Declining Search Frictions, Unemployment, and Growth

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For a search-theoretic model of the labor market, we seek conditions for the existence of a balanced growth path (BGP), where unemployment, vacancy, and worker's transitions rates remain constant in the face of improvements in the production and search technologies. A BGP exists iff firm-worker matches are inspection goods and the quality of a match is drawn from a Pareto distribution. Declining search frictions contribute to growth with an intensity determined by the tail coefficient of the Pareto distribution. We develop a strategy to measure the rate of decline of search frictions and their contribution to growth.

I. Introduction

The leading theory of unemployment and vacancies is the search model of the labor market, first sketched in Stigler (1961, 1962) and then fully developed in Diamond (1982), Mortensen (1982), and Pissarides (1985).

We are grateful to the editor and four referees for many helpful suggestions. We are grateful to Ioana Marinescu and Nicolas Petrosky-Nadeau for generously sharing their data with us. We also thank Mark Aguiar, Manuel Amador, Satyajit Chatterjee, Jason Faberman, Loukas Karabarbounis, John Kennan, Pete Klenow, Sephora Mangin, Claudio Michelacci,

Electronically published October 15, 2020
[*Journal of Political Economy*, 2020, vol. 128, no. 12]

The theory argues that unemployment and vacancies coexist because limited information prevents unemployed workers and vacant jobs from immediately locating each other. To overcome limited information, firms spend resources to advertise their vacancies, and workers spend time to collect and process the ads released by firms. The efficiency with which firms advertise their vacancies and workers collect and process job ads determines the extent of search frictions, that is, the speed at which workers come into contact with vacancies and, in turn, the level of unemployment, vacancies, and the mismatch between employers and employees.

A natural exercise is confronting the search theory of the labor market with data about unemployment and vacancies from times when the extent of frictions is likely to be different: the last 90 years of US history. Over this period, the introduction and diffusion of communication and information technologies—such as the radio, the landline phone, the internet, and the smart phone—is likely to have widened the audience that can be reached by a firm's ad. Moreover, progress in public and private transportation is likely to have widened the audience of workers willing to entertain a job opening in a given location. Both phenomena have plausibly increased the speed at which unemployed workers become aware of relevant vacancies.

Figure 1 shows the Beveridge curve—the scatter plot of unemployment and vacancy rates—over the period between 1927 and 2018 in the United States.[1] We can see the counterclockwise movements of the Beveridge curve at the business cycle frequency, which have been well documented and rationalized (see, e.g., Kaplan and Menzio 2016; Gavazza, Mongey, and Violante 2018; Sniekers 2018). What we find remarkable, though,

Ben Moll, Chris Moser, Ezra Oberfield, Barbara Petrongolo, Mathieu Taschereau-Dumouchel, Chris Tonetti, Todd Schoellman, Venky Venkateswaran, Randy Wright, and audience members at several conferences for their comments. Data are provided as supplementary material online.

[1] Figures 1 and 2 are constructed using the time series for unemployment and vacancies in Petrosky-Nadeau and Zhang (2013). All details are in app. A. Here, it is worth warning our readers that the vacancy rate is constructed from four different series: the MetLife help wanted index (newspaper ads, April 1929–August 1960), the Conference Board help wanted index (newspaper ads, January 1951–July 2006), Barnichon's help wanted index (mix of newspaper and online ads, January 1995–December 2012), and the JOLTS job openings (survey of establishments, December 2000–December 2018). The four series are merged through rescaling. Specifically, a series is rescaled so that it takes the same value as the previous one at a particular point in time (January 1960 for the second series, January 1995 for the third series, January 2000 for the fourth series). Once rescaled, consecutive series closely track each other during the entire period of overlap. This suggests that the meaning of a 1% change remains the same across different series. The unified series is a vacancy index. The index is divided by the labor force and then turned into a vacancy rate by using the observation in Zagorsky (1998) that the vacancy rate was 2.05% in 1965. Reassuringly, for the period of overlap, our vacancy rate is very close to a vacancy rate computed directly from JOLTS.

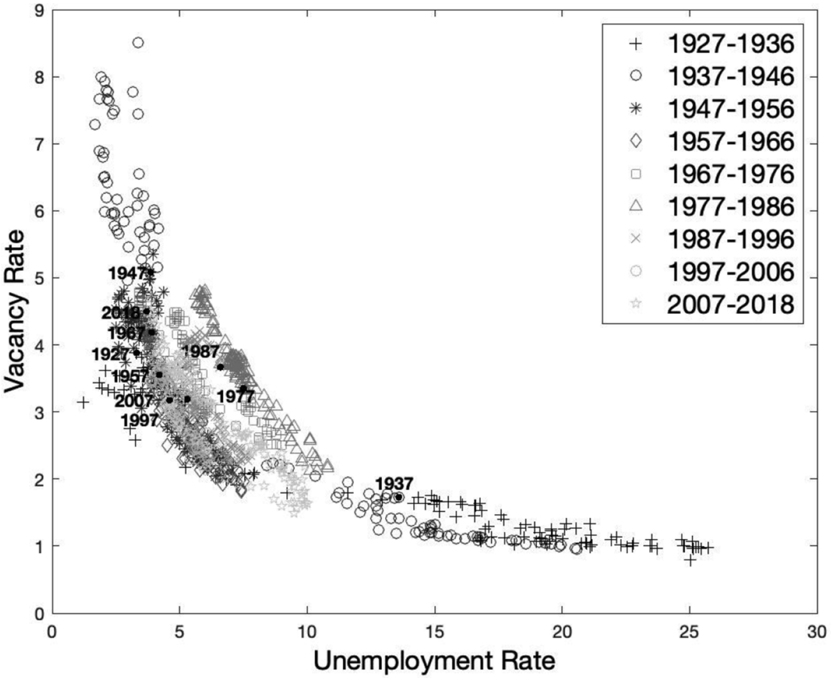

FIG. 1.—Beveridge curve in the United States, 1927–2018.

is the lack of any systematic secular movement of the curve. The Beveridge curve in 2018 is exactly where it was in the late 1940s. There are also no secular movements along the curve. We can see in figure 2 that unemployment and vacancy rates feature large fluctuations at the business cycle frequency, which have recently been the subject of much research (see, e.g., Shimer 2005; Hall 2005, 2017; Menzio and Shi 2011). Unemployment and vacancy rates also feature lower-frequency fluctuations, presumably driven by changes in the demographic and occupational structure of the economy. Unemployment and vacancy rates, however, do not have an overriding secular trend. The rate at which unemployed workers become employed (UE rate) and the rate at which employed workers become unemployed (EU rate) also display business cycle and lower-frequency fluctuations but do not have an overriding secular trend, as we can see from figure 3.[2]

[2] The UE and EU rates are constructed as in Menzio and Shi (2011). The UE and EU rates corrected for time aggregation (as in Shimer 2005) are similar. The trend of the EU rate is slightly positive from 1949 to 1985 and slightly negative afterward. The UE rate shows no trend between 1949 and 2000 and a decreasing trend afterward (see, e.g., Davis and Haltiwanger 2014).

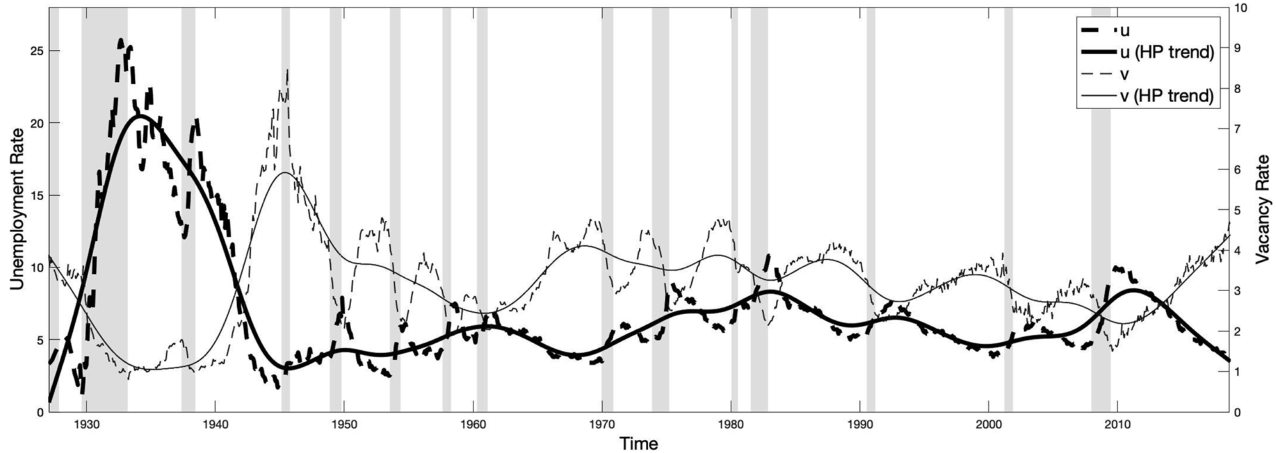

FIG. 2.—Unemployment and vacancy rates in the United States, 1927–2018.

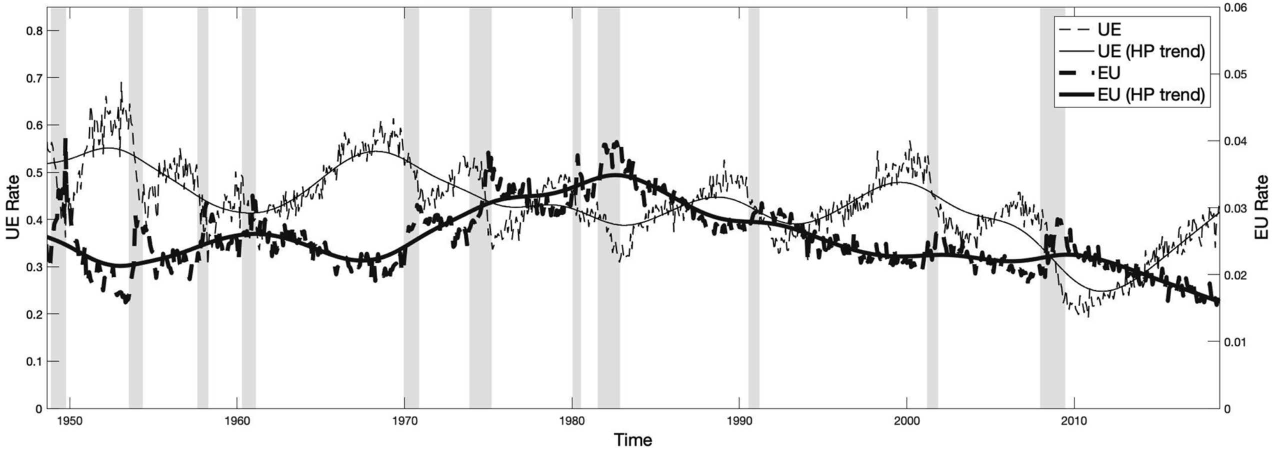

FIG. 3.—UE and EU rates in the United States, 1948–2018.

If search frictions in the labor market have diminished over the last 90 years, why do we not see a secular inward shift of the Beveridge curve, a secular negative trend in the unemployment rate, and a secular rise in the UE rate?[3] One possibility is that search frictions are not the cause of unemployment and vacancies. A second possibility is that the decline in search frictions has not been large enough to create secular trends. A third possibility is that declining search frictions have countervailing effects on unemployment, vacancies, and transition rates that happen to offset each other. We explore this third possibility.

We consider a model of the labor market in the spirit of Mortensen and Pissarides (1994), with progress in the production technology and declining search frictions. Progress in the production technology is modeled as a growth rate g_y in the component of labor productivity that is common to all firm-worker matches. Declining search frictions are modeled as a growth rate g_A in the rate at which a worker meets a vacancy. We assume that firm-worker matches are inspection goods in the sense that when a worker and a vacancy meet, they observe the idiosyncratic component of productivity of their match and, on the basis of this information, decide whether to start an employment relationship. We seek a balanced-growth path (BGP) for this economy, that is, an equilibrium along which unemployment, vacancies, UE, and EU rates are constant over time. We focus on a BGP because it is a description of an economy in which unemployment, vacancies, and transition rates have approximately no trend. Moreover, the conditions for the existence of a BGP are a useful benchmark to understand temporary and persistent deviations of the economy from stationarity.

The main result of the paper is a set of necessary and sufficient conditions for the existence of a BGP, together with a characterization of the BGP. A BGP exists iff (*a*) the quality of a firm-worker match is a sample from a Pareto distribution with some tail coefficient α; and (*b*) the worker's benefit from unemployment and the firm's cost of maintaining a vacancy grow at the same rate as average productivity. The assumption that matches are inspection goods could be considered the third condition for existence, as there is no BGP if matches are experience goods. The BGP has the following properties: (1) unemployment, vacancies, UE, and EU rates are constant; (2) the distribution of employed workers across match qualities is the sampling distribution truncated at a cutoff that grows at the rate g_A/α; (3) average productivity grows at the rate $g_y + g_A/\alpha$.

[3] The industrial organization literature has made a similar observation with respect to price levels and price dispersion. The introduction of online trade, in fact, does not seem to have lowered prices or eliminated price dispersion (see, e.g., Baye, Morgan, and Scholten 2005; Ellison and Ellison 2018).

The intuition behind the main result is simple. The decline in search frictions leads to an increase in the reservation quality—that is, the lowest match quality for which workers and firms are willing to start or continue an employment relationship—as it makes it easier for workers and firms to locate alternative trading partners. Hence, the decline in search frictions has two countervailing effects on the UE rate. On the one hand, it increases the rate at which workers meet vacancies. On the other hand, by increasing the reservation quality, it lowers the probability that a meeting between a worker and a vacancy results in an employment relationship. Iff the sampling distribution of quality is Pareto and the unemployment benefit grows at the same rate as average productivity, the two effects cancel out and the UE rate remains constant. The cross-sectional distribution of employed workers across qualities is the sampling distribution truncated at the reservation quality. Since the sampling distribution is Pareto and the reservation quality grows at a constant rate, the EU rate remains constant. Because UE and EU rates are constant, so is unemployment. The vacancy-to-unemployment ratio remains constant iff the cost of maintaining a vacancy grows at the same rate as the benefit, which is equal to the growth rate of average productivity.

The decline in search frictions contributes to the growth of average productivity by increasing the reservation quality. The contribution is g_A/α, where $1/\alpha$ denotes the thickness of the right tail of the distribution from which workers and firms sample the quality of their match, and hence it controls the return to faster search. The finding that declining search frictions contribute to productivity growth by reducing the mismatch between firms and workers formalizes one of the original insights of search theory. In fact, Stigler (1962, 104) observes, "The better informed the labor market, the closer each worker's product to its maximum at any given time" and "In a regime of ignorance, Enrico Fermi would have been a gardener, Von Neumann a checkout clerk at a drugstore."

The main result of the paper carries over to two natural generalizations of the environment: search on the job and population growth. For both generalizations, the necessary and sufficient conditions for the existence of a BGP are exactly the same as in the baseline. The properties of a BGP, though, are slightly different. With search on the job, the distribution of employed workers across match qualities is not the truncated sampling distribution but a truncated Fréchet. With population growth, the effective rate of decline of search frictions is not g_A, but $g_A + \beta g_N$, where g_N is the growth rate of population and β is the coefficient that controls the return to scale of the search process. Hence, with population growth, the contribution to declining search frictions to productivity growth is not g_A/α but $(g_A + \beta g_N)/\alpha$.

The second result of the paper is about identification. If the economy is moving along a BGP, one cannot infer the rate $g_A + \beta g_N$ at which search frictions decline by looking at the time trends of unemployment, vacancy, UE, and EU rates. Indeed, these variables are constant over time irrespective of $g_A + \beta g_N$. Moreover, if the conditions for a BGP are satisfied, one cannot infer the return to scale β in the search process by looking at the difference in unemployment, vacancy, UE, and EU rates across markets with different size. Indeed, these variables are uncorrelated with market size irrespective of β.[4] We show that one can measure $g_A + \beta g_N$ as the growth rate of the number of candidates that a firm considers for a vacancy before filling it. Similarly, one can measure β from the elasticity of the number of candidates per vacancy with respect to the size of the market where the vacancy is located. Last, we show that one can measure the coefficient α of the sampling distribution as the tail coefficient of the wage distribution for inherently identical workers. We then carry out a rough implementation of our identification strategy. We find a 2.2% decline in search frictions between 1980 and 2010, with 5/6 due to improvements in search technology, g_A, and 1/6 due to increasing returns to scale in search, βg_N. We find that the contribution of declining search frictions to productivity growth, $(g_A + \beta g_N)/\alpha$, is nonnegligible.[5] Similarly, the contribution of increasing returns to scale in the search process to the wage gap between large and small cities is nonnegligible.

