

## REVISITING THE HYPOTHESIS OF HIGH DISCOUNTS AND HIGH UNEMPLOYMENT\*

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We revisit the hypothesis that cyclical fluctuations in unemployment are caused by shocks to the discount rate. We use a simple but rich search-theoretic model of the labour market in which the UE, EU and EE rates are all endogenous. Analytically, we show that an increase in the discount rate lowers the UE rate and, under some natural conditions, it lowers the EU rate. Quantitatively, we show that an increase in the discount rate from 4% to 10% generates a 3.5% decline in the UE rate and a 6% decline in the EU rate. These findings are at odds with the actual behaviour of the US labour market over the business cycle, which features a negative comovement between the UE and EU rates.

In a standard search-theoretic model of the labour market (see, e.g., Mortensen and Pissarides, 1994), agents make two decisions: firms decide whether to open a vacancy in order to hire an additional worker, and firm–worker pairs decide whether or not to break up in order to let the worker seek employment elsewhere. The firms’ decision to open a vacancy trades off the upfront cost of recruiting with the delayed benefit of employing an additional worker. The firm–worker pairs’ decision to break up trades off the upfront cost of foregone wages and profits with the delayed benefit of higher wages, once the worker finds a better job elsewhere. Both of these decisions are investments, in the sense that they trade off an upfront cost with a delayed benefit.

As all investments do, the firms’ decision to open a vacancy and the firm–worker pairs’ decision to break up are sensitive to the rate at which agents discount future payoffs relative to current payoffs. Specifically, if the rate at which agents discount the future increases, firms will open fewer vacancies and firm–worker pairs will become more reluctant to break up. The decline in the number of open vacancies will tend to lower the tightness of the labour market and, in turn, the rate at which unemployed workers become employed (the UE rate). The tightening of the conditions under which firm–worker pairs break up will tend to lower the rate at which employed workers become unemployed (the EU rate).

In this paper, we formalise the argument above and examine its implications for the hypothesis, put forward by Hall (2017), that cyclical fluctuations of the labour market might be caused by shocks to the rate at which agents discount the future. In turn, shocks to the discount rate may be caused by changes in the expected growth rate of the marginal utility of consumption (Hall, 2017) or by changes in an exogenous debt constraint (Eggertsson and Woodford, 2003; Kehoe *et al.*, 2019).

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We consider a search-theoretic model of the labour market in the spirit of Menzio and Shi (2011). In the model, firm–worker matches are heterogeneous, in the sense that different matches have a different quality. Firm–worker matches are experience goods, in the sense that their quality is initially unknown and it is discovered on the job. The model has rich predictions, in the sense that the rate at which unemployed workers become employed (UE rate), the rate at which employed workers become unemployed (EU rate), and the rate at which workers move from one employer to another (EE rate) are all determined endogenously. Workers move from unemployment to employment when they find a vacant job. Workers move from one employer to another employer, as they try to move from low to high-quality matches. Workers move from employment to unemployment when the quality of their match turns out to be low enough. The search process by which firms and workers come together is directed by the terms of the employment contracts offered by firms.

In the first part of the paper, we characterise, analytically, the effect of a discount rate shock. We find that an increase in the discount rate lowers the tightness of the submarket visited by unemployed workers, i.e., it lowers the ratio between the number of vacancies that offer the wage demanded by unemployed workers and the number of unemployed workers. For this reason, an increase in the discount rate lowers the UE rate. We also find a necessary and sufficient condition under which an increase in the discount rate lowers the reservation quality of a firm–worker pair, i.e., it lowers the productivity level below which a firm–worker pair finds it optimal to break up. If and only if the condition is satisfied, an increase in the discount rate lowers the EU rate. The condition is simple and intuitive: the difference between the rate at which a worker finds a new job when unemployed and the rate at which a worker finds a new job when employed at the reservation quality must exceed the arrival rate of an idiosyncratic shock to the quality of a match.

In the second part of the paper, we characterise, quantitatively, the effect of a discount rate shock. We calibrate the model to the US labour market, using data from the Survey of Income and Program Participation (SIPP). In particular, we calibrate the model to match the average UE, EU and EE rates, as well as the relationship between the EU and EE rates and job tenure. For the calibrated version of the model, we find that the necessary and sufficient condition under which an increase in the discount rate increases the EU rate is satisfied. Using the calibrated version of the model, we measure the response of the labour market to an increase in the discount rate from 4 to 10%. We find that the UE rate falls by 3.5%. The EU rate falls by 3% on impact and, over time, reaches a level that is about 6% lower than before the shock. The unemployment rate barely moves, as the decline in the UE rate is offset by the decline in the EU rate.

Our theoretical and quantitative findings highlight a challenge for the hypothesis that labour market fluctuations are driven by discount rate shocks. In response to a discount rate shock, the UE and EU rates move in the same direction—as they are both manifestations of investment decisions—and, hence, they have offsetting effects on unemployment. Empirically, though, the UE and EU rates move in opposite directions at the business cycle frequency and, hence, they both contribute to unemployment fluctuations. The hypothesis of labour market fluctuations driven by aggregate productivity shocks does not suffer from this problem. Using the same model, we find that, in response to a 5% decline in aggregate productivity, the UE rate falls by 10%, the EU rate increases by 20%, and unemployment rises by 30%. Intuitively, the UE and EU rate move in opposite directions because a negative productivity shock lowers the firms' benefit from employing an additional worker and lowers the firm–worker pairs' opportunity cost of breaking up.

In the last part of the paper, we run a battery of robustness checks. First, we consider a version of the baseline model in which the search process is random rather than directed. As in the baseline model, we find that a discount rate shock generates a counterfactually positive comovement between the UE and EU rates. Second, we consider a version of the baseline model in which the quality of a new match is known and evolves over time, either according to a jump process or according to a continuous Ornstein–Uhlenbeck process. Again, we find that a discount rate shock generates positive comovement between the UE and EU rates. Third, we consider a life-cycle version of the model in which workers accumulate human capital on the job. Intuitively, this extension of the model is important because, when workers accumulate human capital on the job, the value of a firm–worker match becomes more backloaded and, thus, more sensitive to changes in the discount rate. Indeed, we find that, for this version of the model, a discount rate shock does generate the same negative comovement between the aggregate UE rate and the aggregate EU rate that we see in the data. A discount rate shock, however, still generates a counterfactually positive comovement between the UE and EU rates for older workers. Intuitively, the fact that the life-cycle profile of wages flattens out implies (according to our model) that the return of experience on human capital is low for older workers. And if the return of experience on human capital is low for older workers, the response of their UE and EU rates is going to be similar to the baseline model.

The contribution of our paper is to use a simple but rich search-theoretic model of the labour market to revisit the hypothesis that labour market fluctuations might be caused by shocks to the discount rate. The first to put forward the high discount hypothesis was Hall (2017), who pointed out that the same shocks to the discount rate that are needed to rationalise the fluctuations of the stock market also rationalise the fluctuations in unemployment.<sup>1</sup> Hall (2017) makes this point using the search-theoretic model of the labour market by Pissarides (1985), a model where the only choice is the firms' decision of how many vacancies to open and, hence, the only endogenous outcome is the UE rate. Here, we use a model in which not only the UE rate, but also the EU and EE rates are endogenous because different firm–worker matches have different quality. The model belongs to a large class of on-the-job search models that have been successfully used to explain the pattern of transitions of workers across employment states (see, e.g., Burdett and Mortensen, 1998; Postel-Vinay and Robin, 2002; Baggers *et al.*, 2014) and, more recently, have been fruitfully used to study aggregate labour market fluctuations (see, e.g., Menzio and Shi, 2011; Lise and Robin, 2017; Baley *et al.*, 2020). By using a richer model, we can derive additional predictions of the high discount hypothesis. One of these predictions—namely that the UE and EU rates would comove over the business cycle—is counterfactual.

Our paper also engages Kehoe *et al.* (2019), a prominent follow-up to Hall (2017). Kehoe *et al.* (2019) point out that, if workers accumulate human capital on the job, the effect of discount rate shocks on the UE rate is magnified.<sup>2</sup> Using a version of the baseline model with human capital accumulation and an explicit life-cycle structure, we find that a positive shock to the discount rate not only generates a large decline in the aggregate UE rate, but also a large increase in the aggregate EU rate. At a disaggregated level, though, the shock still generates positive

<sup>1</sup> Kaplan and Menzio (2016) propose a model that simultaneously generates a decline in the stock market and an increase in the unemployment rate.

<sup>2</sup> Kehoe *et al.* (2019) use a search-theoretic model in which workers are heterogeneous with respect to their human capital. In their model, the UE rate is endogenous. The model also generates an endogenous destruction of firm–worker matches. However, since all the matches between a worker with a given human capital and a firm are identical, when a worker leaves a match he does not enter unemployment (as he has no reason to search for another match that is the same as the one he just left). Rather, the worker exits the labour force. Therefore, their model does not endogenise the EU rate.

comovement between the UE and EU rates for older workers. Moreover, the shock generates larger unemployment response for younger than for older workers. While the first prediction is consistent with the cyclical behaviour of the US labour market, the second and the third one are not.

It is far from us to think that our paper provides final evidence against the high discounts hypothesis. Rather, we think that our paper illustrates a problematic implication of simple formulations of the high discount hypothesis and, in doing so, we hope that our paper may provide useful guidance for future research on the topic.<sup>3</sup> A formulation of the high discount hypothesis that might be consistent with the cyclical behaviour of both UE and EU rates is one in which discount rates affect the cost of some inputs of production. For example, Eckstein *et al.* (2020) consider a search-theoretic model in which firms need to hire workers and rent capital in order to produce. They show that an increase in the rental rate of capital<sup>4</sup> lowers the net income generated by a firm–worker match and, hence, it causes a decline in the UE rate. Since an increase in the rental rate of capital lowers the income produced by a firm–worker match, just as a decline in aggregate productivity does, we would expect it to also cause an increase in the EU rate.

## 1. Theory

In this section, we propose a search-theoretic model of the labour market to understand the effect of fluctuations in the rate at which agents discount future income. Firm–worker matches are heterogeneous, in the sense that different matches have a different quality. Firm–worker matches are experience goods, in the sense that their quality is initially unknown and it is discovered on the job. The model has rich predictions, in the sense that the UE, EU and EE rates are all determined endogenously. Workers move from unemployment to employment when they find a vacant job. Workers move from one employer to another employer, as they try to move from low to high-quality matches. Workers move from employment to unemployment when the quality of their match turns out to be low enough. As in Menzio and Shi (2011), the search process by which firms and workers come together is directed by the terms of the employment contracts offered by firms. Using this model, we derive conditions under which an increase in the discount rate leads not only to a decline in the UE rate, but also to a decline in the EU rate.

