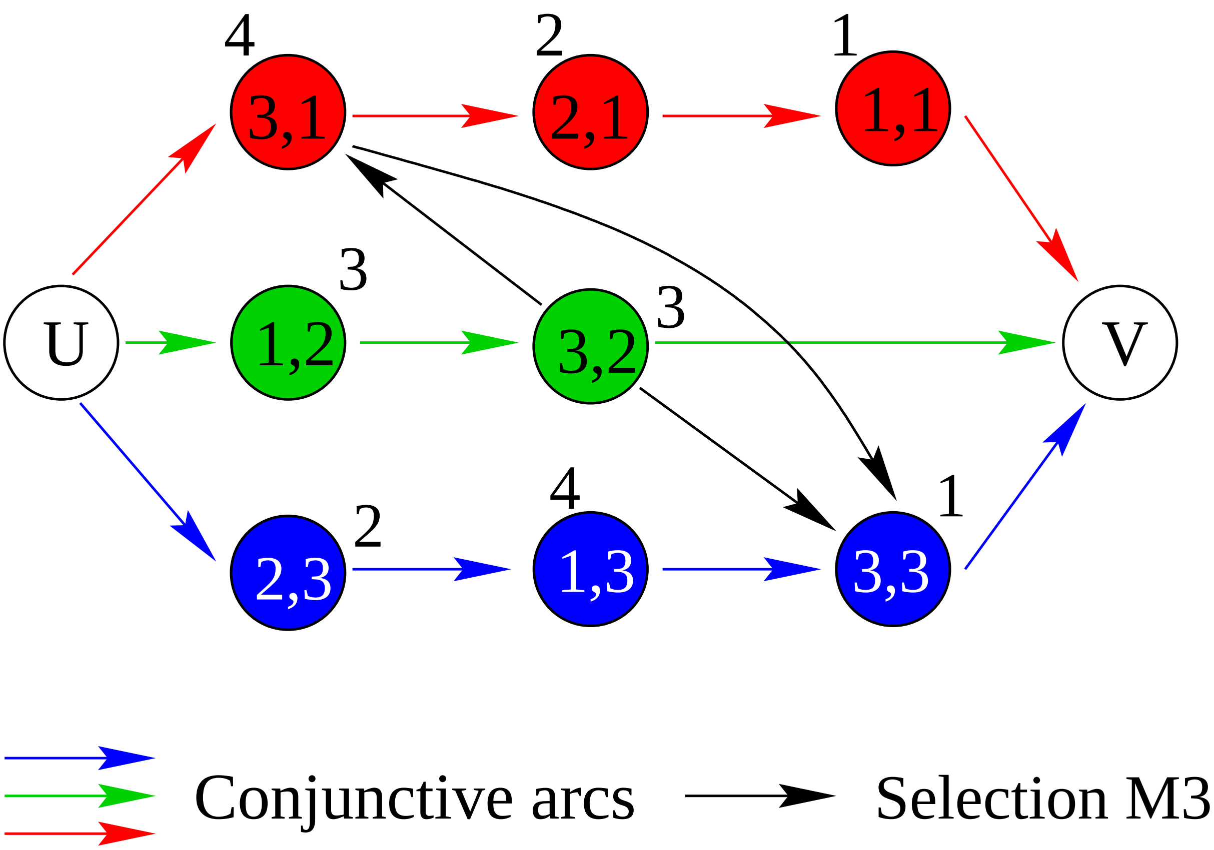
$$C_{max}(M_o \cup k) \geq C_{max}(M_0) + L_{max}(k)$$

Technical realization - Example 1

Given:

- $M_0 = \{M3\}$
- on $M3$: $(3,2) \rightarrow (3,1) \rightarrow (3,3)$

Graph G':

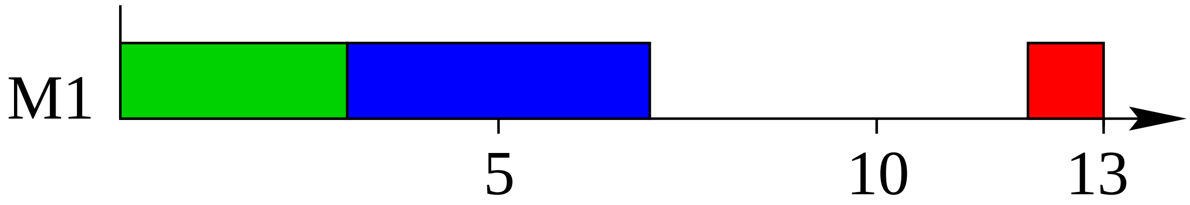


$C_{max}(M_0) = 13$

Technical realization - Example 2

Machine M1:

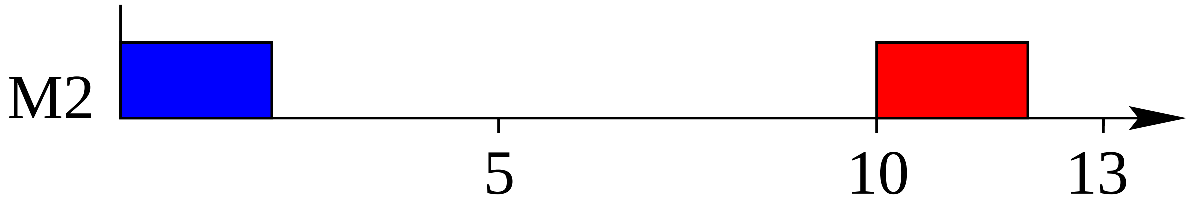
(i,j)	$(1,1)$	$(1,2)$	$(1,3)$
r_{ij}	12	0	2
Δ_{ij}	1	13	5
d_{ij}	13	3	12



$L_{max}(M1) = 0$

Machine M2:

(i,j)	$(2,1)$	$(2,3)$
r_{ij}	10	0
Δ_{ij}	3	7
d_{ij}	12	8



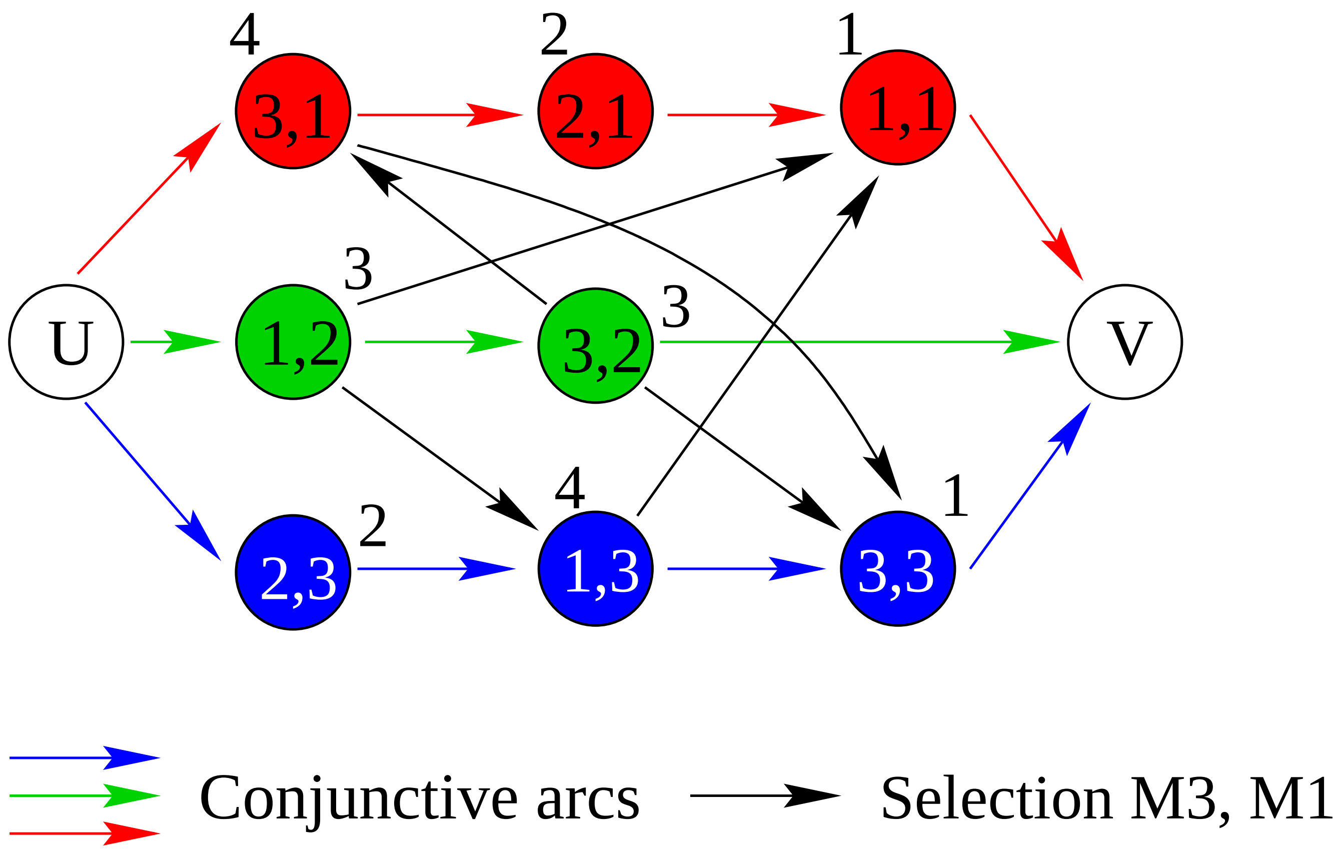
$L_{max}(M2) = 0$

Technical realization - Example 3

Choose machine M1 as the machine to fix the schedule:

add $(1,2) \rightarrow (1,3) \rightarrow (1,1)$ to G'

$$M_0 = \{M1, M3\}$$



$$C_{max}(M_0) = 13$$

Reschedule Machines

Idea: try to reduce the makespan of the schedule for the machines in M_0

Realization:

- consider the machines from M_0 one by one
- remove the schedule of the chosen machine and calculate a new schedule based on the earliest starting times and delays resulting from the other machines of M_0 and the job orders

Technical realization

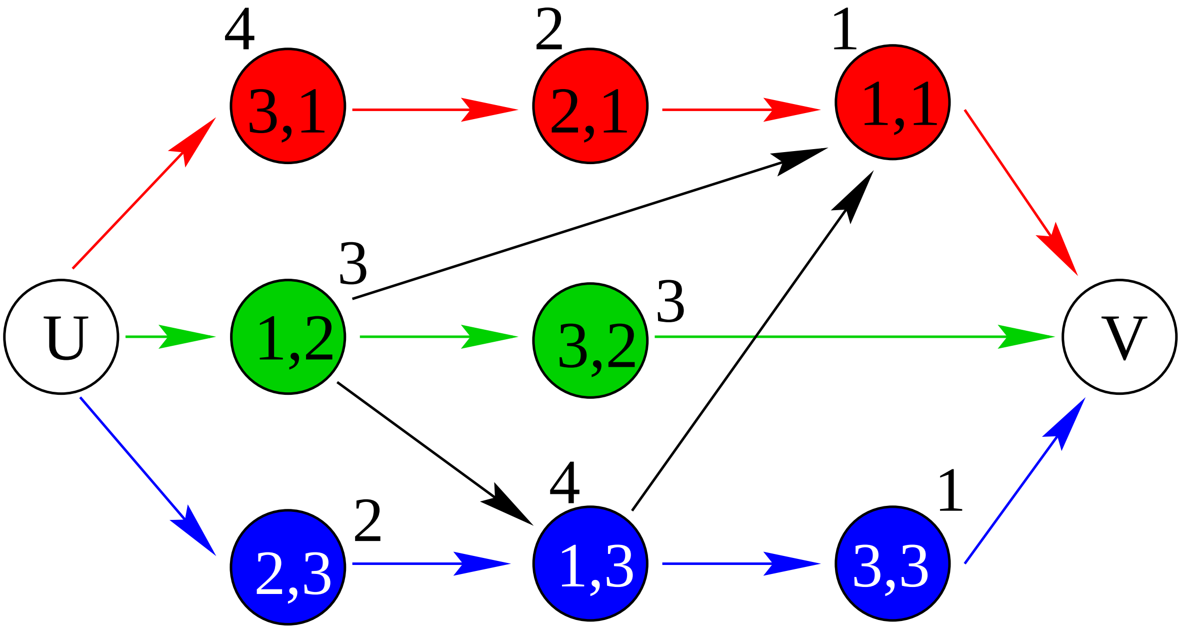
For a chosen machine $l \in M_0 \setminus \{k\}$ do:

- remove the arcs corresponding to the selection on machine l from G'
- call new graph G''
- calculate values $r_{ij},\ \Delta_{ij}$ and d_{ij} in graph G''
- reschedule machine l according to the optimal schedule of the single machine maximum lateness problem with release and due dates

Technical realization - Example 1

$M_0 \setminus \{k\} = \{M3\}$, thus $l = M3$

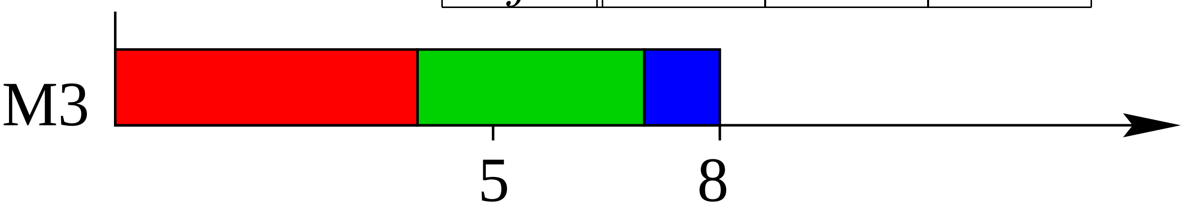
graph G'':



Conjunctive arcs — Selection M1

$C_{max}(G'') = 8$;