The main goal of our paper is to find conditions for a BGP in a search-theoretic model of the labor market in which frictions become smaller over time. Most of the literature seeking conditions for a BGP is applied to the neoclassical growth model (see, e.g., King, Plosser, and Rebelo 1988; Grossman et al. 2017). This literature starts from some stylized facts and uses these facts to derive restrictions on the fundamentals of the economy, such as the utility and the production functions. These restrictions are useful not only as an explanation for the stylized facts but also as a benchmark to understand how to make sense of deviations from the BGP. This is the spirit of our paper too. A smaller part of the literature seeks conditions for a BGP in search-theoretic models (see, e.g., Aghion and Howitt 1994; Pissarides 2000). However, this part of the literature has focused on environments in which the production technology rather than the search technology improves over time.

[4] In the data, unemployment, vacancies, UE, and EU rates are indeed uncorrelated with the size of the local labor market (see, e.g., Petrongolo and Pissarides 2006; Martellini 2019).

[5] The finding that declining search frictions contribute to productivity growth formalizes and quantifies one of the original ideas of Stigler (1962). The finding is related to recent work by Hsieh et al. (2019), who argue that the decline in the discrimination of women and minorities in the labor market might account for somewhere between one-quarter and one-half of the overall increase in US productivity over the past 50 years. Both findings work through a common mechanism: declining distortions in the labor market lower the mismatch between workers and jobs or occupations. In our paper, distortions are caused by information frictions. In theirs, distortions are caused by discrimination.

The premise of our paper is the conjecture that improvements in information technology and in transportation over the last 90 years have reduced search frictions. While we do not have a direct measure of search frictions over time, there is some evidence to support our conjecture. First, we find that applications per vacancy, a proxy of the frequency of firm-worker meetings, increased substantially between 1980 and 2010 in the United States. Second, Bhuller, Kostol, and Vigtel (2019) exploit exogenous temporal and spatial variation in the access to broadband internet across Norway to measure the effect that this technology has had on local labor markets. They find that when broadband internet becomes available, firms report fewer problems in finding workers, workers find employment more quickly, and the wage of newly hired workers increases. In particular, the fraction of firms reporting problems with recruiting falls by 13%, the average duration of vacancies falls by 7%, the UE rate increases by 2% percentage points, and, most importantly, the wage of workers hired out of unemployment increases by 3%. These findings are consistent with the predictions of our model in response to a discrete jump in the efficiency of search.

We find that a necessary condition for a BGP is that the quality distribution from which firms and workers sample is Pareto. In this sense, our paper relates to a recent literature on endogenous growth that has found a central role for Pareto distributions in the construction of BGPs. Perla and Tonetti (2014) study a model of imitation, in which firms can either produce with their current technology or copy the technology of another randomly selected firm. They show that if the initial distribution of technologies is Pareto, there is an equilibrium in which the economy endogenously grows at a constant rate. The role of the Pareto distribution, though, is different from in our model.[6] Lucas and Moll (2014), Benhabib, Perla, and Tonetti (2017), and Buera and Oberfield (2018) are growth models similar to Perla and Tonetti (2014). In a model of endogenous growth through innovation, Kortum (1997) seeks conditions under which the innovation rate is constant even though the number of researchers is growing. This question is analogous to ours, that is, seeking conditions under which the UE rate is constant even though the rate at which unemployed workers meet vacancies is growing. He argues that the innovation rate is constant because the increase in the number of researchers is offset by the decline in the probability that a researcher finds an idea better than the best available one. This answer has the same flavor as ours; that is, the UE rate is constant because the increase in the meeting

[6] In Perla and Tonetti (2014), firms have different technologies and can either produce or copy the technology of another firm, randomly sampled from those that produce. By construction, in Perla and Tonetti, the rate at which copying firms become productive is exogenous, as every new draw of technology is acceptable. In our model, the UE rate is endogenous and it is constant only if the sampling distribution is Pareto.

rate is offset by the decline in the probability of finding an acceptable match. The economics behind the two answers, though, is different.[7]

II. Baseline Model

In this section, we consider a version of the canonical search-theoretic model of the labor market by Mortensen and Pissarides (1994), in which firm-worker matches are inspection goods in the sense that when they meet, a firm and a worker get to observe the productivity of their match before deciding whether to start an employment relationship. We derive necessary and sufficient conditions for the existence of a BGP, a path along which unemployment, vacancies, UE, and EU rates remain constant in the face of improving production and search technologies.

A. *Environment*

The labor market is populated by a continuum of workers with measure 1 and by a continuum of firms with positive measure. The objective of a worker is to maximize the present value of labor income discounted at the rate $r > 0$, where income is a wage w_t if the worker is employed and an unemployment benefit b_t if he is unemployed. The objective of a firm is to maximize the present value of profits discounted at the rate r. A firm operates a technology that turns the flow of labor supplied by a worker into a flow $y_t z$ of output, where y_t is the component of productivity that is common to all firm-worker pairs and z is the component that is idiosyncratic to a specific firm-worker pair.

The labor market is subject to search frictions. Unemployed workers need to search the market to locate vacant jobs. Firms need to search the market to locate workers for their vacancies, which are maintained at the flow cost $k_t > 0$. The outcome of the search process is a flow $A_t M(u_t, v_t)$ of random bilateral meetings between unemployed workers and vacant jobs, where u_t and v_t are the measures of unemployed workers and vacant firms, M is a constant return to scale function, and A_t is the efficiency of search.[8] An unemployed worker meets a vacancy at the rate

[7] In Kortum (1997), the innovation rate is the measure of researchers times the probability that a researcher draws an idea better than the best available one. This is the probability that a draw from the sampling distribution is higher than the best past draw. In our model, the UE rate is the rate at which an unemployed worker meets a vacancy times the probability that the quality of the firm-worker match is above the reservation cutoff. This is the probability that a draw from the quality distribution exceeds the value of sampling again. The success cutoff in Kortum (1997) is backward looking (the best draw in the past). The success cutoff in our model is forward looking (the option of continuing search).

[8] We assume that search is random. The assumption is not crucial, as the conditions and properties of a BGP would be exactly the same if search were directed as in Moen (1997) or Menzio and Shi (2010, 2011).

$A_t p(\theta_t)$, where $\theta_t \equiv v_t/u_t$ is the labor market tightness, and $p(\theta) \equiv M(1,\theta)$ is a strictly increasing and concave function such that $p(0) = 0$ and $p(\infty) = \infty$. A vacancy meets an unemployed worker at the rate $A_t q(\theta_t)$, where $q(\theta) = p(\theta)/\theta$ is a strictly decreasing function such that $q(0) = \infty$ and $q(\infty) = 0$.

Upon meeting, a firm and a worker draw the component of productivity z that is specific to their match from a cumulative distribution function (CDF) F. After observing z, the firm and the worker decide whether to match. If they do, the firm and the worker bargain over the terms of an employment contract and start producing a flow $y_t z$ of output. Production continues until the match is broken off. If they do not match, the worker remains unemployed and the firm's job remains vacant.

The terms of the employment contract are determined by the axiomatic Nash bargaining solution; that is, they maximize the product between the worker's gains from trade taken to the power γ and the firm's gains from trade taken to the power $1-\gamma$. The worker's gains from trade are the difference between the value of the match to the worker and his disagreement point, which we take to be the value of unemployment. The firm's gains from trade are the difference between the value of the match to the firm and its disagreement point, which we take to be the value of a vacancy. The contract specifies, directly or indirectly, a path for the worker's wage and a breakup date. We assume that the contract has enough contingencies to guarantee that the breakup date maximizes the joint value of the match.[9] Given this assumption, the Nash bargaining solution allocates a fraction γ of the total gains from trade to the worker and $1-\gamma$ to the firm.

The environment is nonstationary. The aggregate component y_t of productivity grows at the rate $g_y \geq 0$, which captures the idea that progress in the production technology allows firms to produce more output with the same inputs. The efficiency A_t of search grows at the rate $g_A > 0$, which captures the idea that progress in information technology makes it easier for workers to locate vacancies and for firms to locate workers.[10] We also assume that the cost of a vacancy grows at the rate g_k and the unemployment benefit grows at the rate g_b.

[9] There are many employment contracts with enough contingencies to guarantee that the joint value of the match is maximized. For example, if the employment contract can specify a wage path and a breakup date, the joint value of the match is maximized. The same is true if the employment contract can specify a wage path but the worker and firm are free to leave the match at any time. The same is true even if the employment contract can specify a wage only over the next interval of time, after which the wage is rebargained (as in Mortensen and Pissarides 1994).

[10] We model progress in the search technology as Hicks neutral. In the case of Hicks-neutral progress, the growth rate g_p of the meeting rate between a worker and a firm is equal to g_A. In the case of input-augmenting search progress, the rate g_p converges to some g_p^*, which depends on g_A and on the shape of M. In the limit as $g_p \to g_p^*$, our theorems hold with g_p^* replacing g_A.

The model is a version of Mortensen and Pissarides (1994), in which matches are inspection goods, in the sense that firms and workers observe z before deciding whether to consummate their match. The assumption that firms and workers have some information about z prior to consummating the match is crucial for the existence of a BGP, as it creates a wedge between the rate at which unemployed workers meet vacancies, which is assumed to grow because of improvements in the search technology, and the rate at which unemployed workers become employed, which is required to be constant along a BGP. If matches were experience goods, in the sense that firms and workers knew nothing about z before consummating their match, a BGP could not exist. In that case, the growth in the rate at which unemployed workers meet vacancies would always cause growth in the UE rate. In the baseline model, we assume that firms and workers perfectly observe z upon meeting. In section III.B, we consider the case in which firms and workers observe only a signal about z.

B. *Definition of a BGP*

In order to define a BGP, we need to introduce some notation. Let $V_t(z)$ denote the joint value of a firm-worker match of quality z, where the joint value is defined as the sum of the worker's present value of income and the firm's present value of profits generated by the worker. Let U_t denote the value of unemployment to a worker. Further, let θ_t denote the tightness of the labor market, u_t the measure of unemployed workers, and $G_t(z)$ the CDF of employed workers across match qualities.

The initial state of the economy is the distribution of workers across employment states at date $t = 0$, that is, u_0 and G_0. A rational expectation equilibrium is a path for V_t, U_t, θ_t, u_t, and G_t such that the agents' decisions are optimal, markets are clear, and the evolution of u_t and G_t is consistent with the agents' decisions and the initial state u_0, G_0. A BGP is an initial state and an associated rational expectation equilibrium such that unemployment, tightness, UE, and EU rates are constant over time, and the distribution G_t grows at some constant rate (in the sense that every quantile of G_t grows at the same constant rate). Note that as in the definition of a steady state, the initial conditions are not taken as given in the definition of a BGP.

We are now in the position to formally define a BGP. The joint value $V_t(z)$ of a firm-worker match of quality z is such that

$$V_t(z) = \max_{d\geq 0}\int_t^{t+d} e^{-r(\tau-t)} y_\tau z d\tau + e^{-rd} U_{t+d}. \tag{1}$$

At date τ, the sum of the worker's income and the firm's profit is equal to the flow of output $y_\tau z$. After d units of time, the firm and the worker break up. After the breakup, the worker's present value of income is U_{t+d} and the firm's present value of profits generated by the worker is 0. Note that V_t is well defined only if the discount rate r exceeds the growth rate of the common component of productivity; that is, $r > g_y$.

The optimal breakup date d must satisfy

$$\begin{aligned} y_{t+d}z + \mathring{U}_{t+d} \ &\leq \ rU_{t+d}, \\ d &\geq 0, \end{aligned} \tag{2}$$

where the two inequalities hold with complementary slackness. The left-hand side of condition (2) is the marginal benefit of delaying the breakup, which is given by the flow of output of the match, $y_{t+d}z$, plus the time derivative of the worker's value of unemployment, $\mathring{U}_{t+d}$. The right-hand side is the marginal cost of delaying the breakup, which is given by the sum of the annuitized values that the worker and the firm can attain by breaking up, rU_{t+d}. Condition (2) states that either $d = 0$ and the marginal cost exceeds the marginal benefit or $d > 0$ and the marginal cost equates the marginal benefit. Note that condition (2) is also sufficient because in any BGP, the right-hand side grows at a faster rate than the left-hand side.

The reservation quality R_t is defined as

$$y_t R_t = rU_t - \mathring{U}_t. \tag{3}$$

From condition (2), it follows that an existing match between a firm and a worker is maintained at date t iff its quality z is greater than R_t. Similarly, a meeting between a firm and a worker leads to a match at date t iff its quality z is greater than R_t. That is, R_t is the lowest quality for which existing matches are maintained and new matches are consummated. Define the surplus $S_t(z)$ of an existing or new match as $V_t(z) - U_t$. Then, $S_t(z)$ is positive if z is greater than the reservation quality R_t. Otherwise, $S_t(z) = 0$.

The value of unemployment to a worker, U_t, is such that

$$rU_t = b_t + A_t p(\theta)\gamma \int_{R_t} S_t(\hat{z})\,dF(\hat{z}) + \mathring{U}_t. \tag{4}$$

The left-hand side is the annuitized value of unemployment to a worker. The right-hand side is the sum of three terms. The first term is the worker's flow income from unemployment. The second term is the worker's option value of searching, which is given by the rate at which the worker meets a vacancy times a fraction γ of the expected surplus of a meeting between the worker and a vacancy. The last term is the time derivative of the worker's value of unemployment. Note that U_t is well defined only if the discount rate exceeds the growth rate of b_t; that is, $r > g_b$.

The tightness of the labor market, θ, is such that

$$k_t = A_t q(\theta)(1-\gamma)\int_{R_t} S_t(\hat{z})\,dF(\hat{z}). \tag{5}$$

The left-hand side is the cost to a firm of maintaining a vacancy. The right-hand side is the benefit of maintaining a vacancy, which is given by the rate at which the vacancy meets a worker times a fraction $1-\gamma$ of the expected surplus of a meeting between the vacancy and a worker. In order for the vacancy-to-unemployment ratio θ to be consistent with the firm's optimal behavior, the cost of maintaining an additional vacancy must equal the benefit.

In a BGP, the UE and EU rates as well as u and v are required to be constant. The requirement is fulfilled iff

$$A_t p(\theta)[1-F(R_t)] = h_{\text{ue}}, \tag{6}$$

$$G_t'(R_t)\mathring{R}_t = h_{\text{eu}}, \tag{7}$$

$$u h_{\text{ue}} = (1-u)h_{\text{eu}}. \tag{8}$$

The UE rate at date t is the product between the rate at which an unemployed worker meets a vacancy and the probability that the quality of their match is above R_t. Condition (6) states that the UE rate is equal to some constant h_{ue} for all $t \geq 0$. The EU rate at date t is the product between the density of the distribution of employed workers at R_t and the time derivative of R_t. Condition (7) states that the EU rate is equal to some constant h_{eu} for all $t \geq 0$. The condition for the stationarity of unemployment u is (8), which states that the flow of workers into unemployment is equal to the flow of workers out of unemployment at all dates $t \geq 0$. Given the stationarity of u and the stationarity of θ implied by condition (5), it follows that vacancies v are stationary as well.