### 1.1. Environment

The economy is populated by a measure 1 of workers. Every worker maximises the present value of income discounted at the rate  $r(\omega)$ , where  $\omega$  denotes the state of the aggregate fundamentals of the economy. When a worker is unemployed, his flow income is  $b$ , where  $b$  represents a combination of unemployment benefits and value of leisure. When a worker is employed, his flow income is some  $w$ , where  $w$  represents the wage received by the worker from his employer. A worker permanently exits the labour market at the rate  $\sigma \geq 0$ . A flow of new workers with measures  $\sigma \cdot dt$  enters the labour market during any interval of time of length  $dt$ .

<sup>3</sup> Borovicka and Borovickova (2018) highlight a different set of problematic implications of the high discount hypothesis.

<sup>4</sup> Eckstein *et al.* (2020) assume that the increase in the rental rate of capital is caused by an increase in the cost of financial intermediation. In this sense, theirs is a model of financial intermediation shocks rather than high discounts. It would be easy, though, to modify their model to make the increase in the rental rate of capital emerge from an increase in the rate at which agents discount future income.

The economy is also populated by a positive measure of firms. Every firm maximises the present value of profits discounted at the rate  $r(\omega)$ . A firm operates a constant return to scale production technology that turns the labour input of one worker into a flow  $y(\omega)z$  of output, where  $y(\omega)$  is a component of labour productivity that is common to all firm–worker matches, and  $z$  is a component of labour productivity that is idiosyncratic to a particular firm–worker match. We shall refer to  $z$  as the *quality of a match*.

The labour market features search frictions. A firm searches for workers by maintaining vacant jobs at the flow cost  $k > 0$ . An unemployed worker searches for vacant jobs with an intensity normalised to 1. An employed worker searches for vacant jobs with an intensity of  $\chi \geq 0$ . The process that brings searching workers and searching firms together is directed by the terms of trade. Specifically, for each of its vacant jobs, a firm chooses the lifetime income  $x$  that it will deliver to a worker hired for the job. Searching workers choose what lifetime income  $x$  to seek from a job. Then firms offering the lifetime income  $x$  and workers seeking the lifetime income  $x$  meet bilaterally. A worker seeking the lifetime income  $x$ , finds a job at the Poisson rate  $p(\theta(x))$ , where  $\theta(x)$  is the ratio between the measure of workers seeking  $x$  and the measure of vacancies offering  $x$ , and  $p(\theta)$  is a strictly increasing, strictly concave function of  $\theta$  such that  $p(0) = 0$ . Similarly, a vacancy offering the lifetime income  $x$  finds a worker at the Poisson rate  $q(\theta(x))$ , where  $q(\theta) = p(\theta)/\theta$  is a strictly decreasing function of  $\theta$  such that  $q(0) = \infty$  and  $q(\infty) = 0$ . We shall refer to  $\theta(x)$  as the *tightness of submarket  $x$* .

When a worker and a firm meet in submarket  $x$ , the firm offers the worker an employment contract worth  $x$  in lifetime income. If the worker rejects the offer, the firm retains its vacancy and the worker returns to his prior employment state (either unemployment or employment at some other firm). If the worker accepts the offer, the firm and the worker start production. The firm and the worker are not immediately aware of the quality  $z$  of their match. Instead, they discover the quality of their match at the rate  $\phi > 0$ . The quality of the match is drawn from a twice differentiable cumulative distribution function  $F(z)$  with a mean normalised to 1. The quality of the match changes at the rate  $\eta \geq 0$ , in which case the firm and the worker have to discover it again. The match becomes unviable for exogenous reasons at the rate  $\delta \geq 0$ , in which case the firm and the worker break up.

We assume that the contracts offered by firms to workers are *bilaterally efficient*, in the sense that they maximise the joint value of the firm–worker match. As discussed in Menzio and Shi (2011), this assumption is consistent with several contractual environments. Consider two cases. In the first case, a contract can specify the worker's wage, the worker's search strategy on the job (i.e., in which submarket to search) and the worker's quitting strategy (i.e., when to move into unemployment) contingent on the history of the match and the economy. In this case, the contract space is rich enough to independently control the allocative decisions of the match and the distribution of the value of the match between the firm and the worker. Given this contractual environment, the firm finds it optimal to offer a contract such that the allocative decisions maximise the joint value of the match, and such that the wages provide the worker with the lifetime income  $x$ . In the second case, a contract can specify a sign-on transfer and then a wage contingent on the history of the match and the economy. The worker is then free to follow his preferred search and quitting strategy. In this case, the firm finds it optimal to offer a contract such that the worker is the residual claimant of output (and, hence, makes allocative decisions to maximise the joint value of the match) and a (negative) transfer such that the worker's lifetime income is  $x$ .

The state  $\omega$  of aggregate fundamentals belongs to some discrete set  $\Omega = \{\omega_1, \omega_2, \dots, \omega_N\}$  and evolves stochastically according to some exogenous process. Specifically, the state of the aggregate fundamentals changes at the rate  $\lambda \geq 0$  and, conditional on changing, it takes on the values  $\hat{\omega} \in \Omega$  with probability  $\Pi(\hat{\omega}|\omega)$ . The state  $\omega$  affects two aggregate fundamentals: the discount factor  $r$  and the common component of productivity  $y$ . One can think of shocks to  $y$  as either shocks to the quantity of output produced by a worker (as in Kydland and Prescott, 1982 or Mortensen and Pissarides, 1994), or as shocks to the price of the output produced by a worker relative to the price of maintaining a vacancy (as in Kaplan and Menzio, 2016). One can think of shocks to  $r$  as either shocks to the expected growth rate of the marginal utility of consumption (as in Hall, 2017) or as shocks to the effective discount rate caused by the tightening and loosening of an exogenous debt constraint<sup>5</sup> (as in Eggertsson and Woodford, 2003 and Kehoe *et al.*, 2019).

## 1.2. Equilibrium

At date  $t$ , the state of the economy is described by the tuple  $\psi = \{\omega, u, n, G\}$ , where  $\omega$  is the state of the aggregate fundamentals,  $u$  is the measure of workers who are unemployed,  $n$  is the measure of workers employed in a match of unknown quality, and  $G(z)$  is the measure of employed workers in a match of quality non-greater than  $z$ . Thus, we have  $u + n + G(\infty) = 1$ . In principle, the equilibrium value and policy functions might depend both on the exogenous fundamentals,  $\omega$ , and on the endogenous distribution of workers across employment states,  $\{u, n, G\}$ . However, as established in Menzio and Shi (2011), the equilibrium is block recursive, in the sense that the value and policy functions only depend on  $\omega$  and not on  $\{u, n, G\}$ . In light of this observation, we denote as  $U(\omega)$  the value of unemployment to a worker, as  $V_0(\omega)$  the joint value of a firm–worker match of unknown quality, as  $V(z, \omega)$  the joint value of a firm–worker match of quality  $z$ . Finally, we let  $\theta(x, \omega)$  denote the tightness of submarket  $x$ .

The value  $U(\omega)$  of unemployment to a worker is such that

$$r(\omega)U(\omega) = b + \max_x [p(\theta(x, \omega))(x - U(\omega))] - \sigma U(\omega) + \lambda \mathbb{E}[U(\hat{\omega}) - U(\omega)]. \quad (1)$$

The above expression is easy to understand. The flow income of an unemployed worker is  $b$ . At rate  $p(\theta(x, \omega))$ , the worker meets a vacancy in submarket  $x$ . In this case, the worker experiences a change in value of  $x - U(\omega)$ . At rate  $\sigma$ , the worker permanently exits the labour market. In this case, the worker experiences a change in value of  $0 - U(\omega)$ . At rate  $\lambda$ , the fundamentals of the economy change. In this case, the worker experiences a change in value of  $U(\hat{\omega}) - U(\omega)$ . Notice that, as search is directed, the worker chooses the submarket  $x$  in which to look for a vacancy.

The joint value  $V(z, \omega)$  of a firm–worker match of quality  $z$  is such that

$$\begin{aligned} r(\omega)V(z, \omega) &= y(\omega)z + \chi \max_x [p(\theta(x, \omega))(x - V(z, \omega))] + \eta [V_0(\omega) - V(z, \omega)] \\ &\quad + \delta [U(\omega) - V(z, \omega)] - \sigma V(z, \omega) \\ &\quad + \lambda \mathbb{E}[\max\{V(z, \hat{\omega}), U(\hat{\omega})\} - V(z, \omega)]. \end{aligned} \quad (2)$$

The flow income of a firm–worker pair of quality  $z$  is  $y(\omega)z$ . At rate  $\chi p(\theta(x, \omega))$ , the worker meets a vacancy in submarket  $x$ . In this case, the firm–worker pair experiences a change in

<sup>5</sup> It would be straightforward, albeit tedious, to embed our model within a large family structure as in Kehoe *et al.* (2019). The same equations would describe the behaviour of family members across employment states. An additional equation describing the evolution of the multiplier on the debt constraint would determine the discount rate  $r$ .

the joint value of  $x - V(z, \omega)$ , as the worker is hired by the vacancy and enjoys a continuation value of  $x$  and the firm loses the worker and enjoys a continuation value of 0. Notice that, as the employment contract between the firm and the worker is bilaterally efficient,  $x$  is chosen to maximise the joint value of the match. At rate  $\eta$ , the quality of the match is reset and has to be rediscovered. In this case, the firm–worker pair experiences a change in the joint value of  $V_0(\omega) - V(z, \omega)$ . At rate  $\delta$ , the match is broken for exogenous reasons. In this case, the firm–worker pair experiences a change in the joint value of  $U(\omega) - V(z, \omega)$ , as the worker becomes unemployed and enjoys a continuation value of  $U(\omega)$  and the firm loses the worker and enjoys a continuation value of 0. At rate  $\sigma$ , the worker permanently exits the labour market. In this case, the firm–worker pair experiences a change in the joint value of  $0 - V(z, \omega)$ . At rate  $\lambda$ , there is a fundamental shock. In this case, the firm–worker pair experiences a change in the joint value of  $\max\{V(z, \hat{\omega}), U(\hat{\omega})\} - V(z, \omega)$ , where the maximum between  $V(z, \hat{\omega})$  and  $U(\hat{\omega})$  is the value of the bilaterally efficient choice between staying together and breaking up.