(i,j)	$(3,1)$	$(3,2)$	$(3,3)$
r_{ij}	0	3	7
Δ_{ij}	7	3	1
d_{ij}	5	8	8

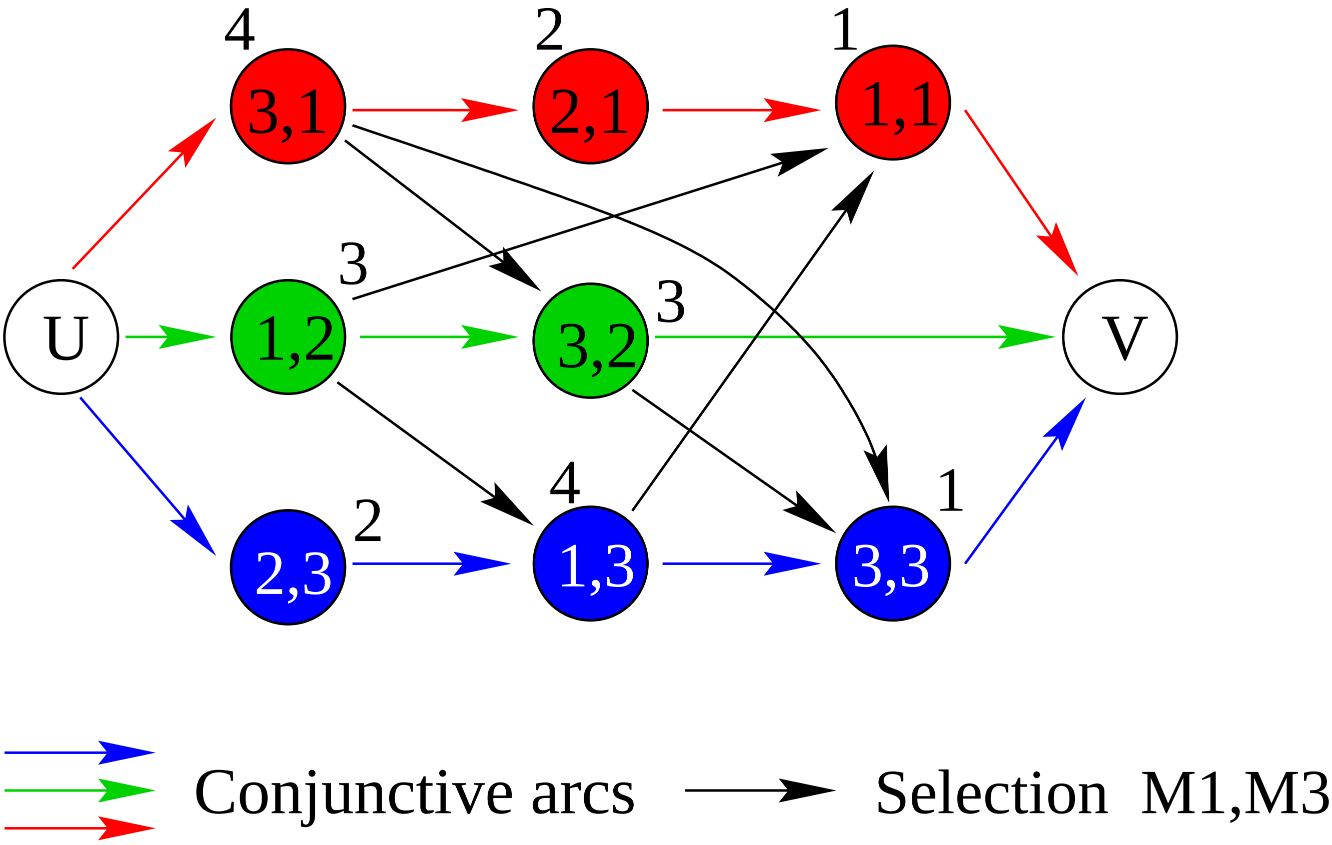


$L_{max}(M3) = 0$

Technical realization - Example 2

add $(3,1) \to (3,2) \to (3,3)$ to G''

New graph:



$C_{max}^{new}(M_0) = 8$

Shifting Bottleneck Heuristic - 1

<u>Step 1</u>: (Initialization)

$M_0 := \emptyset$;
$G :=$ graph with all conjunctive arcs;
$C_{max}(M_0) :=$ length longest path in G:

<u>Step 2</u>: (Analyze unscheduled machines)

FOR ALL $i \in M \setminus M_0$ DO
  FOR ALL operation (i, j) DO
    $r_{ij} :=$ length longest path from U to (i, j) in G;
    $d_{ij} := C_{max}(M_0) -$ length longest path from (i, j) to V in $G + p_{ij}$;
  minimize L_{max} for single machine problem on machine i subject to release dates r_{ij}, due dates d_{ij};
  $L_{max}(i) :=$ minimum lateness on i;

Shifting Bottleneck Heuristic - 2

Step 3: (Bottleneck selection)

determine k such that

$$L_{max}(k) = \max_{i \in M \setminus M_0} L_{max}(i);$$

schedule machine k according to the optimal solution in Step 2;

add corr. disjunctive arcs to G;

$M_0 := M_0 \cup \{k\}$;

Shifting Bottleneck Heuristic - 3

Step 4: (Resequencing of machines)

FOR ALL $i \in M_0 \setminus \{k\}$ DO

delete disjunctive arcs corresponding to machine k from G;

FOR ALL operation (i, j) DO

$r_{ij} :=$ length longest path from U to (i, j) in G;

$d_{ij} := C_{max}(M_0) -$ length longest path from (i, j) to V in $G + p_{ij}$;

minimize L_{max} for single machine problem on machine i subject to release dates r_{ij}, due dates d_{ij};

insert corr. disjunctive arcs to G;