In a BGP, the distribution $G_t(z)$ of employed workers across match qualities is required to grow at some constant rate. Formally, the constant growth condition for $G_t(z)$ is $z_t(x) = z_0(x)\exp(g_z t)$ for all $x \in [0,1]$ and all $t \geq 0$, where $zt(x)$ is the xth quantile of G_t and g_z is some endogenous growth rate. The condition is satisfied iff

$$(1-u)G_t'(z_t(x))z_t(x)g_z + uA_t p(\theta)[F(z_t(x)) - F(R_t)] = (1-u)G_t'(R_t)\mathring{R}_t. \tag{9}$$

The left-hand side is the flow of workers into matches with quality lower than an xth quantile growing at the rate g_z. The first term on the left-hand side is the flow of workers employed in a match of quality z that, in the next instant, falls below the growing xth quantile. The second term is the flow of unemployed workers who, in the next instant, become employed in a match of quality z below the xth quantile. The right-hand side is the flow of workers out of matches with quality lower than the xth quantile. This is the flow of workers who leave employment because the quality of their match, in the next instant, falls below the growing reservation quality R_t. Condition (9) thus guarantees that the measure of workers in matches with quality lower than an xth quantile growing at the rate g_z remains constant over time.

C. *Necessary Conditions for a BGP*

We now derive some conditions on the fundamentals of the economy that are necessary for the existence of a BGP. First, we derive a necessary condition on the distribution F from which firms and workers sample the quality of their match. The stationarity condition (6) for the UE rate implies

$$g_A = \frac{F'(R_t)}{1 - F(R_t)} R_t g_z. \tag{10}$$

The left-hand side is the elasticity with respect to t of the rate at which an unemployed worker meets a vacancy. This elasticity is the growth rate g_A of the efficiency of the search technology. The right-hand side is the negative of the elasticity with respect to t of the probability that the match between an unemployed worker and a vacancy has a quality above R_t. Since R_t is the 0th quantile of the distribution G_t, R_t grows at the rate g_z and the elasticity is $[F'(R_t)/(1 - F(R_t))]R_t g_z$. The UE rate remains constant over time only if the left-hand and the right-hand sides of condition (10) are equal. That is, the UE rate remains constant over time only if the increase in the rate at which an unemployed worker meets a vacancy is exactly offset by the decline in the probability that their match is good enough to be consummated.

Condition (10) is effectively a differential equation for the sampling distribution F, as R_t grows over time from R_0 to ∞. The unique solution to this differential equation that satisfies the boundary condition $F(\infty) = 1$ is

$$F(z) = 1 - \left(\frac{z_\ell}{z}\right)^\alpha, \tag{11}$$

where $\alpha = g_A/g_z$ and z_ℓ is an arbitrary lower bound nongreater than R_0. Since condition (10) is necessary, it follows that a BGP may exist only if the sampling distribution F is the one given in expression (11), which is a Pareto distribution with some tail coefficient α. It is important to notice that $\alpha = g_A/g_z$ is not a restriction on the tail coefficient of the sampling distribution, as g_z is an endogenous object. Instead, $\alpha = g_A/g_z$ should be interpreted as stating that in any BGP, the endogenous growth rate g_z must be equal to the ratio between the exogenous, arbitrary growth rate

g_A of the efficiency of search and the exogenous, arbitrary tail coefficient α of the sampling distribution.

Second, we derive a necessary condition on the growth rate g_b of the worker's unemployment benefit and on the growth rate g_k of the firm's vacancy cost. The optimality condition (3) for the reservation quality can be written as

$$\begin{aligned} y_t R_t &= b_t + A_t p(\theta)\gamma \int_{R_t} S_t(z)\, dF(z) \\ &= b_t + \frac{\gamma}{1-\gamma}\theta k_t, \end{aligned} \tag{12}$$

where the first line makes use of the Bellman equation (4) for the value of unemployment U_t to substitute out $rU_t - \mathring{U}_t$ and the second line makes use of the optimality condition (5) for the market tightness θ to substitute out the expected surplus of a match. The left-hand side of condition (12) is the output of a marginal match, and it grows at the rate $g_y + g_z$. The first term on the right-hand side of condition (12) is the worker's unemployment benefit, and it grows at the rate g_b. The second term is the worker's option value of searching, which, in equilibrium, must be proportional to the firm's vacancy cost and hence grows at the rate g_k. Since condition (12) must hold for all $t \geq 0$, a BGP may exist only if the left-hand and the right-hand sides of condition (12) grow at the same rate. That is, a BGP may exist only if g_b and g_k grow at the rate $g_y + g_z$.

LEMMA 1 (Necessary conditions for a BGP). Let $g_A > 0$ and $g_y \geq 0$ be arbitrary growth rates for the production and search technologies.

1. A BGP may exist only if (*a*) the distribution F is Pareto with an arbitrary coefficient α; (*b*) the growth rate of the vacancy cost, g_k, and the growth rate of the unemployment benefit, g_b, are equal to $g_y + g_z$; and (*c*) the discount rate r is greater than $g_y + g_z$.
2. In any BGP, the growth rate g_z of the distribution G_t is equal to g_A/α.

Let us make a few comments about the necessary conditions for the existence of a BGP. The requirement that F is Pareto does not imply that there is a great deal of heterogeneity in the productivity of different firm-worker matches. Indeed, the variance of the productivity of different matches can be made arbitrarily small if the coefficient α is sufficiently large. The requirement that F is Pareto does imply that there are some firm-worker matches that are arbitrarily productive. This might seem implausible to some of our readers. Note, however, that the F distribution must be Pareto if we want the economy to remain on a BGP indefinitely. If, in contrast, we only want the economy to remain on a BGP up to some period T, F must be Pareto over the interval $[R_0, R_T]$ but, to the right of R_T, F may take any shape as long as it has the same expected value for z as a Pareto.

The requirement that g_b and g_k are equal to $g_y + g_z$ would seem, at first blush, to imply that the existence of a BGP is a knife-edge result that obtains only if the exogenous growth rates of unemployment benefits and vacancy costs take a particular value. If that were the case, our theory of a BGP would not be very satisfactory. However, as we shall see in the next few pages, the growth rate of b_t and k_t that is necessary for the existence of a BGP is exactly the growth rate of wages, productivity, and output per capita. Hence, if the input to produce vacancies is labor and if unemployment benefits are proportional to average wages or average productivity, k_t and b_t endogenously grow at precisely the rate $g_y + g_z$. In appendix B, we develop such a version of the model.

D. Existence and Uniqueness of a BGP

Let us assume that the sampling distribution F is Pareto with tail coefficient α, the growth rate g_k of the vacancy cost and the growth rate g_b of the unemployment benefit are equal to $g_y + g_z$, and the discount rate r is greater than $g_y + g_z$. We now show that a BGP exists and is unique.

The first step is to solve for the expected surplus of a meeting between a firm and a worker. To this aim, note that the surplus $S_t(z)$ of a firm-worker match with quality $z > R_t$ is

$$S_t(z) = \int_t^{t+d_t(z)} e^{-r(\tau-t)}(y_\tau z - y_\tau R_\tau)\,d\tau, \tag{13}$$

where $d_t(z) \equiv \log(z/R_t)/g_z$ is the optimal duration of the match. The expression above states that the surplus of a match with quality z is equal to the present discounted value of the difference between the flow output $y_\tau z$ of the match and the flow output $y_\tau R_\tau$ of a marginal match between dates t and $t + d_t(z)$. The expression above is obtained by taking the difference between the joint value of the match, which is given by equation (1), and the worker's value of unemployment, which, in light of condition (3), is given by the present value of the flow output of a marginal match between dates t and $t + d_t(z)$ plus the discounted value of unemployment at date $t + d_t(z)$.

Using the fact that y_t grows at the rate g_y and R_t grows at the rate g_z, we solve the integral on the right-hand side of equation (13) and find that

$$S_t(z) = y_t\left\{\frac{z}{r-g_y}\left[1-\left(\frac{R_t}{z}\right)^{(r-g_y)/gz}\right] - \frac{R_t}{r-g_y-g_z}\left[1-\left(\frac{R_t}{z}\right)^{(r-g_y-g_z)/gz}\right]\right\}. \tag{14}$$

The expected surplus of a meeting between a firm and a worker is the expectation of the surplus $S_t(z)$ with respect to the quality distribution of F. Using equation (14) and the fact that F is a Pareto with tail coefficient α, we find that if $\alpha > 1$, the expected surplus of a meeting between a firm and a worker is

$$\int_{R_t} S_t(\hat{z}) F'(\hat{z}) d\hat{z} = \Phi y_t R_t^{-(\alpha-1)}, \tag{15}$$

where Φ is a positive constant that depends on parameters. In words, the expected surplus of a meeting is proportional to the product between the aggregate component of productivity and the reservation idiosyncratic component of productivity taken to the power of $-(\alpha - 1)$. Hence, the expected surplus of a meeting grows over time at the rate $g_y - (\alpha - 1)g_z$. If $\alpha \leq 1$, the expected surplus of a meeting is not well defined. Thus, we proceed under the assumption $\alpha > 1$.

The second step is solving for the reservation quality. Using equation (15) to substitute out the expected surplus of a meeting between a firm and a worker, we can write the optimality condition (12) for the reservation quality R_t as

$$y_t R_t = b_t + A_t p(\theta) \gamma \Phi y_t R_t^{-(\alpha-1)}. \tag{16}$$

Let R_0 be a solution of equation (16) for $t = 0$. Then $R_t = R_0 \exp(g_z t)$ is a solution of equation (16) for all $t > 0$ iff $g_z = g_A/\alpha$. To see why this is the case, note that the left-hand side of equation (16) grows at the rate $g_y + g_z$. The first term on the right-hand side of equation (16) grows at the rate g_b, which is assumed to be equal to $g_y + g_z$. The second term on the right-hand side grows at the rate $g_A + g_y - (\alpha - 1)g_z$. The left-hand and right-hand sides of equation (16) grow at the same rate iff $g_z = g_A/\alpha$.

The third step is solving for the tightness of the labor market. Using equation (15) to substitute out the expected surplus of a meeting between a firm and a worker, we can write the optimality condition (5) for the tightness θ as

$$k_t = A_t q(\theta)(1 - \gamma) \Phi y_t R_t^{-(\alpha-1)}. \tag{17}$$

Let θ be a solution of equation (17) for $t = 0$. Then, θ is also a solution of equation (17) for all $t > 0$. To see why this is the case, note that the left-hand side of equation (17) grows at the rate g_k, which is assumed to be equal to $g_y + g_z$. The right-hand side of equation (17) grows at the rate $g_A + g_y - (\alpha - 1)g_z$. The two growth rates are equal because $g_z = g_A/\alpha$.

The fourth step is solving for the initial distribution of employed workers across match qualities. Using the stationarity condition (8) for unemployment to substitute the flow of workers out of employment with the flow of workers into employment in (9) and using the fact that F is a Pareto distribution with coefficient α, we can rewrite the balanced growth condition for the distribution G_t of employed workers as

$$(1-u)G_t'(ze^{g_z t})ze^{g_z t}g_z = uA_t p(\theta)\left(\frac{z_\ell}{ze^{g_z t}}\right)^\alpha, \tag{18}$$

where $g_z = g_A/\alpha$. At $t = 0$, condition (18) is a differential equation for the initial distribution G_0 of employed workers across qualities that depends on the unemployment rate u. The unique G_0 and u that solve the differential equation and satisfy the boundary conditions $G_0(R_0) = 0$ and $G_0(\infty) = 1$ are

$$G_0(z) = 1 - \left(\frac{R_0}{z}\right)^\alpha,$$
$$u = \frac{g_A}{g_A + A_0 p(\theta)(z_\ell/R_0)^\alpha}. \tag{19}$$

The initial distribution G_0 of employed workers is the sampling distribution F truncated at the initial reservation quality R_0. Then, the distribution G_t grows at the constant rate $g_z = g_A/\alpha$. In fact, it is easy to verify that $G_t(z\exp(g_z t)) = G_0(z)$ satisfies the balanced growth condition (18) for all $t > 0$.

The last step is to verify the stationarity conditions for the UE, EU, and unemployment rates. The UE and EU rates are

$$h_{ue} = A_t p(\theta)\left(\frac{z_\ell}{R_t}\right)^\alpha = A_0 p(\theta)\left(\frac{z_\ell}{R_0}\right)^\alpha, \tag{20}$$

$$h_{eu} = G_t'(R_t)R_t g_z = g_A. \tag{21}$$

The UE rate is stationary. To see why, note that the rate at which an unemployed worker meets a vacancy grows at the rate g_A, the probability that the quality of the meeting exceeds the reservation R_t falls at the rate αg_z, and the two rates are equal to each other because $g_z = g_A/\alpha$. The EU rate is also stationary. To see why, note that $G_t(z\exp(g_z t)) = G_0(z)$ implies $G_t'(z\exp(g_z t)) = G_0'(z)\exp(-g_z t)$; hence, $G_t'(R_t)R_t g_z$ is equal to the constant $G_0'(R_0)R_0 g_z$. In light of equation (19) and $g_z = g_A/\alpha$, it follows that $G_0'(R_0)R_0 g_z$ is equal to g_A. Given the stationary values of the UE and EU rates in equations (20) and (21), it is immediate to see that the unemployment rate in equation (19) satisfies the stationarity condition (8).

In the previous steps, we have shown that all the equilibrium conditions for a BGP are satisfied as long as there are a reservation quality R_0 and a tightness θ that satisfy the optimality conditions (16) and (17) for $t = 0$. We have also shown that the BGP is uniquely pinned down up to R_0 and θ. We now turn to solving for R_0 and θ. The solution to condition (16) for R_0 exists and is unique for all $\theta \geq 0$, and we denote it as $R_0^*(\theta)$. It is easy to verify that $R_0^*(0) = b_0/y_0$, $R_0^{*\prime}(\theta) > 0$, and $R_0^*(\infty) = \infty$. The solution to condition (17) for θ exists and is unique for all $R_0 \geq 0$, and we denote it as

$\theta^*(R_0)$. It is easy to verify that $\theta^*(0) = \infty$, $\theta^{*\prime}(R_0) < 0$, and $\theta^*(\infty) = 0$. From these observations, it follows that there exists one and only one pair $(R_0, \theta) \in \mathbb{R}^2_+$ that solves conditions (16) and (17). Hence, a BGP exists and is unique.

THEOREM 1 (Existence and properties of a BGP). Let $g_A > 0$ and $g_y \geq 0$. A BGP exists iff (*a*) F is Pareto with coefficient $\alpha > 1$; (*b*) $g_b, g_k = g_y + g_A/\alpha$; and (*c*) $r > g_y + g_A/\alpha$. If a BGP exists, it is unique and such that

i. u, θ, h_{ue}, and h_{eu} are constant, with

$$h_{ue} = A_0 p(\theta)[1 - F(R_0)],$$
$$h_{eu} = g_A;$$

ii. $G_t(z\exp(g_z t)) = G_0(z)$, with $g_z = g_A/\alpha$ and

$$G_0(z) = \frac{F(z) - F(R_0)}{1 - F(R_0)};$$

and

iii. labor productivity grows at the rate $g_y + g_A/\alpha$.

Theorem 1 states that a BGP exists if and only if the sampling distribution of match quality is Pareto with some coefficient $\alpha > 1$ and unemployment benefits and vacancy costs both grow at the same rate as labor productivity. In a BGP, unemployment, market tightness, vacancies, UE, and EU rates all remain constant over time even though the efficiency of the search technology keeps growing at the rate g_A. While improvements in the search technology do not lower unemployment, they do contribute to labor productivity growth.

Let us provide some intuition for the UE rate being constant over time. Growth in the efficiency of the search technology has two effects on the UE rate. On the one hand, it increases the rate at which an unemployed worker meets a firm. On the other hand, it increases the worker's option value of unemployment and the reservation match quality, and for this reason, it lowers the probability that a match between an unemployed worker and a firm is consummated. When the sampling distribution F is Pareto with coefficient α and the unemployment benefit grows at the same rate as labor productivity, the reservation quality grows at the rate g_A/α, and the probability that a firm-worker match is consummated falls at the rate g_A. Hence, the two effects that the growth in the efficiency of the search technology has on the UE rate exactly cancel each other out.

Next, let us explain why the EU rate remains constant over time. Employed workers are initially distributed across match qualities according to the sampling distribution F truncated at the reservation quality R_0. As the reservation quality grows, the employed workers who are in matches with a quality that falls behind R_t become unemployed, and the employed workers who survive are distributed according to F truncated at R_t. The unemployed workers who become employed are also distributed according to F truncated at R_t. Hence, the overall distribution G_t of employed workers is equal to F truncated at R_t. Since G_t has always the same shape, the flow of employed workers who become unemployed remains constant over time. Since the UE and EU rates are constant, so is the unemployment rate.