The joint value  $V_0(\omega)$  of a firm–worker match of unknown quality is such that

$$r(\omega)V_0(\omega) = y(\omega) + \phi\mathbb{E}[\max\{V(z, \omega), U(\omega)\} - V_0(\omega)] + \chi \max_x [p(\theta(x, \omega))(x - V_0(\omega))] + \delta [U(\omega) - V_0(\omega)] - \sigma V_0(\omega) + \lambda\mathbb{E}[\max\{V_0(\hat{\omega}), U(\hat{\omega})\} - V_0(\omega)]. \quad (3)$$

The expected flow income of a firm–worker pair of unknown quality is  $y(\omega)$ . At rate  $\phi$ , the firm–worker pair discovers the quality  $z$  of their match. In this case, the firm–worker pair experiences a change in value of  $\max\{V(z, \omega), U(\omega)\} - V_0(\omega)$ , where the max between  $V(z, \omega)$  and  $U(\omega)$  is the value of the bilaterally efficient choice between staying together and breaking up. The remaining terms on the right-hand side of (3) have a direct counterpart in (2) and need no further explanation.

The tightness  $\theta(x, \omega)$  of submarket  $x$  is such that

$$k \geq q(\theta(x, \omega))(V_0(\omega) - x) \quad (4)$$

and  $\theta(x, \omega) \geq 0$ , where the two inequalities hold with complementary slackness. The left-hand side of (4) is the cost to a firm from maintaining a vacancy in submarket  $x$ . The right-hand side is the benefit to a firm from maintaining a vacancy in submarket  $x$ , which is given by the product between the rate at which the firm fills its vacancy,  $q(\theta(x, \omega))$ , and the value to the firm from filling its vacancy,  $V_0(\omega) - x$ . Condition (4) states that, if some vacancies are maintained in submarket  $x$  and, hence,  $\theta(x, \omega)$  is strictly positive, then the cost of a vacancy must equal the benefit. If no vacancies are maintained in submarket  $x$ , the cost of a vacancy must exceed the benefit.

The search problems in (1), (2) and (3) have a common structure and their solution can be characterised at once. Consider the search problem for a worker who is currently in an employment state with arbitrary value  $v$  (where  $v$  is  $U$  if the worker is unemployed and  $V$  if he is employed). The search problem can be written as

$$D(v, \omega) = \max_{x, \theta} p(\theta)(x - v), \text{ s.t. } \theta = \theta(x, \omega). \quad (5)$$

In words, (5) states that the worker chooses the lifetime income  $x$  promised by the vacancy and the tightness  $\theta$  of the submarket in which the vacancy is located so as to maximise  $p(\theta)(x - v)$ , taking as given the equilibrium market tightness function  $\theta(x, \omega)$ . For any  $\theta > 0$ , (4) implies that the value  $x$  offered by the vacancy is  $V_0(\omega) - k/q(\theta)$  and, hence, the objective function in (5)

is equal to  $p(\theta)(V_0(\omega) - v) - k\theta$ . For any  $\theta = 0$ ,  $p(\theta) = 0$  and, hence, the objective function in (5) is equal to zero and, also, equal to  $p(\theta)(V_0(\omega) - v) = k\theta$ .

In light of the previous observations, we can rewrite (5) as

$$D(v, \omega) = \max_{\theta} p(\theta)(V_0(\omega) - v) - k\theta. \quad (6)$$

In words, (6) states that the worker chooses the tightness  $\theta$  of the submarket in which to search so as to maximise the product between the rate at which he meets a new firm,  $p(\theta)$ , and the joint value of a meeting with a new firm,  $V_0(\omega) - v$ , net of the cost of maintaining  $\theta$  vacancies,  $k\theta$ . The formulation in (6) makes it clear that the market tightness function  $\theta(x, \omega)$  is such that, when the worker is deciding where to search, he internalises both the firms' cost of maintaining vacancies and the firms' benefit from filling vacancies in different submarkets.

Using (6), we can characterise the solution to the search problems in (1), (2) and (3). When unemployed, the worker searches in a submarket with tightness  $\theta_u(\omega)$  such that

$$k \geq p'(\theta_u(\omega))(V_0(\omega) - U(\omega)) \quad (7)$$

and  $\theta_u(\omega) \geq 0$ , where the two inequalities hold with complementary slackness. Condition (7) is intuitive. The optimal tightness  $\theta_u(\omega)$  is such that the cost to the firm of maintaining an additional vacancy is equated to the benefit to the worker and the firm of increasing the rate at which they meet.

When employed in a match of quality  $z$ , the worker searches in a submarket with tightness  $\theta_e(z, \omega)$  such that

$$k \geq p'(\theta_e(z, \omega))(V_0(\omega) - V(z, \omega)), \quad (8)$$

and  $\theta_e(z, \omega) \geq 0$ , where the two inequalities hold with complementary slackness. Condition (8) implies that, when a worker is employed in a match of quality  $z$  with  $V(z, \omega) \geq V_0(\omega)$ , he will search in a submarket with zero tightness, as the joint value of the match between the worker and his current employer is greater than the joint value of a match between the worker and a new employer. For the same reason, when a worker is currently employed in a match of unknown quality, he will search in a submarket with zero tightness.

Lastly, we characterise the solution of the problem of a firm–worker pair discovering the quality of their match. The firm–worker pair remains together if the quality  $z$  of their match is such that  $V(z, \omega) \geq U(\omega)$ , and it breaks up if  $z$  is such that  $V(z, \omega) < U(\omega)$ . Let the *reservation quality*  $R(\omega)$  be defined as the quality that makes the pair indifferent between remaining together and breaking up, i.e.,

$$V(R(\omega), \omega) = U(\omega). \quad (9)$$

Since the joint value of the match  $V(z, \omega)$  is strictly increasing in  $z$ , it follows that the firm–worker pair remains together if the quality  $z$  of their match exceeds the reservation quality  $R(\omega)$ , and it breaks up if the quality of their match falls short of the reservation quality.

We define the *surplus*  $S(z, \omega)$  of a firm–worker match of quality  $z$  as the difference between the joint value of the match,  $V(z, \omega)$ , and the value of unemployment,  $U(\omega)$ . Subtracting (1) from



(2) and using (6), we obtain the following expression for  $S(z, \omega)$ :

$$\begin{aligned} & (r(\omega) + \delta + \sigma + \lambda)S(z, \omega) \\ &= y(\omega)z - b + \chi [p(\theta_e(z, \omega))(S_0(\omega) - S(z, \omega)) - k\theta_e(z, \omega)] \\ & \quad + \eta [S_0(\omega) - S(z, \omega)] - [p(\theta_u(\omega))S_0(\omega) - k\theta_u(\omega)] + \lambda \mathbb{E} [\max\{S(z, \hat{\omega}), 0\}]. \end{aligned} \tag{10}$$

Similarly, we define the surplus  $S_0(\omega)$  of a firm–worker match of unknown quality as  $V_0(\omega) - U(\omega)$ . Subtracting (1) from (3) and using (6), we obtain the following expression for  $S_0(\omega)$ :

$$\begin{aligned} & (r(\omega) + \delta + \sigma + \lambda)S_0(\omega) \\ &= y - b + \phi \int_{R(\omega)} [S(z, \omega) - S_0(\omega)] dF(z) \\ & \quad - [p(\theta_u(\omega))S_0(\omega) - k\theta_u(\omega)] + \lambda \mathbb{E} [\max\{S_0(\hat{\omega}), 0\}]. \end{aligned} \tag{11}$$

Using (10) and  $S(R(\omega), \omega) = 0$ , we obtain an explicit formula for the reservation quality  $R(\omega)$ :

$$\begin{aligned} yR(\omega) &= b + (1 - \chi) [p(\theta_u(\omega))S_0(\omega) - k\theta_u(\omega)] \\ & \quad - \eta S_0(\omega) - \lambda \mathbb{E} [\max\{S(R(\omega), \hat{\omega}), 0\}]. \end{aligned} \tag{12}$$

The left-hand side of (12) is the reservation quality multiplied by the aggregate component of productivity (i.e., the flow income from employment at the reservation quality). The right-hand side is the sum of four terms. The first term is the flow income from unemployment. The second term is the difference between the flow value of searching while unemployed rather than while employed at the reservation quality. The third term is the negative of the flow value of an idiosyncratic productivity shock while employed at the reservation quality. The last term is the negative of the difference between the flow value of an aggregate fundamental shock while unemployed rather than while employed at the reservation quality.

As the reader can see, the equilibrium conditions (10) and (11) for the value functions  $S_0$  and  $S$ , and the equilibrium conditions (7), (8) and (12) for the policy functions  $\theta_u$ ,  $\theta_e$  and  $R$  are independent of the distribution of workers across employment states. For this reason, the equilibrium is block recursive. The distribution of workers across employment states evolves according to the policy functions. Specifically, during any interval of time in which the aggregate fundamentals remain unchanged, the distribution of workers across employment states evolves according to the differential equations

$$\dot{u} = \sigma + (1 - u)\delta + n\phi F(R(\omega)) - u [\sigma + p(\theta_u(\omega))] \tag{13}$$

$$\dot{n} = up(\theta_u(\omega)) + \int_{R(\omega)} (\eta + \chi p(\theta_e(z, \omega))) dG(z) - n (\delta + \phi + \sigma) \tag{14}$$

$$\dot{G}(z) = n\phi [F(z) - F(R(\omega))] - \int_{R(\omega)}^z (\delta + \sigma + \eta + \chi p(\theta_e(\hat{z}, \omega))) dG(\hat{z}). \tag{15}$$

The laws of motion above are easy to understand. For instance, the change  $\dot{u}$  in the measure of unemployed workers is equal to the flow of workers who enter the labour market, plus the flow of workers who move from employment into unemployment for either exogenous or endogenous reasons, net of the flow of workers who exit unemployment either because they exit the labour market or because they find a job. Similarly, (14) and (15) express the change in the measure of

workers who are employed in a match of unknown quality,  $\hat{n}$ , and the change in the measure of workers in a match of quality non-greater than  $z$ ,  $\hat{G}(z)$ , as the difference between flows in and flows out.