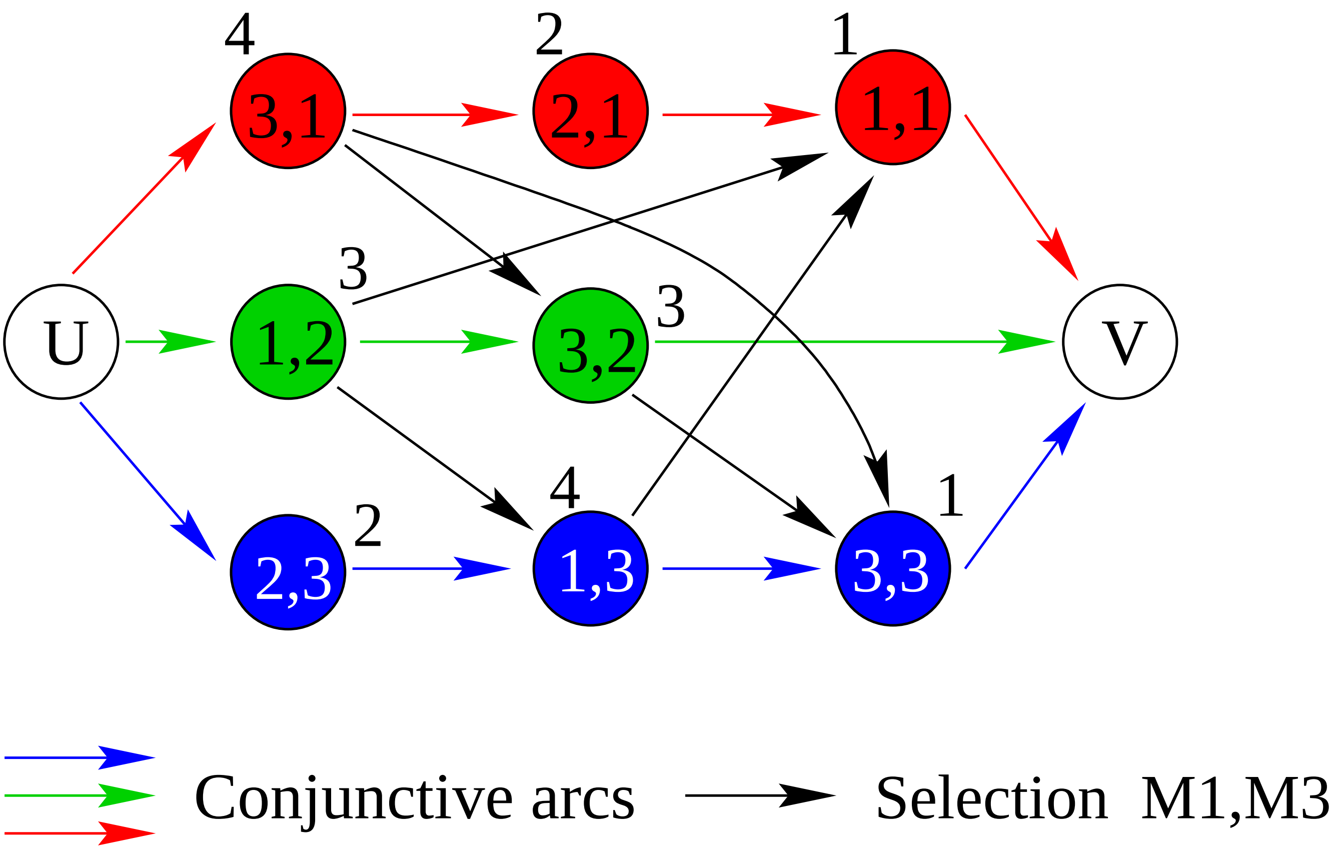
Step 5: (Stopping condition)

IF $M_0 = M$ THEN Stop

ELSE go to Step 2;

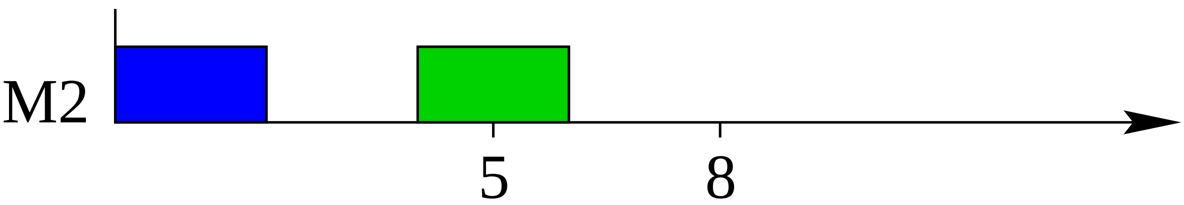
SBH - Example - cont. 1

$M_0 = \{M1, M3\}$; thus $M2$ is bottleneck graph G:



$C_{max}(G) = 8$;