Finally, let us explain why the tightness of the labor market remains constant over time. The benefit of a vacancy is given by the product between the rate at which the vacancy meets a worker, which grows at the rate g_A for a constant tightness θ, and the expected surplus of a meeting between the vacancy and a worker, which grows at the rate $g_y - (\alpha - 1)g_A/\alpha$. When the cost of a vacancy grows at the rate $g_y + g_A/\alpha$, the benefit and the cost grow at exactly the same rate for a constant tightness θ.

In a BGP, improvements in the search technology translate into labor productivity growth. To see this, note that the average labor productivity is given by

$$\int_{R_t} y_t z G_t'(z)\, dz = \frac{\alpha}{\alpha - 1} y_t R_t. \tag{22}$$

Average labor productivity is proportional to the product of the common component of productivity, y_t, and the reservation idiosyncratic component of productivity, R_t. Hence, average labor productivity grows at the rate $g_y + g_A/\alpha$, the sum of the growth rate of the aggregate component of productivity and the growth rate of the efficiency of search divided by the tail coefficient of the sampling distribution F. Growth in the efficiency of search translates into labor productivity growth because it allows firms and workers to become pickier with respect to their match quality. The rate at which growth in the efficiency of search translates into labor productivity growth depends on the thickness $1/\alpha$ of the tail of the sampling distribution F, as this thickness determines the return to faster search.

III. Generalizations and Variations

In this section, we extend theorem 1 to two natural generalizations of the baseline environment. We consider a generalization in which workers may search off and on the job and one in which the measure of workers may grow over time and the search process may have nonconstant returns to scale. We then derive versions of theorem 1 for alternative specifications of the baseline environment. The first variant of the environment is such that workers and firms observe only noisy signals about the quality of their match. The second variant considers alternative bargaining solutions. The third variant is such that workers and firms are ex ante heterogeneous.[11]

A. Generalizations

1. Search on the Job

We first want to extend the baseline model to allow workers to search the labor market both when they are unemployed and when they are employed, albeit with different intensity. This is a crucial extension. First, search on the job is empirically relevant. The rate at which workers move directly from one employer to another is around 1.5% a month, which is almost as high as the rate at which workers move from employment into unemployment. Second, search on the job affects the key trade-offs facing workers and firms. An unemployed worker's decision to accept or reject a job offer depends on whether he can keep searching for a better job once he becomes employed. A firm's decision of how many vacancies to open depends on how many searching workers are unemployed (and hence have a weak outside option) and how many are employed (and hence have a stronger outside option).

We consider a version of the model in which unemployed workers search for jobs with an intensity normalized to 1 and employed workers search with an intensity of $\rho \in [0, 1]$. Firms search for workers by opening vacancies. The outcome of the search process is a flow $A_t M(s_t, v_t)$ of random, bilateral meetings between workers and vacancies, where $s_t \equiv u_t + \rho(1 - u_t)$ is the intensity-weighted measure of searching workers. An unemployed worker meets a vacancy at the rate $A_t p(\theta_t)$, where $\theta_t \equiv v_t / s_t$. An employed worker meets a vacancy at the rate $\rho A_t p(\theta_t)$. When a worker and a vacancy meet, they observe the quality z of their match and decide whether to consummate the match. If they do, they bargain over the terms of a bilaterally efficient contract. If the worker is unemployed, his outside option is the value of unemployment. If the worker is employed, his outside option is the joint value of the match with his current employer.[12]

We find that the necessary and sufficient conditions for the existence of a BGP are the same in the model with search on the job as in the

[11] We also examined versions of the model with endogenous search effort. Suppose that the flow payoff for an unemployed worker is $v(b_t, e_t)$, where e_t denotes the fraction of time devoted to search. First, we show that—under the conditions of theorem 1—there exists a BGP in which effort, UE, EU, u, and v rates are constant iff $v(b_t, e_t)$ has the form $b_t \phi(e_t)$. This condition is analogous to one of the necessary conditions for the existence of a BGP in the neoclassical growth model (see King, Plosser, and Rebelo 1988). Second, we show that—except for knife-edge cases—there exists no function $v(b_t, e_t)$ that supports a BGP in which the UE rate is constant because the search effort e_t falls at the rate g_A while the reservation quality R_t remains constant. That is, income effects alone cannot generically support a BGP.

[12] This is a common assumption (see, e.g., Cahuc, Postel-Vinay, and Robin 2006; Bagger, Fontaine, Postel-Vinay, and Robin 2014; Herkenhoff, Lise, Menzio, and Phillips 2018).

baseline model. Since the stationarity condition for the UE rate is the same as in the baseline model, the sampling distribution F must be Pareto. Given that F is Pareto, we can show that the expected surplus of a meeting between a firm and an unemployed worker grows at the same rate $g_y - (\alpha - 1)g_z$ as in the baseline model. Similarly, the expected surplus of a match between a firm and a randomly selected employed worker grows at the rate $g_y - (\alpha - 1)g_z$. This is true even though the surplus of a meeting includes the worker's option of searching on the job.

The reservation quality is equal to the unemployment benefit plus a fraction $1 - \rho$, rather than a fraction 1, of the option value of searching while unemployed. Since the option value of searching while unemployed grows at the rate $g_A + g_y - (\alpha - 1)g_z$, the reservation quality grows at the constant rate $g_z = g_A/\alpha$ and the UE rate remains constant iff the unemployment benefit grows at the rate $g_b = g_y + g_A/\alpha$. This is the same condition as in the baseline because even though search on the job affects the level of the reservation quality, it does not affect its growth rate. The benefit of a vacancy is equal to the meeting rate times an average between the expected surplus of meeting an unemployed worker and the expected surplus of meeting an employed worker. Since the expected surplus of both meetings grows at the rate $g_y - (\alpha - 1)g_z$, the benefit of a vacancy grows at the rate $g_A + g_y - (\alpha - 1)g_z = g_y + g_A/\alpha$. The tightness θ thus remains constant iff the vacancy cost grows at the same rate as the benefit; that is, $g_k = g_y + g_A/\alpha$. This is the same condition as in the baseline model because even though search on the job affects the composition of workers encountered by a firm, it does not affect the growth rate of the surplus of those meetings. Given the proper initial conditions for u and G_0, unemployment remains constant over time, and the distribution of employed workers grows at the constant rate $g_z = g_A/\alpha$.

The properties of a BGP are essentially the same as in the baseline model. The only difference is that the distribution G_t of employed workers across match qualities is not equal to the sampling distribution F truncated at the reservation quality R_t. Instead, because of search on the job, the distribution G_t is a Fréchet truncated at R_t. The shape parameter of the Fréchet is α, the tail coefficient of the sampling distribution F. The scale parameter of the Fréchet depends on the intensity of search on the job and on the tightness of the labor market.

THEOREM 2 (On-the-job search). Let $g_A > 0$ and $g_y \geq 0$. A BGP exists iff (*a*) F is Pareto with coefficient $\alpha > 1$; (*b*) $g_b, g_k = g_y + g_A/\alpha$; and (*c*) $r > g_y + g_A/\alpha$. Any BGP is such that

i. u, θ, h_{ue}, and h_{eu} are constant, with

$$h_{ue} = A_0 p(\theta)(1 - F(R_0)),$$

$$h_{eu} = A_0 p(\theta)(1 - F(R_0))\rho \frac{H(R_0)}{1 - H(R_0)};$$

ii. $G_t(z\exp(g_z t)) = G_0(z)$, with $g_z = g_A/\alpha$ and

$$G_0(z) = \frac{H(z) - H(R_0)}{1 - H(R_0)},$$
$$H(z) = \exp\left\{-\left[\frac{A_0 p(\theta)\rho}{g_A}\left(\frac{z_\ell}{z}\right)^\alpha\right]\right\};$$

and

iii. labor productivity grows at the rate $g_y + g_A/\alpha$.

Proof. The proof is given in appendix C. QED

2. Population Growth

Next, we want to extend the baseline model to allow for population growth. If the search process has constant returns to scale, the assumption of constant population is essentially without loss of generality. If, in contrast, the search process has nonconstant returns to scale, population growth does matter. The extension reveals an important and natural link between technological improvements in the search technology and returns to scale in the search process.

We consider a version of the baseline model in which population grows at some constant rate and the search process features arbitrary returns to scale. The measure of workers in the labor market at date t is N_t, which grows at the constant rate $g_N \geq 0$. The flow $g_N N_t$ of newborn workers enters the market in the state of unemployment. Unemployed workers and vacant jobs search for each other. The outcome of the search process is described by a flow $A_t N_t^\beta M(N_t u_t, N_t v_t)$ of random bilateral meetings between unemployed workers and vacant jobs, where u_t is the unemployment rate, v_t is the vacancy rate, and M is some increasing, constant returns to scale function. The coefficient β controls the returns to scale of the search process. If $\beta > 0$, the process has increasing returns to scale. If $\beta < 0$, the process has decreasing returns to scale. If $\beta = 0$, the process is scale independent.[13]

The crucial observation is that the version of the model with population growth and nonconstant returns to search is identical to the baseline model, except that the efficiency of the search process is given by $A_t N_t^\beta$

[13] We model increasing returns to scale as $AN^\beta M(N_u, N_v)$, where M is a constant returns to scale function and $\beta > 0$. Alternatively, one could model increasing returns to scale as $A\hat{M}(Nu, Nv)$, where $\hat{M}$ itself has increasing return to scale. The first formulation implies that the flow of meetings increases more than proportionally with market size N, and the flow of meetings increases proportionally with u and v. The second formulation implies that the flow of meetings increases more than proportionally (and with the same constant of proportionality) with both N and u, v. To the extent that u and v are constant along a BGP, the two formulations are conceptually equivalent. In general, they are not.

rather than by A_t. Thus, the efficiency of the search process grows at the rate $g_A + \beta g_N$ rather than at the rate g_A. That is, with population growth and nonconstant returns to scale, the efficiency of the search process grows not only because of technological improvements in search, as captured by g_A, but also because of the increasing market size, as captured by βg_N.

THEOREM 3 (Population growth). Let $g_A \geq 0$, $g_N \geq 0$, and $g_y \geq 0$, with $g_A + \beta g_N > 0$. A BGP exists iff (*a*) F is Pareto with coefficient $\alpha > 1$; (*b*) $g_b, g_k = g_y + (g_A + \beta g_N)/\alpha$; and (*c*) $r > g_y + (g_A + \beta g_N)/\alpha$. Any BGP is such that

i. u, θ, h_{ue}, and h_{eu} are constant, with

$$h_{\text{ue}} = A_0 N_0^{\beta} p(\theta)(1 - F(R_0)),$$
$$h_{\text{eu}} = g_A + \beta g_N;$$

ii. $G_t(z\exp(g_z t)) = G_0(z)$, with $g_z = (g_A + \beta g_N)/\alpha$ and

$$G_0(z) = \frac{F(z) - F(R_0)}{1 - F(R_0)};$$

and

iii. labor productivity grows at the rate $g_y + (g_A + \beta g_N)/\alpha$.

Proof. The proof is given in appendix D. QED

B. Variations

1. Imperfect Signals

In the baseline model, we assume that matches are perfect inspection goods. Here, we consider an alternative specification of the model in which matches are imperfect inspection goods, in the sense that the firm and the worker observe a noisy signal about the quality of their match upon meeting. Specifically, let ζ denote the signal about the quality of the match and with F_1 the CDF of signals. On the basis of ζ, the firm and the worker decide whether to consummate the match. If they do, the quality of the match is observed after t^* units of time. Let $z = \zeta\epsilon$ denote the quality of the match, where ϵ is a random variable with mean 1 distributed according to a CDF F_2.

The key condition for the existence of a BGP is that the distribution of signals F_1 is Pareto with coefficient α. In the model with noisy signals, there is a reservation signal Q_t that controls the creation of an employment relationship and a reservation quality R_t that controls the destruction of an employment relationship of a known quality. For the UE rate to be constant, the distribution F_1 of signals needs to be Pareto. Given that ζ is distributed as a Pareto and that z is a random variable proportional to

ζ, the expected surplus of a meeting between a firm and a worker grows at the constant rate $g_y - (\alpha - 1)g_z$, the reservation signal Q_t grows at the rate g_A/α, and the UE rate is constant. The reservation quality R_t grows also at the rate g_A/α. Thus, the probability that a match with signal $\zeta > Q_t$ turns out to be of quality $z < R_{t+t^*}$ is constant over time, and so is the EU rate.

PROPOSITION 1 (Imperfect signals). A BGP exists iff (*a*) F_1 is Pareto with coefficient $\alpha > 1$; (*b*) $g_b, g_k = g_y + g_A/\alpha$; and (*c*) $r > g_y + g_A/\alpha$. Any BGP is such that (1) u, θ, h_{ue}, and h_{eu} are constant; (2) $G_t(z\exp(g_z t)) = G_0(z)$, $Q_t = Q_0\exp(g_z t)$, and $R_t = R_0\exp(g_z t)$, with $g_z = g_A/\alpha$; and (3) labor productivity grows at the rate $g_y + g_A/\alpha$.

Proof. The proof is given in appendix E. QED

2. Bargaining

In the baseline model, we assume that the outcome of the bargain between a worker and a firm is the axiomatic Nash solution, given that the outside option of the worker is the value of unemployment and the outside option of the firm is the value of a vacancy. This is the standard assumption in the literature (see, e.g., Pissarides 1985; Mortensen and Pissarides 1994; Shimer 2005). Hall (2005, 2017) and Hall and Milgrom (2008) have, however, advocated for different bargaining solutions.

We consider a version of the model in which the bargaining outcome between a firm and a worker in a match of quality z at date t is such that the worker captures a fraction $\gamma_t(z)$ of the gains from trade, where $\gamma_t(z)$ is a function of t and z. As long as the worker's share of the gains from trade is such that $\gamma_t(z\exp(g_z t)) = \gamma_0(z)$, a BGP exists under the same conditions as in the baseline model. In words, γ_t must have the property that the worker's share of the gains from trade is the same at date 0 in a match of quality z and at date t in a match of quality $z\exp(g_z t)$. The property guarantees that the worker's expected gain from a meeting with a firm and the firm's expected gain from a meeting with a worker grow at the rate $g_y - (\alpha - 1)g_z$, as they do in the baseline model. The property is satisfied by several bargaining solutions proposed in the literature: (1) alternating offer games with a risk of breakdown, (2) alternating offer games with a time delay (in the spirit of Hall and Milgrom 2008, and (3) wage norms that grow at the same rate as the economy (in the spirit of Hall 2005).[14]

[14] Specifically, suppose that the firm and the worker bargain over the wage every dt units of time. The bargaining protocol involves alternating offers and takes place in virtual time. If, after an offer is rejected, there is a small probability that the firm and the worker separate forever, the outcome is a wage that gives to the worker a constant fraction of the gains from trade; i.e., $\gamma_t(z) = \phi$. If, after an offer is rejected, there is a small probability that the firm and the worker cannot produce for the next dt units of time but the two parties remain in contact with each other, the outcome is a wage $w_t(z) = \max\{b_t + \phi(y_t z - b_t), y_t R_t\}$. That

PROPOSITION 2 (Bargaining). Let the worker's share $\gamma_t(z)$ of the gains from trade be such that $\gamma_t(z\exp(g_z t)) = \gamma_0(z)$. Then, a BGP exists iff conditions *a*, *b*, and *c* in theorem 1 hold. A BGP satisfies properties i, ii, and iii in theorem 1.

Proof. The proof is given in appendix F. QED

3. Ex Ante Heterogeneity

In the baseline model, we assume that workers and firms are ex ante homogeneous but the quality of the match between different workers and different firms is ex post heterogeneous. The assumption is common (see, e.g., Pissarides 1984; Moscarini 2005; Menzio and Shi 2011). The assumption is heuristically motivated as a reduced-form representation of fundamental differences among workers and firms that interact to determine the quality of the match between a particular worker and a particular firm. What are then the BGP restrictions on the production function that maps the workers' and firms' types into quality?