When the state of the aggregate fundamentals changes from  $\omega$  to  $\hat{\omega}$ , there might be some discontinuity in the evolution of the distribution of workers across employment states. Letting  $+$  denote measures immediately after the aggregate shock, we have

$$u(+) = u + [G(R(\hat{\omega})) - G(R(\omega))] \tag{16}$$

$$n(+) = n \tag{17}$$

$$G(z+) = G(z) - G(R(\hat{\omega})) \text{ if } z > R(\hat{\omega}), 0 \text{ else.} \tag{18}$$

### 1.3. High Discounts

We now want to analytically characterise the effect of an increase in the discount rate  $r$  on the equilibrium value and policy functions and, in turn, on the labour market. Specifically, we consider the effect of a change in the aggregate state of the economy from  $\omega$  to some  $\hat{\omega}$ , with  $y(\hat{\omega}) = y(\omega)$  and  $r(\hat{\omega}) = r(\omega) + dr$ , where  $dr > 0$  is arbitrarily small. Since  $y(\hat{\omega}) = y(\omega)$  and  $r(\hat{\omega}) = r(\omega) + dr$ , we express the effect of moving from state  $\omega$  to state  $\hat{\omega}$  on the value functions and on the policy functions in terms of their derivatives with respect to  $r$ . To keep the analysis simple, we assume that shocks to the aggregate fundamentals are unanticipated and permanent, in the sense that the switching rate  $\lambda$  is zero.<sup>6</sup>

We start by examining the effect of the discount rate on the value functions. Let  $\partial S_0/\partial r$  denote the derivative of the surplus of a match of unknown quality with respect to  $r$  and  $\partial S(z)/\partial r$  the derivative of the surplus of a match of quality  $z$  with respect to  $r$ . The derivative  $\partial S_0/\partial r$  is such that

$$(r + \delta + \sigma + \phi + p(\theta_u)) \frac{\partial S_0}{\partial r} = -S_0 + \phi \int_R \frac{\partial S(z)}{\partial r} dF(z), \tag{19}$$

where the expression above is obtained by differentiating (11) with respect to  $r$  and by making use of the optimality condition (7) for  $\theta_u$ . The derivative  $\partial S(z)/\partial r$  is such that

$$(r + \delta + \sigma + \eta + \chi p(\theta_e(z))) \frac{\partial S(z)}{\partial r} = -S(z) - (p(\theta_u) - \chi p(\theta_e(z)) - \eta) \frac{\partial S_0}{\partial r}, \tag{20}$$

where the expression above is obtained by differentiating (10) with respect to  $r$  and by making use of the optimality condition (8) for  $\theta_e(z)$ . Solving (20) with respect to  $\partial S(z)/\partial r$  and substituting the solution into the right-hand side of (19), we find that  $\partial S_0/\partial r < 0$ . That is, an increase in the discount rate lowers the surplus of a firm–worker match of unknown quality. Intuitively, an increase in the discount rate reduces the difference between the present value of the income generated by a firm and a worker that are matched together relative to the present value of the income generated by a firm and a worker that are unmatched.

<sup>6</sup> Throughout the analysis, we maintain the assumption that the value and policy functions take non-degenerate values in the aggregate state  $\omega$ . Specifically, we assume that the surplus  $S_0$  of a firm–worker match with unknown quality is strictly positive, so that the tightness  $\theta_u(\omega)$  is strictly positive. Similarly, we assume that the reservation quality  $R(\omega)$  is on the support of  $F$ , so that changes in the reservation quality have an effect on outcomes. Relaxing these assumptions would simply turn the strict inequalities in (30)–(33) into weak inequalities.

Next, we examine the effect of the discount rate on the policy functions. Let  $\partial\theta_u/\partial r$  denote the derivative of the tightness of the submarket where unemployed workers look for vacancies with respect to  $r$ . The derivative  $\partial\theta_u/\partial r$  is such that

$$0 = p'(\theta_u)\frac{\partial S_0}{\partial r} + p''(\theta_u)\frac{\partial\theta_u}{\partial r}. \quad (21)$$

Since  $p'(\cdot) > 0$ ,  $p''(\cdot) < 0$  and  $\partial S_0/\partial r < 0$ , (21) implies  $\partial\theta_u/\partial r < 0$ . That is, an increase in the discount rate lowers the tightness of the submarket where unemployed workers look for vacancies. This finding is also intuitive. As established above, an increase in the discount rate lowers the surplus of a firm–worker match of unknown quality, i.e., it lowers the difference between the joint value of a new firm–worker match and the value of unemployment. Consequently, an increase in the discount rate induces unemployed workers to search for vacancies in a submarket with lower tightness. This is the effect of high discounts highlighted by Hall (2017). In Hall (2017), this effect is illustrated in a model where the search process is random. The same effect is at work here, in a model where the search process is directed.

The derivative  $\partial R/\partial r$  of the reservation quality  $R$  with respect to the discount rate  $r$  is such that

$$y\frac{\partial R}{\partial r} = [(1 - \chi)p(\theta_u) - \eta]\frac{\partial S_0}{\partial r}, \quad (22)$$

where the expression above makes use of the optimality condition (8) for  $\theta_u$ . Since  $\partial S_0/\partial r < 0$ , an increase in the discount rate lowers the reservation quality if  $(1 - \chi)p(\theta_u) - \eta > 0$ , and raises the reservation quality if  $(1 - \chi)p(\theta_u) - \eta < 0$ . Also this finding is easy to understand. The value of unemployment includes a current payoff—the flow income  $b$ —and a future payoff—the value of searching. The joint value of a match of quality  $R$  includes a current payoff—the flow income  $yR$ —and a future payoff—the value of searching and the value of an idiosyncratic productivity shock. The future payoff is larger in the value of unemployment than in the joint value of a match of quality  $R$  iff  $(1 - \chi)p(\theta_u) - \eta > 0$ . Thus, iff  $(1 - \chi)p(\theta_u) - \eta > 0$ , an increase in  $r$  lowers the value of unemployment more than the joint value of a match of quality  $R$ , and  $R$  must fall to maintain the indifference between unemployment and employment at the reservation quality. This effect of high discounts is novel, since Hall (2017) considers a model in which all firm–worker matches are identical.

Let us look in some detail at the condition

$$(1 - \chi)p(\theta_u) - \eta > 0. \quad (23)$$

Condition (23) is effectively a condition under which the decision of breaking the match is an investment, in the sense that breaking the match involves future benefits and upfront costs. The condition is certainly satisfied if workers cannot search on the job ( $\chi = 0$ ) and the quality of a firm–worker match is permanent ( $\eta = 0$ ). More generally, the condition is laxer when the job-finding rate  $p(\theta_u)$  for unemployed workers is higher, when the relative efficiency  $\chi$  of search on the job is lower, and when the rate  $\eta$  at which the quality of a match changes is lower. Empirically, the condition is likely to be satisfied. Indeed, the job-finding rate for unemployed worker is about 26% per month, the job-finding rate for employed workers is around 2% per month—which suggests that  $\chi$  is low—and the rate at which matches break-up is very low at long tenures—which suggests that  $\eta$  is low. Later on, we will make these observations precise. For now, we will proceed under the assumption that condition (23) holds.

Lastly, we examine the effect of an increase in the discount rate on the UE and EU rates. To this aim, suppose that, at the time of the positive shock to the discount rate, the distribution of workers across employment states is at its stationary level  $\{u^*, n^*, G^*\}$ . From (13)–(15), it follows that

$$u^* = \frac{\sigma + h_{eu}^*}{\sigma + h_{eu}^* + h_{ue}^*} \tag{24}$$

$$\frac{n^*}{1 - u^*} = \frac{\delta + \sigma + \eta + h_{ee}^*}{\delta + \sigma + \eta + \phi(1 - F(R))} \tag{25}$$

$$G^*(z) = \frac{1}{\delta + \sigma + \eta} \left[ n^* \phi [F(z) - F(R)] - \chi \int_R^z p(\theta_e(\hat{z})) dG^*(\hat{z}) \right], \tag{26}$$

where  $h_{ue}^*$ ,  $h_{eu}^*$  and  $h_{ee}^*$  denote the UE, EU and EE rates at the stationary distribution and are given by

$$h_{ue}^* = p(\theta_u) \tag{27}$$

$$h_{eu}^* = \delta + \frac{n^*}{1 - u^*} \phi F(R) \tag{28}$$

$$h_{ee}^* = \frac{\chi}{1 - u^*} \int_R p(\theta_e(z)) dG^*(z). \tag{29}$$

At the time of the positive shock to the discount factor, there is no instantaneous change in the distribution of workers across employment states. The UE and EU rates, however, do change because of the instantaneous change in the policy functions. In particular, the change in the UE and EU rates relative to  $h_{ue}^*$  and  $h_{eu}^*$  is

$$\frac{\partial h_{ue}}{\partial r} = p'(\theta_u) \frac{\partial \theta_u}{\partial r} < 0 \tag{30}$$

$$\frac{\partial h_{eu}}{\partial r} = \frac{n^*}{1 - u^*} \phi F'(R) \frac{\partial R}{\partial r} < 0. \tag{31}$$

On impact, both the UE and the EU rate fall. Intuitively, an increase in the discount rate lowers the tightness of the submarket where unemployed workers look for vacancies and, thus, it leads to a decline in the UE rate. An increase in the discount rate lowers the reservation quality and, thus, it leads to a decline in the EU rate. In the data, however, recessions start with a sharp increase in the EU rate coupled with a decline in the UE rate (see, e.g., Menzio and Shi, 2011).

Over time, the distribution of workers across employment states evolves according to the laws of motion (13)–(15). Once the distribution reaches its new steady state, the change in the UE and EU rates relative to  $h_{ue}^*$  and  $h_{ee}^*$  is

$$\frac{\partial h_{ue}^*}{\partial r} = p'(\theta_u) \frac{\partial \theta_u}{\partial r} \tag{32}$$

$$\begin{aligned} \frac{\partial h_{eu}^*}{\partial r} &= \frac{n^*}{1 - u^*} \frac{\delta + \sigma + \eta + \phi}{\delta + \sigma + \eta + \phi(1 - F(R))} \phi F'(R) \frac{\partial R}{\partial r} \\ &+ \frac{\phi F(R)}{\delta + \sigma + \eta + \phi(1 - F(R))} \frac{\partial h_{ee}^*}{\partial r}. \end{aligned} \tag{33}$$

The derivative in (32) implies that the UE rate is lower at the new than at the old steady state. The derivative in (33) implies that the EU rate is lower at the new than at the old steady state, as long as the EE rate declines. Overall, a positive discount rate shock cannot possibly lead to

a stationary equilibrium where the UE and EE rates are lower and the EU rate is higher. In the data, though, recessions feature precisely lower UE and EE rates, and a higher EU rate (see, e.g., Menzio and Shi, 2011).

We summarise the analysis of the baseline model in the following proposition.

**PROPOSITION 1.** (*High Discounts*) Consider an unanticipated and permanent positive shock to the discount factor. If  $(1 - \chi)p(\theta_u) - \eta > 0$ , then:

- (i) On impact, the shock lowers the UE and EU rates;
- (ii) In steady state, the shock either lowers the UE and EU rates, or it lowers the UE rate and increases the EU and EE rates.