(i,j)	$(2,1)$	$(2,3)$
r_{ij}	4	0
Δ_{ij}	3	7
d_{ij}	7	3

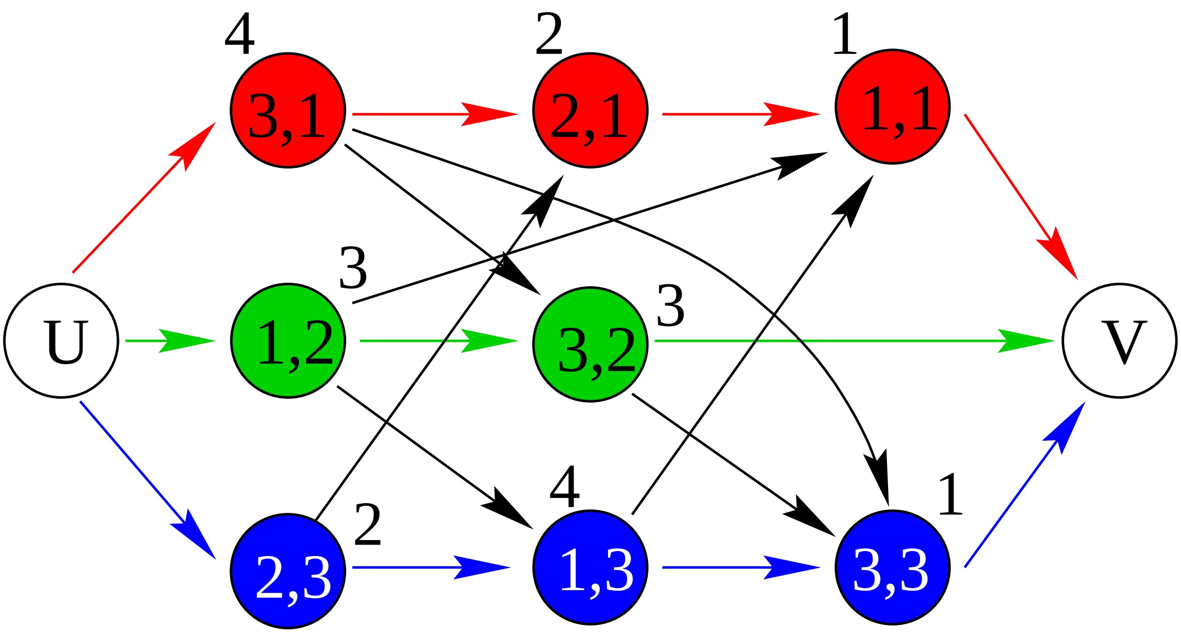


$L_{max}(M2) = -1$

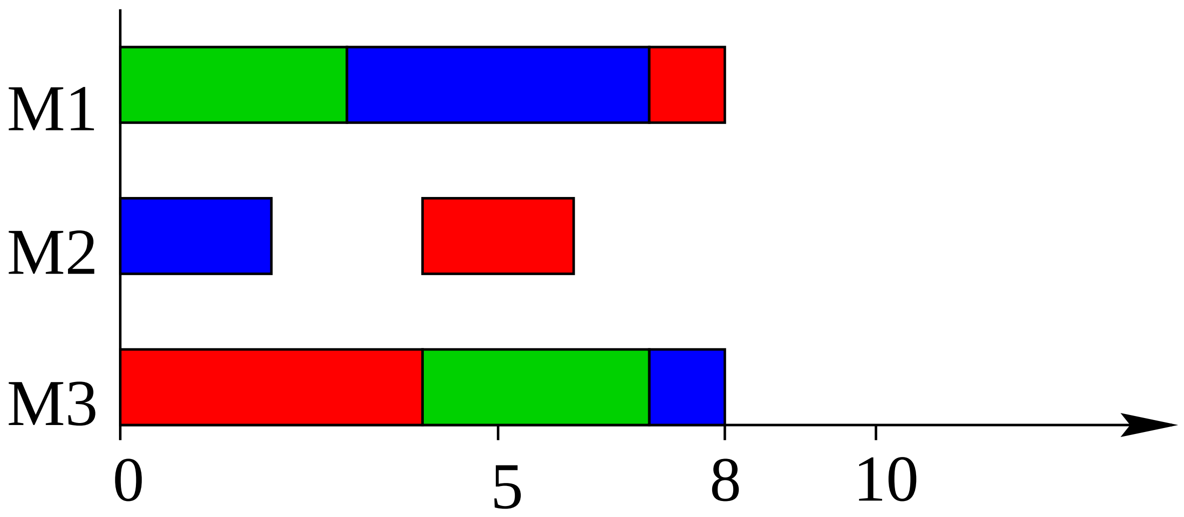
SBH - Example - cont. 2

$M_0 = \{M1, M2, M3\}$, $C_{max}(M_0) = 8$

Graph G:



Corresponding Schedule:



Makespan Cmax=8

An Important Subproblem

Within the SHB the following one-machine problem occurs frequently:

Given:

- release dates r_i
- due dates d_i

Goal: Find a nonpreemptive schedule with minimal maximum lateness

Remarks:

- this problem was also used within branch and bound to calculate lower bounds
- the problem is NP-hard
- there are efficient solution methods for smaller instances
- the actual one-machine problem is a bit more complicated than stated above (see foll. example)

Example Delayed Precedences 1

Jobs: J1	(red)	(1,1)-->(2,1)
J2	(green)	(2,2)-->(1,2)
J3	(blue)	(3,3)
J4	(yellow)	(3,4)

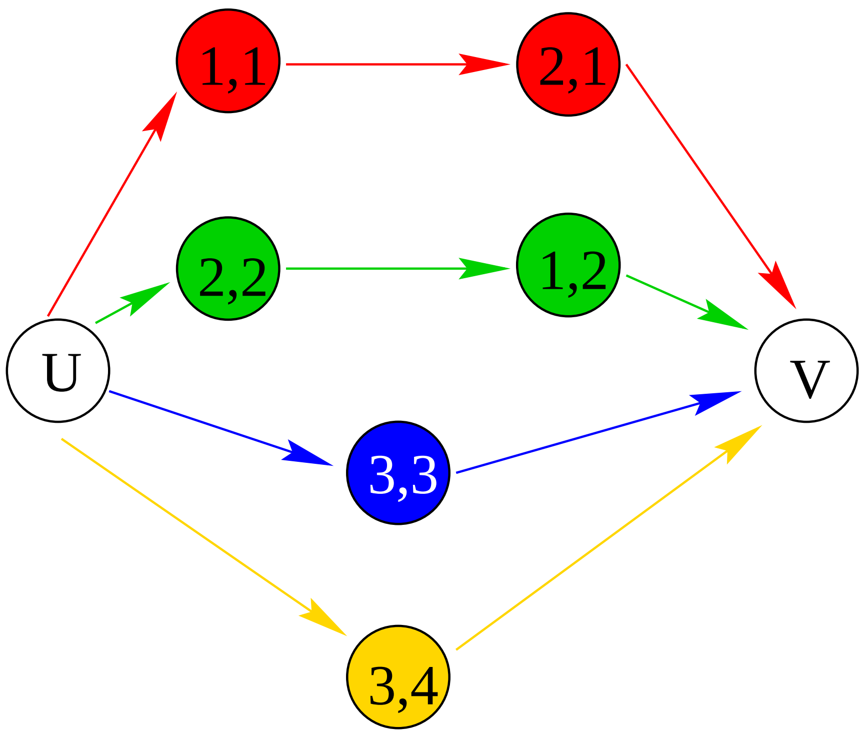
Processing Times: $p_{11} = 1, \;\; p_{21} = 1$

$p_{22} = 1, \;\; p_{12} = 1$

$p_{33} = 4$

$p_{34} = 4$

Initial graph G:

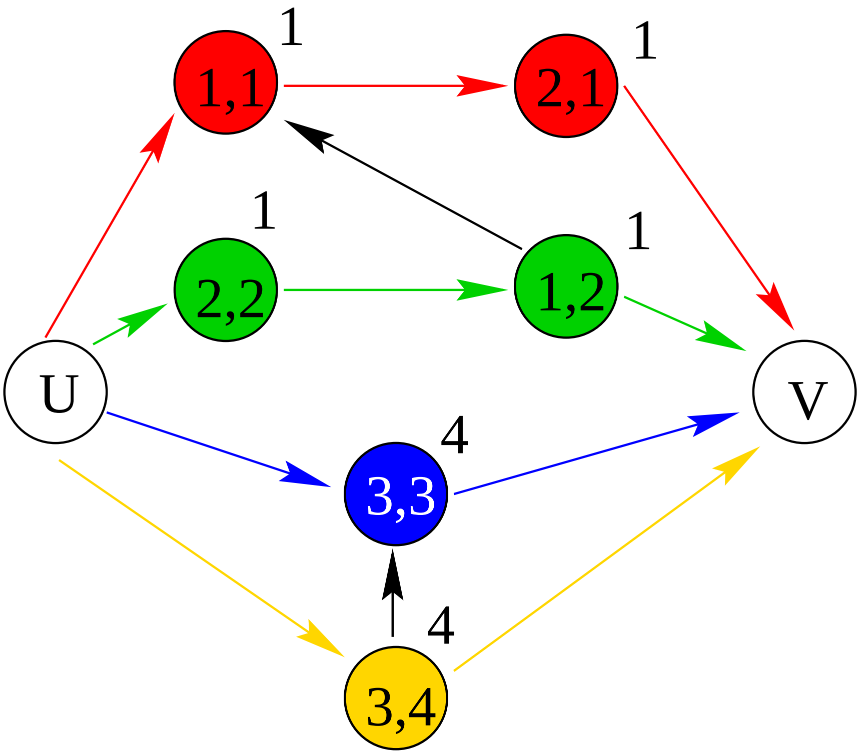


Example Delayed Precedences 2

After 2 iterations SBH we get:

$M_0 = \{M3, M1\}$
$(3,4) \rightarrow (3,3)$ and $(1,2) \rightarrow (1,1)$

Resulting graph G: ($C_{max}(M_0) = 8$)