Consider an alternative specification of the model in which workers and vacancies are ex ante heterogeneous and the quality of their match is determined by the interaction of their types. Specifically, workers are ex ante heterogeneous with respect to their type i, which is distributed uniformly along a circle of perimeter 1. Firms create vacancies that are ex ante heterogeneous with respect to their type j, also located along the unit circle. The quality of a match between a worker of type i and a firm of type j is $z(\delta)$, where δ denotes the shortest distance between i and j along the circle and z is a decreasing function.

Since worker and firm heterogeneity is horizontal, it is natural to focus on a symmetric equilibrium in which unemployed workers and vacant jobs are both uniformly distributed along the unit circle. In a symmetric equilibrium, the distance between an unemployed worker and a vacant job is uniform over the interval $[0, 1/2]$; hence, the distribution of qualities across matches between a randomly selected unemployed worker and a randomly selected vacant job is given by

$$P(z) = 1 - 2\delta^{-1}(z). \tag{23}$$

Clearly, the model with ex ante heterogeneous workers and firms is isomorphic to the baseline model, with the endogenous CDF P taking the place of the exogenous sampling distribution F. Therefore, a BGP

is, the worker and the firm share the flow income unless the worker's individual rationality constraint binds. Alternatively, suppose that wages are determined by social norms subject to individual rationality constraints. Specifically, suppose that the social norm is a $w_t^* > y_t R_t$, which grows at the rate $g_y + g_z$, and the wage is $w_t(z) = \min\{w_t^*, y_t z\}$.

exists if and only if b_t and k_t grow at the same rate as labor productivity and P is a Pareto with same tail coefficient $\alpha > 1$ or, equivalently, if and only if the production function $z(\delta)$ is

$$z(\delta) = z_\ell (2\delta)^{-1/\alpha}. \tag{24}$$

Proposition 3 (Ex ante heterogeneity). A BGP exists iff (*a*) the production function $z(\delta)$ has the form $z_\ell(2\delta)^{-1/\alpha}$ for $\alpha > 1$; (b) $g_b, g_k = g_y + g_A/\alpha$; and (*c*) $r > g_y + g_A/\alpha$. Any BGP satisfies properties i, ii, and iii in theorem 1.

IV. Identification and Calculations

We conclude by discussing some empirical implications of our theory. If the conditions for a BGP are satisfied, the fact that u, v, h_{ue}, and h_{eu} have no clear secular trend is not informative about the growth rate g_A in the search technology, nor is it informative about the returns to scale βg_N of the search process. For the same reason, the fact that u, v, h_{ue}, and h_{eu} are not systematically different across large and small cities does not convey information about the returns to scale of the search process. Is there a way, then, to identify the growth rate in the search technology, the returns to scale to the search process, and their contribution to economic growth?

A. *Identification*

According to our theory, the overall decline in search frictions can be inferred by looking at the growth rate in the number of workers that a firm meets before filling its vacancy. A firm meets an average of n_t workers before filling its vacancy, where

$$n_t = A_t N_t^\beta q(\theta) \cdot \frac{1}{A_t N_t^\beta q(\theta)[1 - F(R_t)]}. \tag{25}$$

The first term on the right-hand side of equation (25) is the number of workers that a firm meets per unit of time. The second term is the time it takes for a firm to fill a vacancy, which is the inverse of the vacancy-filling rate. The first term grows at the rate $g_A + \beta g_N$. The second term is constant, as $A_t N_t^\beta p(\theta)[1 - F(R_t)]$ is constant and $q(\theta) = p(\theta)/\theta$. Therefore, n_t grows at the rate $g_z + \beta g_N$.

The returns to scale in the search process can be inferred by looking at the number of workers that a firm meets before filling its vacancy in large and small markets. Consider two markets with the same search technology A_t but different populations $N_{1,t}$ and $N_{2,t}$. The average number of workers $n_{1,t}$ and $n_{2,t}$ entertained by firms in the two markets is such that

$$\frac{n_{1,t}}{n_{2,t}} = \frac{A_t N_{1,t}^{\beta} q(\theta_1)}{A_t N_{2,t}^{\beta} q(\theta_2)} \cdot \frac{A_t N_{2,t}^{\beta} q(\theta_2)[1 - F(R_{2,t})]}{A_t N_{1,t}^{\beta} q(\theta_1)[1 - F(R_{1,t})]}. \tag{26}$$

Assume that in the two markets, unemployment benefits and vacancy costs are equal to the same fraction of local wages (as they would be in the version of the model with endogenous b and k developed in app. B). Under these assumptions, the tightness in the two markets is the same, that is, $\theta_1 = \theta_2$, and the reservation quality in the two markets is such that $R_{t,1}/R_{t,2} = (N_{t,1}/N_{t,2})^{\beta/\alpha}$. Hence, $n_{t,1}/n_{t,2}$ is equal to $(N_{t,1}/N_{t,2})^{\beta}$.

The contribution of declining search frictions to growth depends on the tail coefficient α of the sampling distribution F. The coefficient α can be inferred from the distribution of wages. Suppose that wages are continuously renegotiated, as is the case in Mortensen and Pissarides (1994). Then, in the model without search on the job, a worker in a match of quality z earns

$$w_t(z) = \gamma y_t z + (1 - \gamma) y_t R_t. \tag{27}$$

The cross-sectional wage distribution $L_t(w)$ is not Pareto. However, the right tail of L_t is well approximated by a Pareto with coefficient α, as $d\log(1 - L_t(w))/d\log w$ converges to $-\alpha$. In the model with search on the job, it is the right-tail distribution of wages for workers hired directly out of unemployment that is well approximated by a Pareto with coefficient α.

The above observations imply the following identification theorem.

THEOREM 4 (Identification). Let data on meetings per vacancy and population, n_t and N_t, cross-sectional data on meetings per vacancy and market size, n_i and N_i, and the wage distribution, L_t, be available. Then β, g_A, and α are identified: (i) β is the elasticity of n_i with respect to N_i; (ii) $g_A + \beta g_N$ is the growth rate of n_t, and g_A is the growth rate of n_t net of βg_N; and (iii) α is the tail coefficient of L_t.

B. Back-of-the-Envelope Calculations

Implementing the identification strategy outlined in theorem 4 presents some challenges at both the conceptual and the data level. In the model, a meeting between a firm and a worker is an event in which the two parties become aware of each other and inspect (either perfectly or imperfectly) the quality of their match. In the model, any meeting between a firm and a worker has a quality that is drawn from the same time-invariant distribution. Then, the growth rate in number of workers that a firm meets before filling its vacancy coincides with the decline in search frictions. The best empirical measure of the number of workers met by a firm is probably the number of applications received by a firm. The measure is far from perfect, as it may either underestimate or overestimate the decline in search frictions. On the one hand, workers may get a signal about the quality of the match when they become aware of the vacancy and, on the basis of such signal, decide whether to send an application. In this case, the threshold for sending an application increases over time—as the hiring threshold rises—and hence applications become better and better, and the growth rate of applications per vacancy underestimates the decline in search frictions. On the other hand, the cost of sending an application may fall over time. In this case, the threshold for sending an application falls, and hence applications become worse and worse, and the growth rate of applications per vacancy overestimates the decline in search frictions.

In terms of data, there is no available time series for applications per vacancies in the United States. There are, however, measures of applications per vacancy at two points in time. Faberman and Menzio (2018) analyze the Employment Opportunity Pilot Project, a survey of US firms conducted in 1980 and 1982 that contains information about job openings, applications, and recruitment outcomes. They find that the average number of applications per vacancy is 24. Marinescu and Wolthoff (2016) analyze data from CareerBuilder, the largest online job site in the United States, which contains over 1 million jobs at a time and is visited by approximately 11 million unique job seekers per month. They find that the average number of applications per vacancy is 59 in the first quarter of 2011 (the focus of their study). In related work, Faberman and Kudlyak (2016) study data from Snag-a-Job, an online search engine that mainly focuses on hourly paid jobs, between September 2010 and September 2011. They find that the average number of applications per vacancy is 31.

The above findings suggest that the number of applications per vacancy increased from 24 in 1982 to somewhere between 31 and 59 in 2011. If we take an average between 31 and 59, these figures imply an average yearly growth rate of 2.2% in applications per vacancy. Subject to the caveats about the possible discrepancy between meetings per vacancy in the model and applications per vacancy in the data, 2.2% is an estimate of the rate $g_A + \beta g_N$ at which search frictions declined between 1982 and 2011.

The data from CareerBuilder also has information on the number of applications per vacancy across different markets in the United States. Ioana Marinescu kindly agreed to run for us a regression of log applications per vacancy on log population in the commuting zone of the vacancy. She estimates a regression coefficient of 0.52. She estimates a similar coefficient after controlling for occupation. Subject to the caveat about the discrepancy between meetings per vacancy in the model and applications per vacancy in the data, 0.52 is an estimate of the returns to scale β in the search process.[15] The 1982 wave of the National Longitudinal Survey of Youth contains information about the number of firms contacted by a worker during his most recent job search. Martellini (2019) regresses the log of the number of firms contacted by a searching worker on the log of the population in the commuting zone of the worker. He estimates a regression coefficient of 0.12. Subject to the same caveat about the mapping between data and model, 0.12 also represents an estimate of β. The average of the two estimates gives us $\beta = 0.32$.

Given our estimates of $g_A + \beta g_N$ and β, we can break down the decline in search frictions into a component due to increasing returns to scale in the search process and a component due to improvements in the search technology. To this aim, note that the US labor force grew from 108 to 152 million people between 1982 and 2011, a yearly growth rate g_N of 1.1%. Then, increasing returns to scale in the search process contribute to a $\beta g_N = 0.35\,\%$ decline in search frictions per year, while improvements in the search technology contribute to a $g_A = 2.2\,\% - 0.35\,\% = 1.85\,\%$ decline. Hence, increasing returns contribute to about one-sixth of the decline in search frictions and technological improvements to approximately five-sixths.

In order to translate the decline in search frictions into a contribution to labor productivity growth, we need an estimate of α, the tail coefficient of the sampling distribution F. As stated in theorem 4, α can be estimated from the shape of the wage distribution. However, this is not a simple task. In the model, workers are inherently identical, and the wage dispersion is entirely caused by differences among workers in the quality of their match. In the data, workers are not inherently identical, and wage dispersion reflects both differences in match quality and differences in skills, human capital, and so on. Thus, to estimate α, we would need to purge the wage data from all fundamental differences among workers, which is a task beyond the scope of this paper. Instead, we shall present the implications of the model for different reasonable values of α.

If $\alpha = 5$, the 90–50 percentile ratio in the distribution of match qualities is 37%. In this case, the decline in search frictions contributes to a 0.44 percentage point increase in labor productivity per year. This is about 23% of the 1.9% yearly growth rate in output per worker in the US nonfarm business sector between 1982 and 2011. If $\alpha = 10$, the 90–50 percentile ratio in the distribution of match qualities is 17%. In this case, the decline in search frictions contributes to a 0.22 percentage point increase in labor productivity per year, which is about 11% of the total yearly growth rate in output per worker. Overall, the contribution

[15] Note that the estimate of β may also be biased if the search technology A_t is systematically different in larger than in smaller commuting zones.

of declining search frictions to economic growth is far from negligible for both $\alpha = 5$ and 10, which are conservative estimates of α. In fact, using a model of search on the job in which workers are heterogeneous in human capital, Martellini (2019) estimates α to be 3.6. Using a model of on-the-job search stratified by industry, Bontemps, Robin, and Van den Berg (2000) estimate a match quality distribution that is Pareto with a tail coefficient of 2.5.

Our estimates of the returns to scale in the search process have implications for understanding the city-size wage premium. If $\alpha = 5$, increasing returns in the search process alone make productivity and wages in a market with 2.2 million people (the average size of US metro areas with more than 0.75 million people) 19% higher than in a market with 0.14 million people (the average size of US metro areas with less than 0.75 million people). This is approximately two-thirds of the empirical wage gap between large and small metro areas in the US (Martellini 2019). If $\alpha = 10$, increasing returns in search alone make productivity and wages 9% higher in a market with 2.2 rather than 0.14 million people. This is approximately one-third of the empirical wage gap. In either case, the contribution of increasing returns to scale to the wage differential between large and small cities is substantial.

Appendix A

Data

A1. Unemployment Rate

The unemployment rate is constructed using the National Bureau of Economic Research (NBER) macro-history files from 1927 to 1947 and the Federal Reserve Economic Data (FRED) from 1948 to 2018. Data before 1948 are obtained by concatenating three series of seasonally adjusted monthly unemployment rates: NBER data series m08292a (January 1929–February 1940), NBER data series m08292b (March 1940–December 1946), and NBER data series m08292c (January 1947–December 1947). Starting from 1948, the time series corresponds to the seasonally adjusted civilian unemployment rate from the Bureau of Labor Statistics (FRED series UNRATE).

A2. Vacancy Rate

The vacancy rate series is the concatenation of four different series: the MetLife help wanted advertising index, NBER data series m08082a (January 1927–December 1959), the help wanted advertising index from the Conference Board (January 1960–December 1994), the composite print and online help wanted index from Barnichon (2010; January 1995–December 2000), and the job openings series from the Job Openings and Labor Turnover Survey (JOLTS; January 2001–October 2018).

The MetLife index includes help wanted ads published in 45 US cities in 100 newspapers (1927 to the early 1940s) or in 60 newspapers (thereafter). The construction of the Conference Board index tightly follows the MetLife index. The three main aspects in which the Conference Board differs from MetLife are the use of 51 newspapers in 51 different cities, the adjustment of the index to account for the different number of Sundays in each month (help wanted ads were usually published on Sundays), and the weighting of the index computed in each city by the city's employment share (see Zagorsky 1998 for additional details). The two series coexisted between January 1951 and August 1960. The two series are merged by rescaling the Conference Board index so that it takes the same value as the MetLife index in January 1960. Once rescaled, the second series closely tracks the first one during the entire period of overlap. This suggests that the meaning of a 1% change remains the same across the two series.

As online advertising became widespread after the mid 1990s, the Conference Board index had increasingly lost its ability to represent the actual dynamics of job vacancies. To address this issue, Barnichon (2010) combines data on print and online help wanted ads. He weights their relative importance by assuming that the diffusion of online postings followed a similar pattern as the diffusion of internet use among US households. This assumption allows him to create a composite print-online index. The composite print-online index is rescaled so as to coincide with the Conference Board index in January 1995. In the period of overlap, the two series diverge, as printed ads became less and less relevant.

Starting from 2001, vacancies are computed using data from the JOLTS, which is a survey of 16,000 establishments. The JOLTS series is turned into a vacancy index. To this aim, the JOLTS series is rescaled so as to take the same value as the vacancy index in January 2001. The rescaled JOLTS series tracks very closely the vacancy index during the period of overlap from January 2001 until December 2016. Again, this suggests that the meaning of a 1% change is the same for the JOLTS series and the vacancy index.

Having constructed a vacancy index for the entire sample period, the index is transformed into a vacancy rate. To this aim, the index is divided by the contemporaneous labor force and then rescaled so as to take the value of 2.05% in 1965, which Zagorsky (1998) documents to be the actual vacancy rate in that year. We think it is very reassuring that in the period of overlap, the vacancy rate constructed by Petrosky-Nadeau and Zhang (2013) closely tracks a vacancy rate directly computed from JOLTS. This observation suggests that the print-online index constructed by Barnichon (2010)—and used to rescale the JOLTS series into a vacancy index—accurately captures the actual behavior of vacancies.

Appendix B

Endogenous Vacancy Cost and Unemployment Benefit

In this appendix, we analyze a version of the baseline model in which the cost of a vacancy and the benefit of unemployment are endogenous. We show that in this version of the model, the vacancy cost and the unemployment benefit grow endogenously at the same rate as the economy. Hence, in this version of the model, the only substantive condition for a BGP is that the distribution of productivity for new firm-worker matches is Pareto.