## 2. Calibration

In this section, we calibrate the model using some crucial features of workers' reallocation in the US labour market: the average UE, EU and EE rates, as well as the relationship between EU and EE rates and job tenure. Using the calibrated model, we simulate the response of the labour market to an increase in the rate at which agents discount future income, and compare it with the response of the labour market to a decline in the aggregate component of productivity. We find that an increase in the discount rate leads to a small decline in the UE and EU rates and to a negligible change in the unemployment rate. In contrast, a decline in the aggregate component of productivity leads to a sizeable decline in the UE rate and a sizeable increase in the EU rate, leading to an even larger change in the unemployment rate. Since the UE rate and the EU rate move in opposite directions over the business cycles, our findings highlight a challenge for the high discount hypothesis of labour market fluctuations.<sup>7</sup>

### 2.1. Parameters and Data

The model is described by a handful of fundamentals. The search process is described by the vacancy cost  $k$ , the flow income of unemployment  $b$ , the job-finding rate function  $p(\theta)$ , and the intensity of search on the job  $\chi$ . The production process is described by the aggregate component of productivity  $y$ , which we normalise to 1, the distribution  $F$  of the idiosyncratic component of productivity, and the rate  $\delta$  at which a match is exogenously destroyed. The learning process is described by the rate  $\phi$  at which a firm-worker pair discovers the quality of their match and by the rate  $\eta$  at which the quality of the match is redrawn. The entry-and-exit process is described by the exit rate  $\sigma$ . We assume that  $p(\theta)$  is of the form  $\theta^\gamma$ , where  $\gamma \in [0, 1]$  is the elasticity of the job-finding rate to the vacancy-to-applicant ratio. We assume that  $F$  is a Weibull distribution with shape  $\alpha$ , scale  $\beta$  and a location parameter chosen so that the average of the distribution is 1. The Weibull distribution encompasses distributions with declining density, hump-shaped density and a thick right tail, and hump-shaped densities and a thick left tail.

We calibrate the parameters of the model using moments constructed from the US Census's SIPP for male workers with a high-school degree and no further degree. We focus on male workers to minimise the discrepancy between the model—where workers enter the labour force

<sup>7</sup> To be very clear, we do not claim that every version of the high discount hypothesis generates a counterfactually positive comovement between the UE and the EU rate. We only claim that there is a natural economic force that tends to push the UE and EU rates in the same direction in response to a discount rate shock. Our simple model illustrates this force and our calibration quantifies this force.

and then exit it permanently—and the data—where workers transition in and out of the labour force. We focus on high-school graduates because they represent the largest as well as the median education group. We refer the reader to Menzio *et al.* (2016) for additional details on the data.<sup>8</sup>

We construct the following moments: (i) the monthly UE rate; (ii) the fraction of workers with  $t$  months of tenure in their job who, in the next month, leave for unemployment (EU rate by tenure) or for another job (EE rate by tenure); (iii) the fraction of workers who, before reaching a tenure of  $t$  months in their job, leave for unemployment (cumulative EU flow by tenure) or for another job (cumulative EE flow by tenure); (iv) the fraction of workers who reach a tenure of  $t$  months (survival by tenure); (v) the monthly EU and EE rates implied by the previous moments.<sup>9</sup>

We calibrate  $k$  so that the average UE rate in the model is the same as in the data (26% per month). We calibrate  $\chi$  so that the average EE rate in the model is the same as in the data (1.7% per month). We calibrate  $\beta$  so that the average EU rate in the model is the same as in the data (0.95% per month). We calibrate  $\phi$ , which controls how quickly a firm–worker pair discovers the quality of its match, so that the fraction of jobs that end within a year is the same in the model as in the data (45%). We calibrate  $\eta$ , which controls the frequency of shocks to the quality of a match, so that the hazard rate for jobs with tenure above 4.5 years is the same in the model and in the data. We calibrate  $\alpha$ , which determines the shape of the Weibull distribution, so as to minimise the distance between the shape of the tenure profiles in the model and in the data.<sup>10</sup> We set  $\delta$  to 0.1% per month, some low, positive value to guarantee that the stationary distribution of workers across employment states is non-degenerate for any choice of the match quality distribution.<sup>11</sup>

Lastly, we need to choose values for the parameters  $\sigma$ ,  $\gamma$  and  $b$ . We set  $\sigma$ , the rate at which workers exit the labour market, to 0.27% per month. This value implies that a worker remains in the labour market for an average of 35 years. We set  $\gamma$ , the elasticity of the job-finding rate with respect to the vacancy-to-applicant ratio, to 1/2. This is the value typically chosen in the literature. We set  $b$ , the flow income of unemployment, so as to equal 70% of the average

<sup>8</sup> It is well known that the UE, EU and EE rates in the SIPP and the Current Population Survey (CPS) microdata are not identical. In the SIPP, workers are interviewed every four months and can report a maximum of two distinct employers for each four-month period. For this reason, the SIPP features slightly lower UE, EU and EE rates than the CPS (see, e.g., Nagypal, 2008). Moreover, we adopt a different definition of unemployment than the BLS. Specifically, we consider workers who are ‘with a job, on layoff’ as employed rather than as unemployed (as they would in the CPS). Since workers ‘with a job, on layoff’ are in a relationship with their employer and they expect to return to their job, we believe that—through the lens of the model—they should be counted as employed. For this reason too, our UE and EU rates are lower than in the CPS microdata.

<sup>9</sup> Let us explain how we construct the EU and EE rates. We compute the stationary distribution of employed workers by tenure, using the monthly UE rate and the fraction of workers who leave their job after  $t$  months of tenure. Using the stationary distribution of employed workers by tenure, we use the EU and EE rates by tenure to calculate the average EU and EE rates. Alternatively, we could have directly computed the fraction of employed workers who, in the next month, move into unemployment and into another job. The two approaches to computing EU and EE rates give similar results for both the EU rate (0.95% versus 0.65%) and for the EE rate (1.8% versus 1.7%). The difference may be due to life-cycle changes in transition rates that cannot be captured by our perpetual youth model, or by other forms of worker heterogeneity that are not included in our model (see, e.g., Gregory *et al.*, 2020). We chose the indirect approach to computing the EU and EE rates because it is consistent with the model. It is worth noting that, if we use the direct measures of the EU and EE rates, our qualitative findings on the effect of an increase in the discount rate do not change.

<sup>10</sup> The logic behind the calibration strategy is straightforward and well established (see, e.g., Menzio and Shi, 2011; Menzio *et al.*, 2016; Gregory *et al.*, 2020). Taking as given the other parameters: an increase in  $k$  lowers the UE rate; an increase in  $\chi$  raises the EE rate; an increase in  $\beta$  raises  $F(R)$  and, thus, the EU rate; an increase in  $\phi$  raises the rate at which the match quality is discovered and, thus, the fraction of jobs that terminate in less than one year; an increase in  $\eta$  raises the rate at which the quality of a match is reset and, hence, the EU and EE rates at long tenures. The shape of the  $F$  distribution determines the density of qualities above  $R$  and, hence, the rate at which the fraction of surviving jobs declines with tenure.

<sup>11</sup> At the cost of some more computational time, we could have chosen  $\delta$  so as to match the EU rate at long tenures. As one can infer from Figure 1, this would have resulted in a slightly lower value for  $\delta$ .

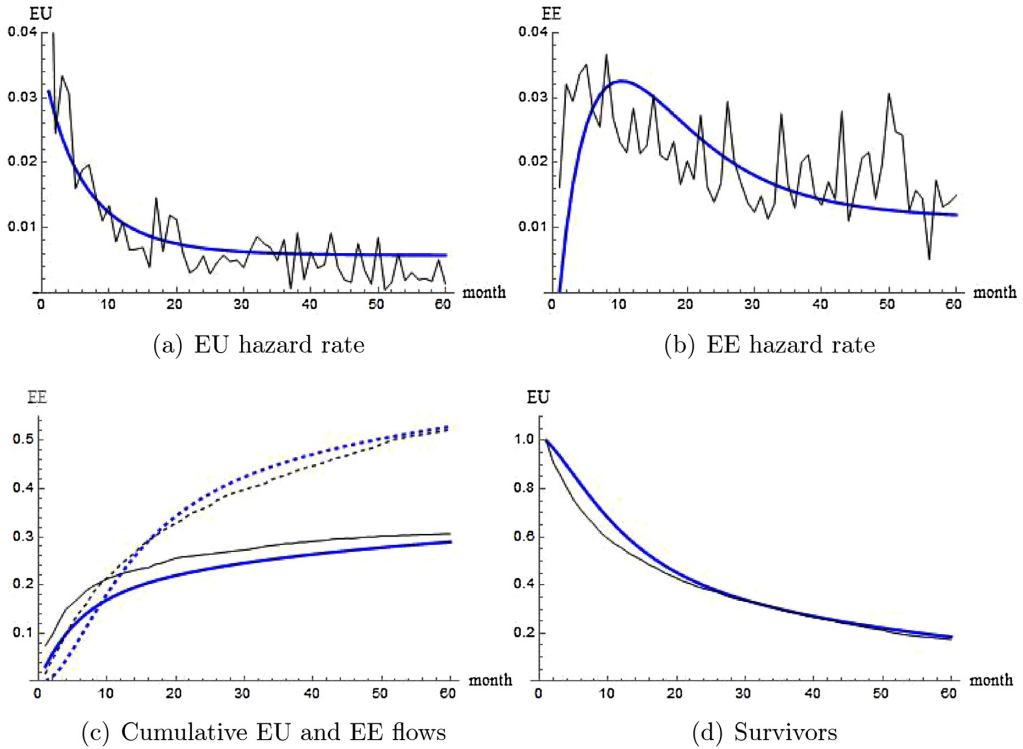


Fig. 1. Tenure Profiles.

Notes: Tenure profiles for male workers with high-school degree in the SIPP (thin black) and in the model (thick blue).

productivity of labour. This percentage has now become the standard target for  $b$  in the literature, in light of Hall and Milgrom (2008). Our findings are qualitatively robust to changes in the value of these parameters.

The calibrated match quality distribution is a Weibull with shape parameter  $\alpha = 1.5$  and scale parameter  $\beta = 0.34$ . This is a distribution with an SD of 21%, a skewness of 1.1, and a kurtosis of 4.4. The calibrated  $\phi$  is 1.89, which implies that, on average, a firm-worker pair discovers the quality of their match in six months. The calibrated  $\eta$  is 0.32, which implies that, on average, the quality of a match changes once every three years. The calibrated  $\chi$  is 0.67, which implies that workers search with 33% lower intensity while employed than while unemployed. The calibrated values of the other parameters are  $k = 0.028$ ,  $\delta = 0.012$ ,  $b = 0.78$  and  $r = 0.04$ .