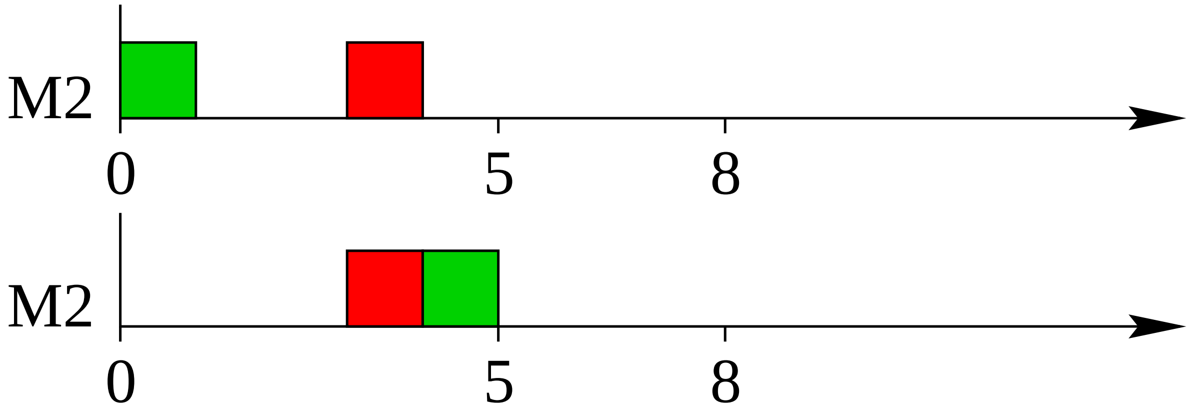


3. iteration: only $M2$ unscheduled

(i,j)	p_{ij}	r_{ij}	d_{ij}
$(2,1)$	1	3	8
$(2,2)$	1	0	5

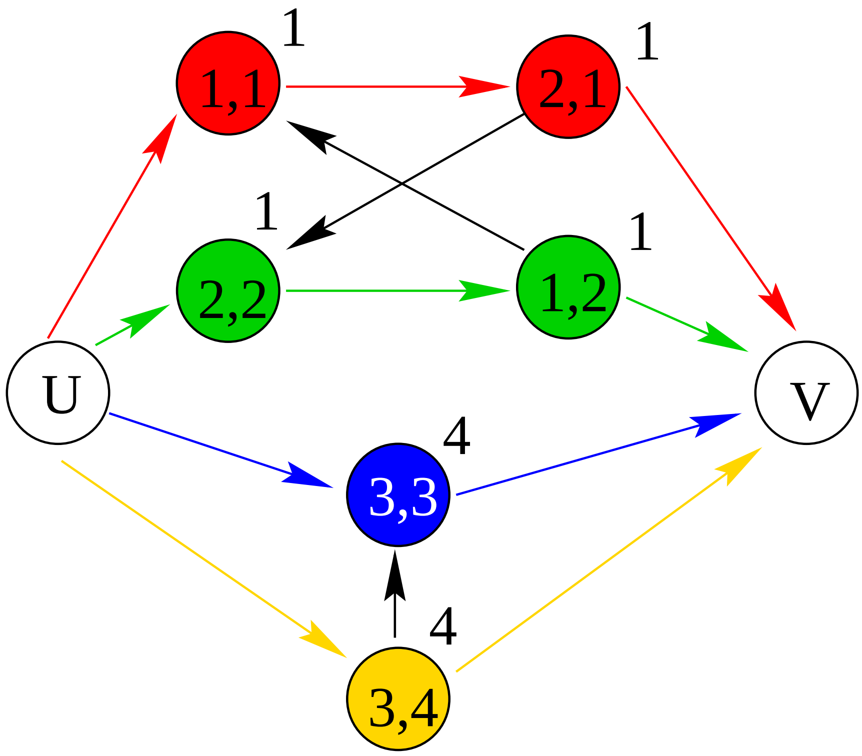
Example Delayed Precedences 3

Possible schedules for $M2$:



Both schedules are feasible and have $L_{max} \leq 0$

But: 2. schedule leads to



which contains a cycle

Delayed Precedences

The example shows:

- not all solutions of the one-machine problem fit to the given selections for machines from M_0
- the given selections for machines from M_0 may induce precedences for machines from $M \setminus M_0$

Example: scheduling operation $(1, 2)$ before $(1, 1)$ on machine M1 induces a delayed precedence constraint between $(2, 2)$ and $(2, 1)$ with length 3

$\rightarrow$ operation $(2, 1)$ has to start at least 3 time units after $(2, 2)$

this time is needed to process operations $(2, 2), (1, 2)$, and $(1, 1)$

Opgaven voor werkcollege

Date: Time:	Thursday, 23 May, 2002 13.45-15.30 (5+6)
Room:	BB 3
Exercises: College 5 College 6	 5.1, 5.3, 5.6 (a) 5.5 (a), 5.7 (b), 5.8