There are two types of firms, production firms and recruitment firms. Production firms are the firms described in section II, which operate a constant returns to scale technology that turns one worker into $y_t z$ units of output, where y_t is the common component of productivity and z is the component of productivity that is idiosyncratic to a firm-worker match. Recruitment firms are firms that create the hiring services required by production firms to maintain their vacancies. In particular, production firms need to purchase 1 unit of hiring services to maintain a vacancy. Recruitment firms create hiring services according to a constant return to scale production function that turns 1 unit of labor into $A_h > 0$ units of hiring services. Recruitment firms hire labor in a frictionless and competitive market and sell hiring services in a frictionless and competitive market. We assume that recruitment firms hire labor in a frictionless market to guarantee that even when every worker is unemployed, the economy does not shut down. Finally, the unemployment benefit is determined by the government as a fraction $\eta > 0$ of the average output of workers employed by production firms. We assume that the unemployment benefit is proportional to average output so as to make it independent of any particular wage determination rule.

Let $w_{h,t}$ denote the wage paid by recruitment firms to their employees. Let $p_{h,t}$ denote the price at which recruitment firms sell hiring services to productions firms. Let $e_{h,t}$ denote the measure of workers who are employed by recruitment firms. The endogenous variables $w_{h,t}$, $p_{h,t}$, and $e_{h,t}$ are such that

$$w_{h,t} = rU_t - \mathring{U}_t, \tag{B1}$$

$$p_{h,t} = \frac{w_{h,t}}{A_h}, \tag{B2}$$

$$e_{h,t} = \frac{u\theta}{A_h}. \tag{B3}$$

Intuitively, the wage $w_{h,t}$ makes an unemployed worker indifferent between taking a job at a recruitment firm and searching for a job at a production firm. The price $p_{h,t}$ makes the profit of a recruitment firm equal to zero. The employment $e_{h,t}$ is such that the aggregate supply of hiring services is equal to the aggregate demand of hiring services.

The joint-value V_t, the reservation quality R_t, and the surplus S_t are such that

$$V_t(z) = \max_{d\geq 0}\int_t^{t+d} e^{-r(\tau-t)} y_\tau z d\tau + e^{-rd} U_{t+d}, \tag{B4}$$

$$R_t = \frac{rU_t - \mathring{U}_t}{y_t}, \tag{B5}$$

$$S_t(z) = V_t(z) - U_t. \tag{B6}$$

The value U_t of unemployment to a worker and the tightness θ of the labor market are such that

$$rU_t = \eta \int_{R_t} y_t \hat{z} dG_t(\hat{z}) + \hat{A}_t p(\theta)\gamma \int_{R_t} S_t(\hat{z}) dF(\hat{z}) + \mathring{U}_t, \tag{B7}$$

$$\frac{y_t R_t}{A_h} = A_t q(\theta)(1-\gamma)\int_{R_t} S_t(\hat{z}) dF(\hat{z}). \tag{B8}$$

Conditions (B4) and (B5) are the same as conditions (1) and (3). The difference between conditions (B7) and (4) is that here the unemployment benefit is a fraction η of the average productivity of labor rather than the exogenous b_t. The difference between conditions (B8) and (5) is that here the cost of a vacancy is the price of a unit of hiring services rather than the exogenous k_t. Note that the price $p_{h,t}$ of a unit of hiring services is equal to $y_t R_t / A_h$ because $w_{h,t} = y_t R_t$ and $p_{h,t} = w_{h,t}/A_h$.

The stationarity conditions for the UE, EU, and unemployment rates are

$$A_t p(\theta)(1 - F(R_t)) = h_{\text{ue}}, \tag{B9}$$

$$G_t'(R_t)\mathring{R}_t = h_{\text{eu}}, \tag{B10}$$

$$u h_{\text{ue}} = \left(1 - u - \frac{u\theta}{A_h}\right) h_{\text{eu}}. \tag{B11}$$

The stationarity conditions for the UE and EU rates are the same as conditions (6) and (7). The difference between conditions (B11) and (8) is that here the flow into unemployment is given by the product between the measure of workers employed in the production sector (rather than the total measure of employed workers) and the EU rate.

The constant-growth condition for the distribution of workers employed in the production sector is such that

$$\begin{aligned} \left(1 - u - \frac{u\theta}{A_h}\right) G_t'(z_t(x)) z_t(x) g_z + u A_t p(\theta)[F(z_t(x)) - F(R_t)] \\ = \left(1 - u - \frac{u\theta}{A_h}\right) G_t'(R_t(x)) R_t(x) g_z. \end{aligned} \tag{B12}$$

The difference between conditions (B12) and (9) is that here the first term on the left-hand side is the measure of workers employed in the production sector (rather than the total measure of employed worker) times the rate at which these workers fall below the xth quantile of the distribution. Similarly, the term on the right-hand side is the measure of workers employed in the production sector times the rate at which these workers become unemployed.

It is easy to show that a BGP may exist only if the sampling distribution F is Pareto with tail coefficient $\alpha > 1$ and the discount rate r is greater than $g_y + g_A/\alpha$. Given these restrictions on the fundamentals, it easy to show that a BGP exists and is unique as long as $\eta < (\alpha - 1)/\alpha$.[16] In the BGP, the reservation quality R_t grows at the rate $g_z = g_A/\alpha$, and R_0 is equal to

[16] This condition is necessary and sufficient for the unemployment benefit to be lower than the reservation quality.

$$R_0 = \left[\frac{A_0 p(\theta)\gamma\Phi}{1 - \eta\alpha/(\alpha - 1)}\right]^{1/\alpha}. \tag{B13}$$

The labor market tightness θ is such that

$$\theta = q^{-1}\left[\frac{R_0^\alpha}{A_h A_0(1 - \gamma)\Phi}\right]. \tag{B14}$$

The UE, EU, and unemployment rates are

$$h_{\text{ue}} = A_0 p(\theta)\left(\frac{z_\ell}{R_0}\right)^\alpha, \tag{B15}$$

$$h_{\text{eu}} = g_A, \tag{B16}$$

$$u = \frac{g_A}{A_0 p(\theta)(z_\ell/R_0)^\alpha + g_A}. \tag{B17}$$

The distribution of workers employed by production firms grows at the rate $g_z = g_A/\alpha$, and G_0 is equal to

$$G_0(z) = 1 - \left(\frac{R_0}{z}\right)^\alpha. \tag{B18}$$

The wage $w_{h,t}$ paid by recruitment firms is equal to $y_t R_t$ and hence grows at the rate $g_y + g_z/\alpha$, with $w_{h,0} = y_0 R_0$. The price $p_{h,t}$ of hiring services is equal to $w_{h,t}/A_h$ and hence grows at the rate $g_y + g_z/\alpha$, with $p_{h,0} = y_0 R_0/A_h$. Employment $e_{h,t}$ at recruitment firms is constant and equal to $u\theta/A_h$.

We have thus established the following proposition.

Proposition 4 (Existence and properties of BGP). Let $g_A > 0$ and $g_y \geq 0$. A BGP exists iff (a) F is Pareto with tail coefficient $\alpha > 1$; (b) $r > g_y + g_A/\alpha$; and (c) $\eta < (\alpha - 1)/\alpha$. If the BGP exists, it is unique and such that

i. u, θ, h_{ue}, and h_{eu} are constant;
ii. $G_t(z\exp(g_z t)) = G_0(z)$, with $g_z = g_A/\alpha$;
iii. labor productivity grows at the rate $g_y + g_A/\alpha$; and
iv. vacancy cost and unemployment benefit grow at the rate $g_y + g_A/\alpha$.

Appendix C

Search on the Job

In this appendix, we define a BGP for the version of the model generalized to allow for the possibility that workers might search off and on the job. We then prove the existence of a BGP and characterize its properties.

C1. Definition of a BGP

The joint value $V_t(z)$ of a firm-worker match with quality z is such that

$$V_t(z) = \max_{d \geq 0}\left\{\int_t^{t+d} e^{-r(\tau-t)}\mu_\tau\left[y_\tau z + A_\tau p(\theta)\rho\gamma\int_z (V_\tau(\hat{z}) - V_\tau(z))dF(\hat{z})\right]d\tau + e^{-rd}\mu_{t+d}U_{t+d}\right\}, \tag{C1}$$

where μ_τ denotes the probability that the match is still active at date τ and is equal to

$$\mu_\tau = \exp\left[-\int_t^\tau A_x p(\theta)\rho[1 - F(z)]\,dx\right].$$

Conditional on the firm-worker match surviving to date τ, the sum of the worker's labor income and the firm's profit is equal to $y_\tau z$. Moreover, at date τ, the worker meets a poaching firm at rate $A_\tau p(\theta)\rho$. If the idiosyncratic productivity $\hat{z}$ of the match between the worker and the poaching firm is greater than z, the worker moves to the poaching firm. In this case, the worker's value is $V_\tau(z) + \gamma(V_\tau(\hat{z}) - V_\tau(z))$ and the incumbent firm's value is zero. Hence, the joint value of the firm-worker match increases by a fraction γ of the gains from trade $V_\tau(\hat{z}) - V_\tau(z)$. If the idiosyncratic productivity $\hat{z}$ of the match between the worker and the poaching firm is smaller than z, the worker stays with the incumbent firm and there is no change in their joint value. Conditional on the firm-worker match surviving to date $t + d$, the worker and the firm voluntarily break up. In this case, the value to the worker is U_{t+d} and the value to the firm is zero. Since the firm-worker match breaks up at the rate $A_x p(\theta)\rho[1 - F(z)]$ at date x, the probability that the match survives until τ is given by $\mu(\tau)$.

The optimal breakup date d must satisfy

$$y_{t+d}z + A_{t+d}p(\theta)\rho\gamma\int_z (V_{t+d}(\hat{z}) - V_{t+d}(z))\,dF(\hat{z}) + \mathring{U}_{t+d} \;\leq\; rU_{t+d},\; d \geq 0, \tag{C2}$$

where the two inequalities hold with complementary slackness. The left-hand side of condition (C2) is the marginal benefit of delaying the breakup of the match, which is the sum of the flow of output of the match, the option value of searching, and the time derivative of the worker's value of unemployment. The right-hand side of condition (C2) is the marginal cost of delaying the breakup of the match, which is given by the annuitized values that the worker and the firm can attain individually.

The reservation quality R_t is defined as

$$y_t R_t = rU_t - \mathring{U}_t - A_t p(\theta)\rho\gamma\int_{R_t} (V_t(\hat{z}) - V_t(R_t))\,dF(\hat{z}). \tag{C3}$$

Condition (C3) implies that a firm and a worker prefer staying together rather than being apart iff the idiosyncratic productivity of their match is greater than R_t. Similarly, a firm and an unemployed worker prefer consummating their match rather than staying apart iff the idiosyncratic productivity of their match is greater than R_t. Note that the reservation quality R_t characterizes the choice of whether a firm and a worker should be together or alone. In contrast, the choice of whether a worker should stay with an incumbent firm or move to a poaching firm is characterized by the ranking of the idiosyncratic productivity of the two available matches.

The surplus $S_t(z)$ of a firm-worker match with idiosyncratic productivity z is defined as

$$S_t(z) = V_t(z) - U_t. \tag{C4}$$

The definition (C4) implies that the surplus of a firm-worker match is strictly positive for $z > R_t$ and equal to zero for all $z \leq R_t$. Hence, a firm and a worker prefer staying together rather than being apart iff the surplus of their match is strictly positive. A firm and an unemployed worker prefer consummating their match rather than searching for alternative partners iff the surplus of their match is strictly positive.

The value of unemployment to a worker, U_t, is such that

$$rU_t = b_t + A_t p(\theta)\gamma \int_{R_t} S_t(\hat{z})dF(\hat{z}) + \mathring{U}_t. \tag{C5}$$

The tightness of the labor market, θ, is such that

$$\begin{aligned} &A_t q(\theta)\frac{u}{u+\rho(1-u)}(1-\gamma)\int_{R_t} S_t(\hat{z})dF(\hat{z}) \\ &+ A_t q(\theta)\tfrac{\rho(1-u)}{u+\rho(1-u)}(1-\gamma)\int_{R_t}\left[\int_z (V_t(\hat{z}) - V_t(z))dF(\hat{z})\right]dG_t(z) = k_t. \end{aligned} \tag{C6}$$

When workers search both off and on the job, a vacancy meets both unemployed and employed workers, and this is reflected in the right-hand side of equation (C6). Conditional on a meeting, the vacancy meets an unemployed worker with probability $u/[u + \rho(1-u)]$. In this case, the firm captures a fraction $1-\gamma$ of the expected gains from trade $S_t(\hat{z})$. The vacancy meets a worker employed in a job of quality z with probability $\{\rho(1-u)/[u+\rho(1-u)]\}G_t'(z)$. In this case, the firm captures a fraction $1-\gamma$ of the gains from trade $V_t(\hat{z}) - V_t(z)$.

The stationarity conditions for UE, EU, and unemployment rates are

$$A_t p(\theta)(1 - F(R_t)) = h_{\mathrm{ue}}, \tag{C7}$$

$$G_t'(R_t)\mathring{R}_t = h_{\mathrm{eu}}, \tag{C8}$$

$$uh_{\mathrm{ue}} = (1-u)h_{\mathrm{eu}}. \tag{C9}$$

The condition $z_t(x) = z_0(x)\exp(g_z t)$ for the constant growth of the distribution G_t of employed workers across match qualities is

$$\begin{aligned} &(1-u)G_t'(z_t(x))z_t(x)g_z + uA_t p(\theta)[F(z_t(x)) - F(R_t)] \\ &\quad = (1-u)G_t'(R_t)R_t g_z + (1-u)G_t(z_t(x))\rho A_t p(\theta)[1 - F(z_t(x))]. \end{aligned} \tag{C10}$$

The left-hand side of condition (C10) is the flow of workers into matches with quality lower than the xth quantile. The first term is the flow of employed workers employed in a match of quality z that, in the next instant, falls below the xth quantile, which grows at the rate g_z. The second term is the flow of unemployed workers who, in the next instant, become employed in a match of quality z below the xth quantile. The right-hand side of condition (C10) is the flow of workers out of matches with quality lower than the xth quantile. It includes the flow of employed workers who become unemployed as well as the flow of workers who—by searching on the job—move out of a match with quality lower than the xth quantile.

C2. Existence of a BGP

It is easy to generalize the proof of lemma 1 to show that a BGP may exist only if (a) F is Pareto with tail coefficient α, (b) g_k and g_b are equal to $g_y + g_z$, and (c) r is greater than $g_y + g_z$. Moreover, in any BGP, the growth rate g_z must be equal to g_A/α. Therefore, we shall assume a, b, and c as we solve for a BGP.

The joint value $V_t(z)$ of a firm-worker match with quality $z > R_t$ and the value U_t of unemployment to a worker can be written as

$$rV_t(z) = y_t z + A_t p(\theta)\rho\gamma \int_z (S_t(\hat{z}) - S_t(z))\,dF(\hat{z}) + \mathring{V}_t(z), \tag{C11}$$

$$rU_t = y_t R_t + A_t p(\theta)\rho\gamma \int_{R_t} S_t(\hat{z})\,dF(\hat{z}) + \mathring{U}_t. \tag{C12}$$

Equation (C11) is obtained by taking the derivative of equation (C1) with respect to t. Equation (C12) is obtained from equation (C5) after substituting in the definition of reservation quality. From equations (C11) and (C12), it follows that the surplus $S_t(z)$ of a firm-worker match with quality $z > R_t$ is given by

$$rS_t(z) = y_t(z - R_t) - A_t p(\theta)\rho\gamma\left[\int_{R_t}^{z} S_t(\hat{z})\,dF(\hat{z}) + S_t(z)(1 - F(z))\right] + \mathring{S}_t(z). \tag{C13}$$

We solve the partial differential equation (C13) by guessing that S_t is such that, when evaluated at an idiosyncratic productivity that grows at the rate g_z, the surplus of a match grows at the rate $g_y + g_z$; that is,

$$S_t(ze^{g_z t}) = S_0(z) \cdot e^{(g_y + g_z)t}. \tag{C14}$$

To verify the guess (C14), let us evaluate the partial differential equation (C13) at $z\exp(g_z t)$ to obtain

$$\begin{aligned} rS_t(ze^{g_z t}) = {} & y_t(ze^{g_z t} - R_t) - A_t p(\theta)\rho\gamma \int_{R_0 e^{g_z t}}^{ze^{g_z t}} S_t(\hat{z})\,dF(\hat{z}) \\ & - A_t p(\theta)\rho\gamma S_t(ze^{g_z t})(1 - F(ze^{g_z t})) + \mathring{S}_t(ze^{g_z t}). \end{aligned} \tag{C15}$$

Using the guess (C14) and the fact that $y_t = y_0\exp(g_A t)$, $A_t = A_0\exp(g_A t)$, $R_t = R_0\exp(g_z t)$, $1 - F(ze^{g_z t}) = (1 - F(z))\exp(-\alpha g_z t)$, and $g_z = g_A/\alpha$, we can rewrite equation (C15) as[17]

$$\begin{aligned} rS_0(z) \cdot e^{(g_y + g_z)t} = {} & \Bigg\{ y_0(z - R_0) - A_0 p(\theta)\rho\gamma \int_{R_0}^{z} S_0(\hat{z})\,dF(\hat{z}) \\ & - A_0 p(\theta)\rho\gamma S_0(z)(1 - F(z)) + \left[(g_y + g_z)S_0(z) - zg_z S_0'(z)\right]\Bigg\} \cdot e^{(g_y + g_z)t}. \end{aligned} \tag{C16}$$

The left-hand side of equation (C16) is an expression that depends on only S_0 and that grows over time at the rate $g_y + g_z$. The right-hand side is an expression

[17] The reader can find more details about the derivation of this and other expressions in Martellini and Menzio (2018).

that depends on only S_0 and that grows over time at the rate $g_y + g_z$. Thus, the guess (C14) satisfies the partial differential equation (C13) for all $t \geq 0$, as long as the initial surplus S_0 satisfies equation (C16) at date $t = 0$.