Figure 1 plots the tenure profiles of the EU and EE rates and the cumulative EU and EE flows by tenure. Even though the model is very parsimonious, it fits the empirical tenure profiles quite well. The assumption that the quality of a match is initially unknown means that selection takes place after, rather than before, the start of the employment relationship. For this reason, the rate at which matches dissolve falls with tenure, as matches of quality  $z < R$  dissolve as soon as their quality is discovered, matches of quality  $z \in [R, Q]$ , with  $Q$  such that  $V(Q) = V_0$ , dissolve as soon as the worker finds a new match, and matches with quality  $z > Q$  are maintained. The assumption that the quality of a match is discovered at a Poisson rate controls the speed at which

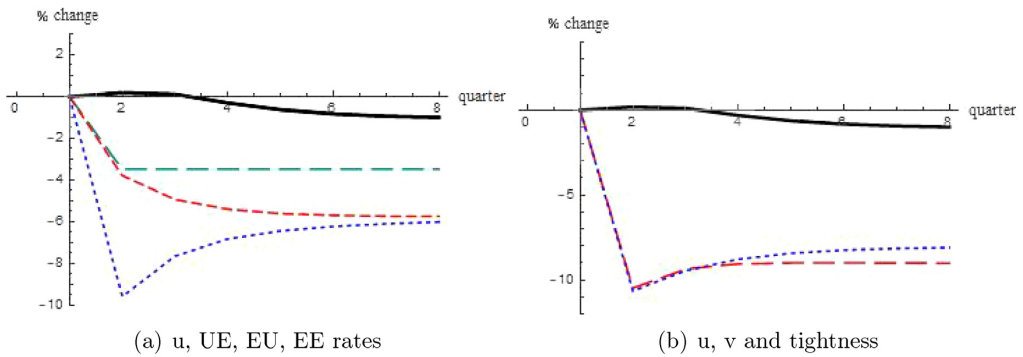


Fig. 2. High Discounts.

Notes: Percentage change relative to steady state for  $u$  (black, solid), UE rate (green, long dash), EU rate (red, medium dash), EE rate (blue, short dash),  $v$  (red, long dash),  $\theta$  (blue, short dash). Transition rates computed by comparing employment state at one-month intervals and then aggregated at quarterly level.

the exit rate declines with tenure. The assumption that the quality of a match is redrawn controls the exit rate even at long tenures.

As expected, condition (23) is satisfied at the calibrated parameter values. The job-finding rate for unemployed workers is  $p(\theta_u) = 0.26 \cdot 12 = 3.12$  per year. The difference between the efficiency of search off and on the job is  $1 - \chi = 0.33$ . The rate at which the quality of a match is reset is  $\eta = 0.32$  per year. Overall,  $(1 - \chi)p(\theta_u) - \eta = 0.69 > 0$ . Since condition (23) holds, Proposition 1 applies to the calibrated model.

## 2.2. Discount Shock

Using the calibrated model, we compute the response of the labour market to a positive discount rate shock. Specifically, we assume that the economy is at the steady state associated with a discount rate  $r$  of 4% per year (the steady state at which the model is calibrated). We then hit the economy with an unanticipated and permanent increase in the discount rate from 4 to 10% per year.<sup>12</sup> The exact magnitude of the discount rate shock is arbitrary, but it does not qualitatively affect our findings.

The left panel in Figure 2 shows the response of the UE, EU and EE rates to the  $r$ -shock. The UE rate falls by 3.5%. The EU rate falls by approximately 3% on impact. Over time, the EU rate keeps falling and reaches a level that is about 6% lower than before the shock. The EE rate falls by 9% on impact. Over time, the EE rate recovers and settles at a level that is about 6% lower than before the shock. The UE rate falls because the increase in  $r$  lowers the tightness  $\theta_u$  of the submarket where unemployed workers look for vacancies. On impact, the EU rate falls because the increase in  $r$  lowers the reservation quality  $R$  and, hence, the firm–worker pairs that

<sup>12</sup> Hall (2017) does not directly consider shocks to the discount rate, but shocks to the marginal utility of income. The shocks to the marginal utility of income are similar to discount rate shocks. For instance, when the economy is in a state where the marginal utility of income is unusually low and, thus, expected to rise, discounting of future income is low. Conversely, when the economy is in a state where the marginal utility of income is unusually high and, thus, expected to fall, discounting of future income is high. We have simulated a version of the model with the same shocks to marginal utility as in Hall (2017) and found qualitatively similar results. Specifically, we found that the UE and EU rate move together, rather than against each other.



discover the quality of their match are less likely to break up. Over time, the EU rate continues to fall because—due to an overall decline in the hiring rate—the fraction of firm–worker pairs that are in a match of unknown quality goes down, and so does the fraction of firm–worker pairs that discover the quality of their match in a given month. On impact, the EE rate falls because the increase in  $r$  lowers the average tightness of the submarkets where employed workers look for vacancies. Over time, the EE rate recovers because—due to the decline in the reservation quality—the distribution of employed workers shifts towards low-quality matches, which have a higher EE rate.

The right panel in Figure 2 shows the response of unemployment, vacancies and aggregate market tightness to the  $r$ -shock. The unemployment rate is subject to two opposing forces. On the one hand, unemployment is pushed up by the decline in the UE rate. On the other hand, unemployment is pushed down by the decline in the EU rate. On impact, the two forces nearly cancel each other and unemployment barely changes. Over time, the second force becomes stronger and stronger and unemployment starts falling. Eventually, unemployment settles down to a level that is 1% lower than before the shock. The vacancy rate falls by about 9%. The aggregate tightness of the labour market, defined as the ratio of vacancies to unemployment, falls by 8%.

Even though the  $r$ -shock does not affect technology directly, it does lead to a decline in labour productivity. In fact, the increase in  $r$  shifts the distribution of employed workers towards matches of lower quality. First, the increase in  $r$  lowers the reservation quality and, for this reason, the rate at which workers leave low-quality matches for unemployment. Second, the increase in  $r$  lowers the EE rate and, for this reason, the rate at which workers leave low-quality matches for other matches. Overall, labour productivity declines by 0.25%. Borrowing the language of Barlevy (2002), we say that an increase in  $r$  has a ‘sullyng’ effect.

The response of the labour market to an  $r$ -shock does not resemble the behaviour of the US labour market in a typical recession. In a typical recession, the UE rate falls by about 30%, the EU rate increases by about 20% and, as a result of both the movement in the UE and EU rates, the unemployment rate increases by around 50%. An  $r$ -shock, in contrast, leads to a decline in both the UE and the EU rate. The decline in the EU rate dominates the decline in the UE rate and the unemployment rate actually falls, albeit by as little as 1%.

### 2.3. Productivity Shock

We now want to compare and contrast the response of the labour market to a positive discount rate shock and the response of the labour market to a negative productivity shock. We assume that the economy is at the steady state associated with a discount rate  $r$  of 4% per year and an aggregate component of productivity  $y$  of 1 (the steady state at which the model is calibrated). We then hit the economy with an unanticipated and permanent negative shock to the aggregate component of productivity of 5%. The exact magnitude of the shock is arbitrary, but it does not qualitatively affect our findings.

The left panel in Figure 3 shows the response of the UE, EU and EE rates to the  $y$ -shock. The UE rate falls by 10%. The EU rate increases by more than 50% on impact and then falls to a level that is 30% higher than before the shock. The EE rate falls by approximately 15%. As discussed in Menzio and Shi (2011), the UE rate falls because the  $y$ -shock lowers the difference between the value of a firm–worker match and the value of unemployment and, hence, it induces unemployed workers to search for vacancies in a submarket with a lower tightness. The EU rate

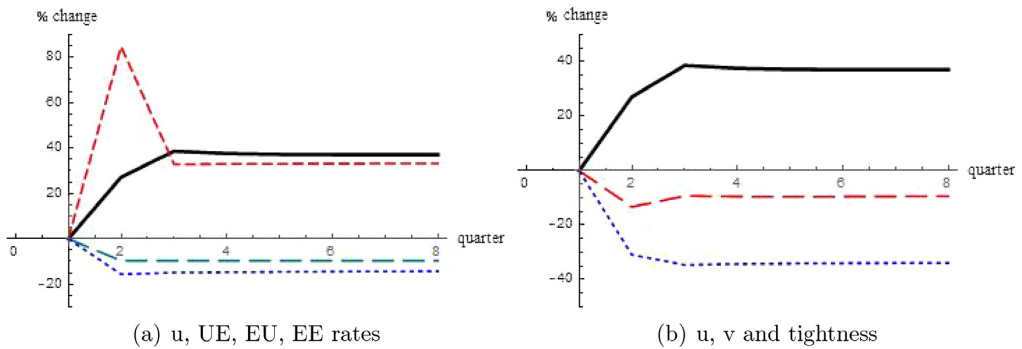


Fig. 3. *Low Productivity.*

Notes: Percentage change relative to steady state for  $u$  (black, solid), UE rate (green, long dash), EU rate (red, medium dash), EE rate (blue, short dash),  $v$  (red, long dash),  $\theta$  (blue, short dash). Transition rates computed by comparing employment state at one-month intervals and then aggregated at quarterly level.

increases because the  $y$ -shock raises the reservation quality, as it lowers the difference between the value of a firm–worker match and the value of unemployment. The EE rate falls because the  $y$ -shock lowers the average tightness of the submarkets where employed workers look for vacancies.

The right panel in Figure 3 shows the response of unemployment, vacancies and aggregate market tightness to the  $y$ -shock. Unemployment increases by almost 40%. The increase in unemployment is caused by both the decline in the UE rate and by the increase in the EU rate. Vacancies fall by approximately 10%, as the result of two opposing forces. On the one hand, both unemployed workers and employed workers look for vacancies in submarkets with a lower tightness. This tends to lower vacancies. On the other hand, the fraction of unemployed workers increases and the fraction of employed workers decreases. Since unemployed workers search in tighter submarkets than employed worker, this tends to increase vacancies. Overall, the first effect dominates and vacancies decline.

We also find that labour productivity falls by 4.5%, which is less than the decline in the aggregate component of productivity  $y$ . The decline in labour productivity is dampened because, in response to the shock, firms and workers increase their reservation quality causing an increase in the average idiosyncratic component of productivity  $z$ . Borrowing the language of Mortensen and Pissarides (1994), we say that a  $y$ -shock has a ‘cleansing’ effect.

Overall, the response of the labour market to the  $y$ -shock reproduces quite closely the behaviour of the labour market in a typical US recession: the UE and EE rate fall, while the EU rate increases; the unemployment rate rises, while the vacancy rate falls. As in the data, the fluctuations in labour market outcomes are much larger than the decline in labour productivity. The percentage decline of the UE rate is 2.5 times larger than the percentage decline in labour productivity, the percentage increase of the EU rate six times larger, the percentage decline in the EE rate three times larger, and the percentage increase in unemployment six times larger. These findings confirm the results in Menzio and Shi (2011), even though the model considered here is richer and the calibration strategy uses more data.