To solve for S_0, we take condition (C16) evaluated at $t = 0$ and we differentiate it with respect to z. We obtain

$$rS_0'(z) = y_0 + \left[g_y - \sigma\gamma(1 - F(z))\right] S_0'(z) - zg_z S_0''(z), \tag{C17}$$

where σ is shorthand for $A_0 p(\theta)\rho$. The solution for $S_0'(z)$ to the differential equation (C17) that satisfies the smooth-pasting condition $S_0'(R_0) = 0$ is

$$S_0'(z) = \frac{y_0}{g_z}\int_{R_0}^{z} \frac{1}{s}\exp\left\{-\frac{1}{g_z}\left[\frac{\sigma\gamma}{\alpha}(F(z) - F(s)) + (r - g_y)\log\left(\frac{z}{s}\right)\right]\right\} ds. \tag{C18}$$

The solution for $S_0(z)$ to the differential equation (C18) that satisfies the value-matching condition $S_0(R_0) = 0$ is

$$S_0(z) = \frac{y_0}{g_z}\int_{R_0}^{z}\left[\int_{R_0}^{x} \frac{1}{s}\exp\left\{-\frac{1}{g_z}\left[\frac{\sigma\gamma}{\alpha}(F(x) - F(s)) + (r - g_y)\log\left(\frac{z}{s}\right)\right]\right\} ds\right] dx. \tag{C19}$$

Thus, the initial surplus S_0 in expression (C19) together with $S_t(z\exp(g_z t)) = S_0(z)\exp(g_y + g_z)t$ provides a solution to the partial differential equation (C13). While other solutions may exist and may be associated with different BGPs, all these other BGPs satisfy the properties in theorem 2.

Using the fact that $S_t(ze^{g_z t}) = S_0(z)\exp(g_y + g_z)t$ and that $G_t(ze^{g_z t}) = G_0(z)$, we can derive some useful properties of the expected gains from trade $\bar{S}_{u,t}$ in a meeting between a firm and an unemployed worker, the expected gains from trade $S_{e,t}(ze^{g_z t})$ in a meeting between a firm and a worker employed in a match with quality $z\exp(g_z t)$, and the expected gains from trade $\bar{S}_{e,t}$ in a meeting between a firm and an employed worker who is randomly drawn from the employment distribution G_t. We can show that all these expected gains from trade increase over time at the rate of $g_y - (\alpha - 1)g_z$; that is,

$$\begin{aligned} S_{e,t}(ze^{g_z t}) &\equiv \int_{ze^{g_z t}} (S_t(\hat{z}) - S_t(ze^{g_z t}))\, dF(\hat{z}) = S_{e,0}(z)e^{[g_y-(\alpha-1)g_z]t}, \\ \bar{S}_{e,t} &\equiv \int_{R_t} S_{e,t}(z)\, dG_t(z) = \bar{S}_{e,0}e^{[g_y-(\alpha-1)g_z]t}, \\ \bar{S}_{u,t} &\equiv \int_{R_t} S_t(\hat{z})\, dF(\hat{z}) = S_{u,0}e^{[g_y-(\alpha-1)g_z]t}. \end{aligned} \tag{C20}$$

Note that the expected gains above are well defined only if the tail coefficient α of the distribution F is greater than 1.

We are now in the position to construct a BGP. The reservation quality R_t is given by

$$y_t R_t = b_t + A_t p(\theta)(1 - \rho)\gamma\bar{S}_{u,t}. \tag{C21}$$

Let R_0 be a solution of equation (C21) for $t = 0$. Then $R_t = R_0\exp(g_z t)$ solves equation (C21) for all $t > 0$ iff $g_z = g_A/\alpha$. To see why this is the case, note that the left-hand side grows at the rate $g_y + g_z$. The first term on the right-hand side grows at the rate $g_y + g_z$. The second term grows at the rate $g_A + g_y - (\alpha - 1)g_z$. Thus, the left- and right-hand sides grow at the same rate iff $g_z = g_A/\alpha$.

The market tightness θ is given by

$$k_t = A_t q(\theta)(1-\gamma)\left[\frac{u}{u+\rho(1-u)}\bar{S}_{u,t} + \frac{\rho(1-u)}{u+\rho(1-u)}\bar{S}_{e,t}\right]. \tag{C22}$$

Let θ be a solution of equation (C22) for $t = 0$. Then the same θ also solves equation (C22) for all $t > 0$. The left-hand side grows at the rate $g_y + g_z$. The right-hand side grows at the rate $g_A + g_y - (\alpha - 1)g_z$. The two growth rates are the same because $g_z = g_A/\alpha$.

The constant-growth condition for the distribution G_t of employed workers across match qualities is

$$(1-u)G_t'(ze^{g_z t})ze^{g_z t}g_z = [u + (1-u)\rho G_t(ze^{g_z t})]A_t p(\theta)\left(\frac{z_\ell}{ze^{g_z t}}\right)^\alpha. \tag{C23}$$

At $t = 0$, condition (C23) is a differential equation for the initial distribution G_0 that depends on the unemployment rate u. The unique solution for G_0 and u that satisfies the differential equation and the boundary conditions $G_0(R_0) = 0$ and $G_0(\infty) = 1$ is

$$G_0(z) = \frac{H(z) - H(R_0)}{1 - H(R_0)}, \text{ with } H(z) = \exp\left\{-\left[\frac{A_0 p(\theta)\rho}{g_A}\left(\frac{z_\ell}{z}\right)^\alpha\right]\right\}, \tag{C24}$$

and

$$u = \frac{\rho H(R_0)}{1 - (1-\rho)H(R_0)}, \tag{C25}$$

To verify that the distribution grows at the constant rate $g_z = g_A/\alpha$, it is sufficient to check that $G_t(ze^{g_z t}) = G_0(z)$ satisfies condition (C23) for all $t > 0$.

The UE and EU rates are given by

$$h_{\mathrm{ue}} = A_t p(\theta)\left(\frac{z_\ell}{R_t}\right)^\alpha = A_0 p(\theta)\left(\frac{z_\ell}{R_0}\right)^\alpha, \tag{C26}$$

$$h_{\mathrm{eu}} = G_t'(R_t)R_t g_z = A_0 p(\theta)\left(\frac{z_\ell}{R_0}\right)^\alpha \frac{H(R_0)}{1 - H(R_0)}. \tag{C27}$$

The UE rate is constant over time, as A_t grows at the rate g_A and $1 - F(R_t)$ grows at the rate $-g_z/\alpha$, which is equal to $-g_A$. The EU rate is constant over time, as $G_t'(R_t)$ grows at the rate $-g_z$ and R_t grows at the rate g_z. Using conditions (C26) and (C27), it is easy to verify that the unemployment rate in condition (C25) equates the flow of workers in and out of unemployment and hence is constant over time as well.

The analysis above implies that a BGP exists as long as there is a reservation quality R_0 and a market tightness θ that solve equations (C21) and (C22) at $t = 0$. We can show that there exists such a pair (R_0, θ) that solves equations (C21) and (C22). Hence, a BGP exists. However, we are not able to show that there exists a unique pair (R_0, θ) that solves equations (C21) and (C22). Hence, there may be multiple BGPs.

Appendix D

Population Growth

In this appendix, we define a BGP for the version of the model generalized to allow for the possibility that population might grow over time and that the search process might have nonconstant returns to scale. We then establish conditions for the existence and properties of a BGP.

D1. Definition of a BGP

The joint value V_t, the reservation quality R_t, and the surplus S_t are such that

$$V_t(z) = \max_{d\geq 0}\int_t^{t+d} e^{-r(\tau-t)} y_\tau z d\tau + e^{-rd} U_{t+d}, \tag{D1}$$

$$y_t R_t = rU_t - \mathring{U}_t, \tag{D2}$$

$$S_t(z) = V_t(z) - U_t. \tag{D3}$$

The value U_t of unemployment to a worker and the tightness θ of the labor market are such that

$$rU_t = b_t + \hat{A}_t p(\theta)\gamma \int_{R_t} S_t(\hat{z}) dF(\hat{z}), \tag{D4}$$

$$k_t = \hat{A}_t q(\theta)(1-\gamma)\int_{R_t} S_t(\hat{z}) dF(\hat{z}). \tag{D5}$$

Conditions (D1)–(D3) are the same as in section II. Conditions (D4) and (D5) are the same as conditions (4) and (5), with $\hat{A}_t \equiv A_t N_t^\beta$ replacing A_t.

The stationarity conditions for the UE, EU, and unemployment rates are

$$\hat{A}_t p(\theta)(1 - F(R_t)) = h_{\text{ue}}, \tag{D6}$$

$$G_t'(R_t)\mathring{R}_t = h_{\text{eu}}, \tag{D7}$$

$$N_t u h_{\text{ue}} = N_t(1-u)(h_{\text{eu}} + g_N). \tag{D8}$$

The stationarity conditions (D6) and (D7) for the UE and EU rates are the same as conditions (6) and (7), with $\hat{A}$ replacing A_t. The stationarity condition (D8) for unemployment is different from condition (8). The unemployment rate is stationary when the flow of workers out of unemployment, $N_t u h_{\text{ue}}$, is equal to the flow of workers entering unemployment from employment, $N_t(1-u)h_{\text{eu}}$, plus the flow of workers entering unemployment from outside the labor market, $N_t g_N$, multiplied by the difference $1-u$ between the unemployment rate of entering and existing workers.

The condition guaranteeing that the employment distribution G_t grows at the constant rate g_z—in the sense that $z_t(x) = z_0(x)\exp(g_z t)$, where $z_t(x)$ denotes the xth quantile of G_t—is

$$\begin{aligned} &N_t(1-u)G_t'(z_t(x))z_t(x)g_z + N_t u\hat{A}_t p(\theta)[F(z_t(x)) - F(R_t)] \\ &\quad = N_t(1-u)G_t'(R_t(x))R_t(x)g_z + N_t g_N(1-u)G_t(z_t(x)). \end{aligned} \tag{D9}$$

The left-hand side of condition (D9) is the flow of workers into matches with quality lower than the xth quantile. The first term is the flow of workers who are employed in a match of quality z who, in the next instant, fall below the xth quantile. The second term is the flow of unemployed workers who, in the next instant, become employed in a match of quality z below the xth quantile. The right-hand side of condition (D9) is the flow of workers out of matches with quality below the xth quantile. The first term is the flow of workers who are employed and, in the next instant, move into unemployment. The second term is the flow of workers entering the labor market times the difference between the fraction of existing workers who are employed in matches below the xth quantile (which is $(1-u)G_t(z_t(x))$) and the fraction of new workers who are employed in matches below the xth quantile (which is zero).

Note that the definition of BGP is the same as in section II, except that (1) A_t is replaced with $\hat{A}_t$ in all the BGP conditions and (2) the stationarity condition for unemployment and the constant-growth condition for the employment distribution are modified to account for the flow of workers entering the labor market.

D2. Existence of a BGP

Following the same steps as in section II, we can show that the necessary conditions for a BGP are (1) F is Pareto with coefficient $\alpha > 1$, (2) g_b and g_k are equal to $g_y + g_z$, and (3) r is greater than $g_y + g_z$. Under these conditions, a BGP exists, is unique, and is such that the reservation quality R_t grows at the rate $g_z = (g_A + \beta g_N)/\alpha$ and R_0 is equal to

$$y_0 R_0 = b_0 + \hat{A}_0 p(\theta)\gamma\hat{\Phi} y_0 R_0^{-(\alpha-1)}, \tag{D10}$$

where $\hat{\Phi}$ is a positive constant that depends on only parameters. The labor market tightness θ is such that

$$k_0 = \hat{A}_0 q(\theta)(1-\gamma)\hat{\Phi} y_0 R_0^{-(\alpha-1)}. \tag{D11}$$

The UE, EU, and unemployment rates are

$$h_{ue} = \hat{A}_0 p(\theta)\left(\frac{z_\ell}{R_0}\right)^\alpha, \tag{D12}$$

$$h_{eu} = g_A + \beta g_N, \tag{D13}$$

$$u = \frac{g_A + (1+\beta)g_N}{\hat{A}_0 p(\theta)(z_\ell/R_0)^\alpha + g_A + (1+\beta)g_N}. \tag{D14}$$

The distribution of employed workers grows at the rate $g_z = g_A/\alpha$, and G_0 is equal to

$$G_0(z) = 1 - \left(\frac{R_0}{z}\right)^\alpha. \tag{D15}$$

Appendix E

Imperfect Signals

In this appendix, we define a BGP for the version of the model in which firms and workers observe only a noisy signal about the quality of their match. We then establish conditions for the existence of a BGP.

E1. Definition of a BGP

The joint value $V_t(z)$ of a match of known quality z is given by

$$V_t(z) = \max_{d \geq 0} \int_t^{t+d} e^{-r(\tau - t)} y_\tau z d\tau + e^{-rd} U_{t+d}. \tag{E1}$$

The reservation quality R_t is defined as

$$y_t R_t = rU_t - \mathring{U}_t. \tag{E2}$$

It is easy to verify from the optimality condition for d in condition (E1) that the reservation quality R_t is the lowest quality for which a match is maintained. That is, a match of known quality z is maintained at date t if $z > R_t$, and it is destroyed if $z \leq R_t$.

The joint value $\tilde{V}_t(\zeta)$ of a match of unknown quality with a signal ζ is given by

$$\tilde{V}_t(\zeta) = \max\left\{ \int_t^{t+t^*} e^{-r(\tau-t)} y_\tau \zeta d\tau + e^{-rt^*} \int V_{t+t^*}(\zeta\epsilon) dF_2(\epsilon), U_t \right\}. \tag{E3}$$

Note that implicit in the formulation of $\tilde{V}_t$ is the assumption that a match cannot be destroyed before the quality is revealed. The assumption is only for the sake of simplicity. The reservation signal Q_t is defined as

$$\tilde{V}_t(Q_t) = U_t. \tag{E4}$$

Since $\tilde{V}_t(\zeta)$ is increasing in ζ, it follows that a firm and a worker consummate their match at date t if $\zeta > Q_t$, and they keep searching the labor market if $\zeta \leq Q_t$.

We define the surplus of a match of know quality z and the surplus of a match of unknown quality with signal ζ as

$$\begin{aligned} S_t(z) &= V_t(z) - U_t, \\ \tilde{S}_t(\zeta) &= \tilde{V}_t(\zeta) - U_t. \end{aligned} \tag{E5}$$

From conditions (E1) and (E2), it follows that $S_t(z) > 0$ if $z > R_t$, and $S_t(z) = 0$ otherwise. From conditions (E3) and (E4), it follows that $\tilde{S}_t(\zeta) > 0$ if $\zeta > Q_t$ and $\tilde{S}_t(\zeta) = 0$ otherwise.