The main shortcoming of the hypothesis that labour market fluctuations are driven by  $y$ -shocks is not lack of amplification—a point made by Shimer (2005) in the context of a much simpler

search-theoretic model, and then prominently repeated by Hall (2017) and Ljungqvist and Sargent (2017). The main shortcoming of the hypothesis of  $y$ -shocks driving cyclical fluctuations in the labour market is that, since 1984, the contemporaneous correlation between labour productivity and unemployment is effectively zero, a fact that is largely due to labour productivity recovering much more quickly than unemployment after a recession. In this sense, the main shortcoming of the hypothesis of  $y$ -shocks driving labour market fluctuations is lack of propagation.<sup>13</sup>

### 3. Robustness

In this section, we run a battery of robustness checks. First, we consider a version of the baseline model in which search is random rather than directed. We find that, as in the baseline model, an increase in  $r$  lowers both the UE and EU rates. Second, we consider a version of the model in which the quality of a new firm–worker match is known and evolves stochastically over time, rather than being unknown and revealed over time. We consider a jump process for quality à la Mortensen and Pissarides (1994), as well as a continuous Ornstein–Uhlenbeck process. In both cases, an increase in  $r$  lowers both the UE and the EU rate. Third, we consider a life-cycle version of the model in which workers accumulate human capital on the job. At the aggregate level, an  $r$ -shock generates the correct negative comovement between UE and EU rates. The shock, however, still generates a counterfactually positive comovement between the UE and EU rates for older workers.

#### 3.1. Random Search

In the baseline model, search is directed by the employment contracts offered by firms to workers. Now we want to consider a version of the model with random search. We assume that an unemployed worker meets a vacancy at the rate  $p(\theta)$ , an employed worker meets vacancy at the rate  $\chi p(\theta)$ , where  $\theta$  denotes the ratio between the measure of vacancies  $v$  and the measure of searching workers  $u + \chi(1 - u)$ . Similarly, a vacancy meets an unemployed worker at the rate  $q(\theta)u/(u + \chi(1 - u))$  and an employed worker at the rate  $q(\theta)\chi(1 - u)/(u + \chi(1 - u))$ . Upon meeting, a firm and a worker bargain over the terms of a bilaterally efficient contract. The outcome of the bargain maximises the Nash product, where the gains from trade accruing to the worker are taken to the power of  $\mu$ , with  $\mu \in [0, 1]$ , and the gains from trade accruing to the firm are taken to the power of  $1 - \mu$ . The gains from trade accruing to either party are defined as difference between the value of the match to that party and their outside option. The outside option of the firm is the value of a vacancy. If the worker is unemployed, his outside option is the value of unemployment. If the worker is employed, his outside option is the joint value of the match with his current employer. These are the same specifications of the bargaining solution as in Cahuc *et al.* (2006) and many subsequent papers.

Under random search, the equilibrium is not block recursive—in the sense that the value and policy functions depend not only on the state of the aggregate fundamentals,  $\omega$ , but also on the distribution of workers across employment states,  $\{u, n, G\}$ . For this reason, the equilibrium cannot be solved outside of a non-stochastic steady state. At a steady state, the reservation quality

<sup>13</sup> Kaplan and Menzio (2016), Gali and van Rens (2021) and Golosov and Menzio (2020) propose alternative theories of labour market fluctuations, which generate low contemporaneous correlation between labour productivity and unemployment and that, unlike the high discount theory, do not imply counterfactual movements in the EU rate.

$R$  is such that

$$yR = b + (1 - \chi)p(\theta)\mu S_0 - \eta S_0. \quad (34)$$

The left-hand side of (34) is the reservation quality multiplied by the aggregate component of productivity (i.e., the flow income from employment at the reservation quality). The right-hand side is the sum of three terms. The first term is the flow income from unemployment. The second term is the difference between the flow value of searching while unemployed rather than while employed at the reservation quality. The third term is the negative of the flow value of an idiosyncratic productivity shock while employed at the reservation quality. Condition (34) suggests that, as long as an increase in the discount rate  $r$  lowers the tightness of the labour market,  $\theta$ , and the surplus of a firm–worker match of unknown quality,  $S_0$ , it will also lower the steady-state reservation quality  $R$  as long as  $(1 - \chi)p(\theta)\mu > \eta$ .

We calibrate the random-search version of the model exactly as in Section 2. We do not attempt to calibrate the worker’s bargaining power,  $\mu$ . The natural target to calibrate  $\mu$  is the average change in a worker’s wage upon transitioning from one job to another job. However, in order to compute the analogue of this target in the model, we would have to make some assumptions about the timing of wage payments from the firm to the worker, assumptions that are of no consequence to the equilibrium allocation. Instead, we carry out the calibration for three different values of the worker’s bargaining power: 0.25, 0.5 and 0.75.<sup>14</sup> For each calibration, we find that a permanent, unanticipated increase in the discount rate from 4% to 10% generates a decline in the steady-state UE, EU and EE rates.

### 3.2. Stochastic Productivity

In the baseline model, the quality of a new match is unknown and it is discovered at some rate  $\phi$ . Now we want to consider a version of the model in which the quality of a new match is known, but changes over time according to some stochastic process.

We first consider a stochastic process similar to the one in Mortensen and Pissarides (1994). In Mortensen and Pissarides, the idiosyncratic productivity of a new match is  $z_0 = z_h$ . At some rate  $\eta \geq 0$ , the idiosyncratic productivity of the match changes and it is randomly drawn from a cumulative distribution function  $F$ , where  $F$  is uniform with support  $[z_\ell, z_h]$ . This stochastic process is meant to capture the idea that a firm designs a new job optimally, so that its productivity is initially at the technological frontier. Over time, though, the economic environment facing the firm might change, so that the productivity of the job might fall below the technological frontier. In order to preserve the spirit of the stochastic process proposed by Mortensen and Pissarides (1994) while relaxing the assumption that the distribution  $F$  is uniform, we assume that  $z_0$  is the 90th percentile of  $F$ , where  $F$  is some Weibull distribution with shape parameter  $\alpha$  and scale parameter  $\beta$ .

In the Mortensen–Pissarides version of the model, the reservation productivity  $R$  is such that

$$yR = b + (1 - \chi)[p(\theta_u)S(z_0) - k\theta_u] - \eta \int_R S(z)dF(z). \quad (35)$$

<sup>14</sup> Let us place these values for the worker’s bargaining power  $\mu$  in context. Cahuc *et al.* (2006) use French data and estimate  $\mu$  by worker’s occupation and sector, and find  $\mu$ ’s ranging from 0 (low-skill occupations in manufacturing) to 0.98 (high-skill occupations in construction). Bagger *et al.* (2014) use Danish data and estimate  $\mu$  to be between 0.29 and 0.31, depending on the worker’s educational attainment. Herkenhoff *et al.* (2018) use US data and estimate  $\mu$  to be 0.65. Gregory (2020) uses German data and estimates  $\mu$  to be 0.66. All of these studies assume that the firm keeps the wage of the worker constant until the worker receives an outside offer that can only be matched by raising the wage.

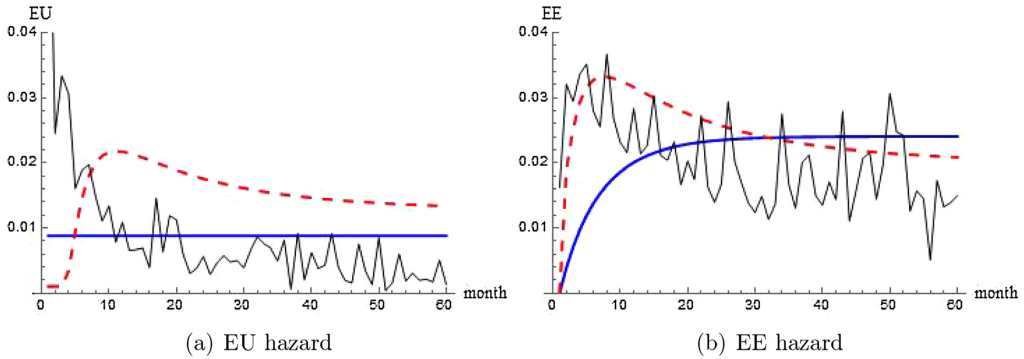


Fig. 4. Tenure Profiles with Stochastic Productivity.

Notes: EU and EE hazard rates by tenure in the data (black), in the Mortensen–Pissarides version of the model (blue, solid), and in the Ornstein–Uhlenbeck version of the model (red, dashed).

The left-hand side of (35) is the reservation quality multiplied by the aggregate component of productivity. The right-hand side is the sum of three terms. The first term is the flow income from unemployment. The second term is the difference between the flow value of searching while unemployed rather than while employed at the reservation quality. The third term is the negative of the flow value of an idiosyncratic productivity shock while employed at the reservation quality. Condition (35) suggests that, as long as  $\chi$  and  $\eta$  are not too large, an increase in the discount rate  $r$  should lower the reservation quality  $R$  and, hence, the EU rate.

We calibrate the Mortensen–Pissarides version of the model using a strategy similar to the one deployed in Section 2. Specifically, we choose the vacancy cost,  $k$ , to match the average UE rate. We choose the intensity of search on the job,  $\chi$ , to match the average EE rate. We choose the scale,  $\beta$ , of the  $F$  distribution to match the average EU rate. We choose the arrival rate,  $\eta$ , of an idiosyncratic productivity shock to match the fraction of jobs with short duration. We choose the shape,  $\alpha$ , of the  $F$  distribution so as to minimise the distance between the various tenure profiles in the model and in the data.

Figure 4 overlays the tenure profiles in the data and in the model. The Mortensen–Pissarides version of the model does not fit the data nearly as well as the baseline model. In particular, this version of the model predicts an EU hazard rate that is independent of tenure and equal to  $\eta F(R)$ . In contrast, the data (as well as the baseline model) features an EU hazard rate that sharply declines with tenure. Figure 5 shows the response of the Mortensen–Pissarides version of the model to a permanent and unanticipated increase in the discount rate from 4% to 10%. As in the baseline model, the UE, EU and EE rates fall in response to the  $r$ -shock. In the data, however, the UE and EU rates move against each other at the business cycle frequency.