The value of unemployment to a worker, U_t, is such that

$$rU_t = b_t + A_t p(\theta)\gamma \int_{Q_t} \tilde{S}_t(\hat{\zeta}) dF_1(\hat{\zeta}). \tag{E6}$$

The tightness of the labor market, θ, is such that

$$k_t = A_t q(\theta)(1-\gamma)\int_{Q_t} \tilde{S}_t(\hat{\zeta})dF_1(\hat{\zeta}). \tag{E7}$$

The expressions in conditions (E6) and (E7) are analogous to those in conditions (4) and (5).

The stationarity conditions for the UE, EU, and unemployment rates are

$$A_t p(\theta)[1 - F_1(Q_t)] = h_{\text{ue}}, \tag{E8}$$

$$\frac{1-u-n}{1-u} G_t'(R_t)\mathring{R}_t + \frac{n_{t-t^*}}{1-u}\int F_2\left(\frac{R_t}{\zeta}\right)dH_{t-t^*}(\zeta) = h_{\text{eu}}, \tag{E9}$$

$$uh_{\text{ue}} = (1-u)h_{\text{eu}}. \tag{E10}$$

In the conditions above, n denotes the measure of employed workers who have yet to find out the quality of their match, n_{t-s} denotes the flow of workers who become employed at date $t-s$, and $H_{t-s}(\zeta)$ is the CDF of their signals. Clearly, we have

$$\begin{aligned} n &= \int_{s=0}^{t^*} n_{t-s}\,ds, \\ n_{t-s} &= uh_{\text{ue}}, \\ H_{t-s}(\zeta) &= \frac{F_1(\zeta) - F_1(Q_{t-s})}{1 - F_1(Q_{t-s})}. \end{aligned} \tag{E11}$$

Condition (E8) is the same as condition (6), except that the firm-worker pair decision's to consummate their match is based on the comparison between the signal ζ and the reservation signal Q_t. Condition (E9) is the same as condition (7), except that here the EU rate includes the flow of employed workers who learn that the quality z of their match is below the reservation quality R_t.

The constant growth condition for G_t is

$$\begin{aligned} &(1-u-n)G_t'(z_t(x))z_t(x)g_z + n_{t-t^*}\int\left[F_2\left(\frac{z_t(x)}{\zeta}\right) - F_2\left(\frac{R_t}{\zeta}\right)\right]dH_{t-t^*}(\zeta) \\ &\quad = (1-u-n)G_t'(R_t)\mathring{R}_t + n_{t-t^*}\int F_2\left(\frac{R_t}{\zeta}\right)dH_{t-t^*}(\zeta). \end{aligned} \tag{E12}$$

Condition (E12) is the same as condition (9), except that the flow of workers into matches of quality below the xth quantile includes the flow of workers who learn that the quality z of their match is above the reservation quality R_t and below the xth quantile $z_t(x)$.

E2. Existence of a BGP

Following the same steps as in section II, we can show that a BGP may exist only if (1) F_1 is Pareto with coefficient $\alpha > 1$, (2) g_b and g_k are equal to $g_y + g_z$, and (3) r is smaller than $g_y + g_z$. Moreover, in any BGP, $g_z = g_A/\alpha$.

Under the necessary conditions above, we can show that the surplus $S_t(z)$ of a firm-worker match with known quality $z > R_t$ is

$$S_t(z) = y_t\left\{\frac{z}{r-g_y}\left[1-\left(\frac{R_t}{z}\right)^{(r-g_y)/gz}\right] - \frac{R_t}{r-g_y-g_z}\left[1-\left(\frac{R_t}{z}\right)^{(r-g_y-g_z)/gz}\right]\right\}. \quad \text{(E13)}$$

The expected surplus at date $t + t^*$ of a firm-worker match created at date t with signal $\zeta e^{g_z t}$ is

$$\begin{aligned}
&\int_{R_{t+t^*}/\zeta e^{g_z t}} S_{t+t^*}(\zeta e^{g_z t}\epsilon)\,dF_2(\epsilon) \\
&= \int_{R_{t^*}/\zeta} y_{t^*} e^{g_y t}\left\{\frac{\zeta e^{g_z t}\epsilon}{r-g_y}\left[1-\left(\frac{R_{t^*}}{\zeta\epsilon}\right)^{(r-g_y)/gz}\right] - \frac{R_{t^*}e^{g_z t}}{r-g_y-g_z}\left[1-\left(\frac{R_{t^*}}{\zeta\epsilon}\right)^{(r-g_y-g_z)/gz}\right]\right\}dF_2(\epsilon) \qquad \text{(E14)} \\
&= \left[\int_{R_{t^*}/\zeta} S_{t^*}(\zeta\epsilon)\,dF_2(\epsilon)\right]e^{(g_y+g_z)t}.
\end{aligned}$$

The surplus $\tilde{S}_t(\zeta e^{g_z t})$ of a firm-worker match created at date t with signal $\zeta e^{g_z t}$ is

$$\begin{aligned}
\tilde{S}_t(\zeta e^{g_z t}) &= \int_t^{t+t^*} e^{-r(\tau-t)}(y_\tau \zeta e^{g_z t} - y_\tau R_\tau)\,d\tau + e^{-rt^*}\int_{R_{t+t^*}/\zeta e^{g_z t}} S_{t+t^*}(\zeta e^{g_z t}\epsilon)\,dF_2(\epsilon) \\
&= \left\{\int_0^{t^*} e^{-r\tau}(y_\tau\zeta - y_\tau R_\tau)\,d\tau + e^{-rt^*}\left[\int_{R_{t^*}/\zeta} S_{t^*}(\zeta\epsilon)\,dF_2(\epsilon)\right]\right\}e^{(g_y+g_z)t} \qquad \text{(E15)} \\
&= \tilde{S}_0(\zeta)e^{(g_y+g_z)t}.
\end{aligned}$$

The expected surplus of a firm-worker meeting at date t is

$$\begin{aligned}
\int_{Q_t} \tilde{S}_t(\zeta)\left(\frac{\zeta_\ell}{\zeta}\right)^\alpha \frac{\alpha}{\zeta}\,d\zeta &= e^{(g_y+g_z)t}\int_{Q_0 e^{g_z t}} \tilde{S}_0(\zeta e^{-g_z t})\left(\frac{\zeta_\ell}{\zeta}\right)^\alpha \frac{\alpha}{\zeta}\,d\zeta \\
&= e^{(g_y+g_z)t}\int_{Q_0} \tilde{S}_0(\tilde{\zeta})\left(\frac{\zeta_\ell}{\tilde{\zeta}e^{g_z t}}\right)^\alpha \frac{\alpha}{\tilde{\zeta}}\,d\tilde{\zeta} \qquad \text{(E16)} \\
&= e^{[g_y-(\alpha-1)g_z]t}\int_{Q_0} \tilde{S}_0(\tilde{\zeta})\left(\frac{\zeta_\ell}{\tilde{\zeta}}\right)^\alpha \frac{\alpha}{\tilde{\zeta}}\,d\tilde{\zeta},
\end{aligned}$$

where the second line is obtained by changing the variable of integration from ζ to $\tilde{\zeta} = \zeta\exp(-g_z t)$.

The reservation quality R_t is such that

$$y_t R_t = b_t + A_t p(\theta)\gamma\int_{Q_t} \tilde{S}_t(\zeta)\,dF_1(\zeta). \qquad \text{(E17)}$$

Let R_0 be a solution of equation (E17) for $t = 0$. Then, $R_t = R_0\exp(g_z t)$ is a solution of equation (E17) for all $t > 0$ iff $g_z = g_A/\alpha$. In fact, the left-hand side of equation (E17) grows at the rate $g_y + g_z$. The first term on the right-hand side grows at the rate $g_y + g_z$. The second term on the right-hand side grows at the rate $g_A + g_y - (\alpha - 1)g_z$. The left- and right-hand sides grow at the same rate iff $g_z = g_A/\alpha$.

The reservation signal Q_t is such that

$$\tilde{S}_t(Q_t) = 0. \qquad \text{(E18)}$$

Let Q_0 be a solution of equation (E18) for $t = 0$. Then, $Q_t = Q_0 \exp(g_z t)$, where $g_z = g_A/\alpha$ is a solution of equation (E18) for all $t > 0$. This follows directly from condition (E15).

The tightness θ is such that

$$k_t = A_t q(\theta)(1-\gamma)\int_{Q_t} \tilde{S}_t(\zeta)dF_1(\zeta). \tag{E19}$$

Let θ be a solution of equation (E19) for $t = 0$. Then, the same θ is also a solution of equation (E19) for all $t > 0$. In fact, the left-hand side of equation (E19) grows at the rate $g_y + g_z$. The right-hand side grows at the rate $g_A + g_y - (\alpha - 1)g_z$. And the two rates are equal, as $g_z = g_A/\alpha$.

Finally, one can easily verify that there exist an initial distribution G_0 and an initial unemployment u such that G_t grows at the constant rate $g_z = g_A/\alpha$, unemployment is constant, and the UE and EU rates are constant.

Appendix F

Alternative Bargaining

We consider a generic bargaining outcome such that the joint value of a firm-worker match is maximized, the fraction of the gains from trade accruing to the worker is $\gamma_t(z)$, and the fraction of the gains from trade accruing to the firm is $1 - \gamma_t(z)$, with $\gamma_t(z) \in [0, 1]$. Along a BGP, the bargaining outcome has the property that $\gamma_t(z\exp(g_z t)) = \gamma_0(z)$, where g_z denotes the endogenous growth rate of the distribution of employed workers across match qualities. It is easy to verify that the bargaining outcomes described in section III satisfy this property.

The joint value V_t, the reservation quality R_t, and the surplus S_t are such that

$$V_t(z) = \max_{d\geq 0}\int_t^{t+d} e^{-r(\tau-t)}y_\tau z d\tau + e^{-rd}U_{t+d}, \tag{F1}$$

$$y_t R_t = rU_t - \mathring{U}_t, \tag{F2}$$

$$S_t(z) = V_t(z) - U_t. \tag{F3}$$

The value U_t of unemployment to a worker and the tightness θ of the labor market are such that

$$rU_t = b_t + A_t p(\theta)\int_{R_t}\gamma_t(\hat{z})S_t(\hat{z})dF(\hat{z}), \tag{F4}$$

$$k_t = A_t q(\theta)\int_{R_t}(1-\gamma_t(\hat{z}))S_t(\hat{z})dF(\hat{z}). \tag{F5}$$

The stationarity conditions for the UE, EU, and unemployment rates are

$$A_t p(\theta)(1 - F(R_t)) = h_{\text{ue}}, \tag{F6}$$

$$G_t'(R_t)\mathring{R}_t = h_{\text{eu}}, \tag{F7}$$

$$uh_{\text{ue}} = (1-u)h_{\text{eu}}. \tag{F8}$$

The constant growth condition for G_t is

$$(1-u)G_t'(z_t(x))z_t(x)g_z + uA_t p(\theta)[F(z_t(x)) - F(R_t)] = (1-u)G_t'(R_t(x))R_t(x)g_z. \tag{F9}$$

As in section II, we can show that the necessary conditions for the existence of a BGP are (1) F is Pareto with coefficient $\alpha > 1$, (2) g_b and g_k are equal to $g_y + g_z$, and (3) r is greater than $g_y + g_z$. Assuming that these conditions hold, we can now prove the existence of a BGP.

The surplus of a match of quality $z > R_t$ is given by

$$S_t(z) = y_t\left\{\frac{z}{r-g_y}\left[1-\left(\frac{R_t}{z}\right)^{(r-g_y)/g_z}\right] - \frac{R_t}{r-g_y-g_z}\left[1-\left(\frac{R_t}{z}\right)^{(r-g_y-g_z)/g_z}\right]\right\}. \tag{F10}$$

Let $\bar{S}_{w,t}$ denote an unemployed worker's expected gain from meeting a firm, and let $\bar{S}_{f,t}$, denote the firm's expected gain from meeting a worker. That is,

$$\bar{S}_{w,t} = \int_{R_t}\gamma_t(z)S_t(z)dF(z), \tag{F11}$$

$$\bar{S}_{f,t} = \int_{R_t}(1-\gamma_t(z))S_t(z)dF(z). \tag{F12}$$

Using condition (F10) to substitute out S_t in condition (F11), we obtain

$$\begin{aligned}
&\bar{S}_{w,t}\\
&= \int_{R_t}\gamma_t(z)y_t\left\{\frac{z}{r-g_y}\left[1-\left(\frac{R_t}{z}\right)^{(r-g_y)/g_z}\right] - \frac{R_t}{r-g_y-g_z}\left[1-\left(\frac{R_t}{z}\right)^{(r-g_y-g_z)/g_z}\right]\right\}\left(\frac{z_\ell}{z}\right)^\alpha\frac{\alpha}{z}dz\\
&= \int_{R_0e^{g_zt}}\gamma_0(ze^{-g_zt})y_t\left\{\frac{z}{r-g_y}\left[1-\left(\frac{R_t}{z}\right)^{(r-g_y)/g_z}\right] - \frac{R_t}{r-g_y-g_z}\left[1-\left(\frac{R_t}{z}\right)^{(r-g_y-g_z)/g_z}\right]\right\}\left(\frac{z_\ell}{z}\right)^\alpha\frac{\alpha}{z}dz\\
&= e^{(g_y+g_z)t}\int_{R_0}\gamma_0(\tilde{z})y_0\left\{\frac{\tilde{z}}{r-g_y}\left[1-\left(\frac{R_0}{\tilde{z}}\right)^{(r-g_y)/g_z}\right] - \frac{R_0}{r-g_y-g_z}\left[1-\left(\frac{R_0}{\tilde{z}}\right)^{(r-g_y-g_z)/g_z}\right]\right\}\left(\frac{z_\ell}{\tilde{z}e^{g_zt}}\right)^\alpha\frac{\alpha}{\tilde{z}}d\tilde{z}\\
&= e^{[g_y-(\alpha-1)g_z]t}\bar{S}_{w,0}.
\end{aligned} \tag{F13}$$

The second line in the expression above makes use of the fact that $\gamma_t(z) = \gamma_0(z\exp(-g_zt))$. The third line is obtained using the fact that $R_t = R_0\exp(g_zt)$ and $y_t = y_0\exp(g_yt)$ and then by changing the variable of integration from z to $\tilde{z} = z\exp(-g_zt)$. Following the same steps, we can show that

$$\bar{S}_{f,t} = e^{[g_y-(\alpha-1)g_z]t}\bar{S}_{f,0}. \tag{F14}$$

The reservation quality R_t and the tightness are such that

$$y_tR_t = b_t + A_tp(\theta)\bar{S}_{w,t}, \tag{F15}$$

$$k_t = A_tq(\theta)\bar{S}_{f,t}. \tag{F16}$$

Let R_0 denote a solution of condition (F15) for $t = 0$. Then, $R_t = R_0\exp(g_zt)$ is a solution of condition (F15) for all $t > 0$ iff $g_z = g_A/\alpha$. Let θ denote a solution of condition (F16) for $t = 0$. Then, the same θ is also a solution of condition (F16) for all $t > 0$.

The constant growth condition (F9) is satisfied for $g_z = g_A/\alpha$ iff G_0 and u are given by

$$G_0(z) = 1 - \left(\frac{R_0}{z}\right)^{\alpha}, \tag{F17}$$

$$u = \frac{g_A}{A_0 p(\theta)(z_\ell/R_0)^{\alpha} + g_A}. \tag{F18}$$

The UE and EU rates are constant and given by

$$h_{\mathrm{ue}} = A_0 p(\theta)\left(\frac{z_\ell}{R_0}\right)^{\alpha}, \tag{F19}$$

$$h_{\mathrm{eu}} = g_A + g_N. \tag{F20}$$

Clearly, given conditions (F19) and (F20), the unemployment rate (F18) is stationary. Finally, note that a BGP exists because there is a solution of equations (F15) and (F16) for R_0 and θ.

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