We next consider a continuous stochastic process for the idiosyncratic productivity of a match. We assume that the productivity of a new match is some  $z_0$ . Over time, the productivity of the match evolves according to the Ornstein–Uhlenbeck process

$$dz = \rho(z^* - z)dt + \sigma dW_t, \tag{36}$$

where  $W_t$  is a standard Wiener process,  $\sigma$  is the SD of the Wiener process,  $\rho$  is a parameter that controls the speed at which  $z$  moves towards a mean-reversion point  $z^*$ . The Ornstein–Uhlenbeck process is, essentially, a continuous-time version of an AR(1) process.

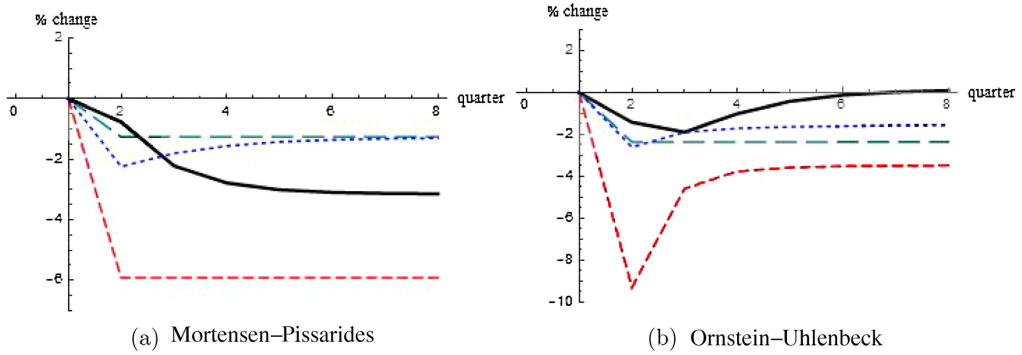


Fig. 5. High Discounts with Stochastic Productivity.

Notes: Percentage change relative to steady state for  $u$  (black, solid), UE rate (green, long dash), EU rate (red, medium dash), EE rate (blue, short dash).

In the Ornstein-Uhlenbeck version of the model, the reservation productivity  $R$  is such that

$$yR = b + (1 - \chi) [p(\theta_u)S(z_0) - k\theta_u] - \frac{1}{2}S''(R)\sigma^2. \tag{37}$$

The left-hand side of (37) is the reservation quality multiplied by the aggregate component of productivity. The right-hand side is the sum of three terms. The first term is the flow income from unemployment. The second term is the difference between the flow value of searching while unemployed rather than while employed at the reservation quality. The last term is the negative of the flow value of an idiosyncratic productivity shock while employed at the reservation quality. The last term depends on the convexity of the surplus and on the SD of the Wiener process. Condition (37) suggests that, as long as  $\chi$  and  $\sigma$  are not too large, an increase in the discount rate  $r$  should lower the reservation quality  $R$  and, in turn, the EU rate.

We calibrate the Ornstein-Uhlenbeck version of the model as follows. We choose the vacancy cost,  $k$ , to match the UE rate. We choose the SD,  $\sigma$ , of the Wiener process to match the fraction of short duration jobs. We choose the mean-reversion point,  $z^*$ , the speed of convergence to the mean-reversion point,  $\rho$ , and the intensity of search on the job,  $\chi$ , so as to minimise the distance between the model-generated and the empirical EU rate, EE rate and tenure profiles.

The Ornstein-Uhlenbeck version of the model does not fit the data as well as the baseline model. In particular, this version of the model predicts an EU rate that is hump-shaped with respect to tenure. In the data, however, the EU rate declines monotonically with tenure. Figure 5 shows the response of the Ornstein-Uhlenbeck version of the model to a permanent and unanticipated increase in the discount rate from 4% to 10%. As in the baseline model, the UE, EU and EE rates fall in response to the  $r$ -shock. In the data, however, the UE and EU rates move in opposite directions.

### 3.3. Human Capital Accumulation

We now consider a life-cycle version of the model in which workers accumulate human capital while on the job. This version of the model is natural, as well as relevant to our stated goal of revisiting the hypothesis that labour market fluctuations are driven by fluctuations in the discount

rate. The version of the model is natural because it is well documented that average wages grow over the life-cycle and that human capital accumulation is the main source of such growth (see, e.g., Heckman *et al.*, 1998; Bagger *et al.*, 2014; Gregory, 2020). The extension is relevant for our goal because, in a recent and prominent follow-up to Hall (2017), Kehoe *et al.* (2019) show that the response of the labour market to a discount rate shock is amplified when an employment relationship does not only produce a flow of output, but also allows the worker to grow his stock of human capital.

We use the model developed and calibrated by Menzio, Telyukova and Visschers (2016, henceforth MTV), which is a life-cycle version of our baseline model in which workers accumulate human capital on the job. Workers enter the labour market when young and exit the labour market when old according to probabilistic functions  $\mu(t)$  and  $\nu(t)$  of the worker's age  $t$ . The labour market is organised in submarkets indexed by the value  $x$  offered by the vacancy to the worker and by the required human capital,  $h$ , and age,  $t$ , of the worker. Workers choose in which submarket to search and firms choose in which submarket to open vacancies, taking as given the equilibrium tightness function  $\theta(x, h, t)$ . When matched, a firm and a worker produce a flow of  $h\gamma z$  units of output. The quality  $z$  of the match is initially unknown, it is discovered at the rate  $\phi$ , and it is re-drawn at the rate  $\eta$ . The wage of the worker is set as a constant fraction of the output flow, where the fraction obviously depends on the offered value  $x$ . The human capital of the worker depends on his months of work-experience  $e$  according to the function  $h = g(e)$ . We refer the reader to MTV for an exhaustive description of the environment, the equilibrium conditions, and the welfare properties of the equilibrium.

The model is calibrated using SIPP data on male workers with a high-school degree, and no further degree. As we did in Section 2, we calibrate the model to match the average UE, EU and EE rates and the EU and EE tenure profiles. The probability of entry into and exit from the labour market is calibrated to match the fraction of workers of age  $t$  entering and exiting the labour market. The human capital accumulation function, which they take to have the functional form  $g(e) = (1 - \rho_1) + \rho_1(1 + e)^{\rho_2}$ , is calibrated to match the life-cycle profile of average wages.<sup>15</sup> Table 1 in MTV contains the calibrated value of the parameters. Here, it suffices to say that the calibrated human capital accumulation function ( $\rho_1 = 4.3$  and  $\rho_2 = 0.065$ ) displays steep decreasing returns to experience, in the sense that a worker's productivity nearly doubles with the first 2.5 years of experience and only increases by an additional 45% with the next 5 years of experience.<sup>16</sup> Tables 2, 3 and 4 in MTV report the fit between the targeted moments and the data. Figures 12, 14 and 16 in MTV show that the calibrated model explains well the profile of the UE, EU and EE rates across different age groups.

Using the calibrated model, we simulate the response of the labour market to a positive discount rate shock. As we did in Section 2, we assume that the economy is at the steady state associated

<sup>15</sup> The functional form for  $g$  is flexible. The parameter  $\rho_1$  controls the slope of the function, and the parameter  $\rho_2$  controls the curvature of the function. The functional form for  $g$  is designed so that the human capital for a worker with no experience is always equal to 1 (a normalisation). The functional form for  $g$  is such that the human capital is monotonically increasing in experience.

<sup>16</sup> It is useful to compare our calibrated human capital function  $g$  with the one in Bagger *et al.* (2014). Both human capital functions are concave. The human capital function in Bagger *et al.* (2014) is increasing for the first 20 years of experience and, then, becomes decreasing. As we are sceptical of negative returns to experience, our human capital function is designed to be non-decreasing and, when calibrated, it only becomes flatter with experience. Our finding that  $r$ -shock generate a counterfactually positive comovement in the UE and EU rates of older workers would only be strengthened if the return to experience was negative for older workers.

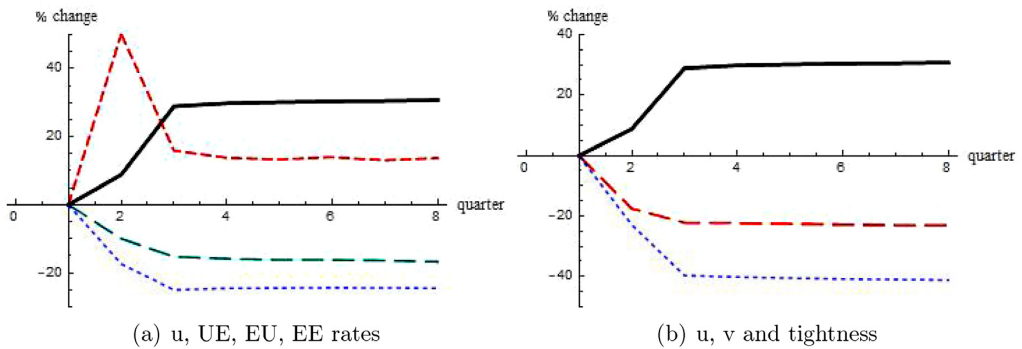


Fig. 6. *High Discounts.*

Notes: Percentage change relative to steady state for  $u$  (black, solid), UE rate (green, long dash), EU rate (red, medium dash), EE rate (blue, short dash),  $v$  (red, long dash),  $\theta$  (blue, short dash).

with a discount rate of 4% per year. We then hit the economy with an unanticipated, permanent increase in the discount rate from 4% to 10%. The left panel in Figure 6 shows the response of the aggregate UE, EU and EE rates to the  $r$ -shock. The right panel in Figure 6 shows the response of the aggregate unemployment and vacancy rates. A comparison between Figures 6 and 2 reveals that the response of the labour market to an  $r$ -shock changes dramatically once we take into account the fact that workers accumulate human capital on the job. In response to the same  $r$ -shock, the aggregate UE rate falls by 18% rather than 3.5%, the aggregate EU rate increases by 15% rather than falling by 6%, the aggregate EE rate falls by 25% rather than 6%. Similarly, the aggregate unemployment rate increases by almost 30% rather than falling by 1%, and the aggregate vacancy rate falls by about 20% rather than 9%. Once we take into account human capital accumulation, not only does the aggregate UE rate become more sensitive to discount rate shocks (a point already made by Kehoe *et al.*, 2019), but the aggregate EU rate moves in the right direction (a point that is novel to the literature).

We now turn to examine the response to the UE, EU and EE rates at a more disaggregated level. Figure 7 reports the response of transition and unemployment rates for workers in the age groups 21–30, 31–40, 41–50 and 51–60. For the youngest group of workers, the UE rate falls by 20% and the EU rate nearly doubles on impact and then settles at a level that is 40% higher than before the shock. The unemployment rate for these workers rises by 60%. For workers in the age group 31–40, the UE rate falls by 10% and the EU rate increases by 30% on impact and then settles at a level that is 5% higher than before the shock. The unemployment rate for these workers increases by about 18%. For the workers in the age group 41–50, the UE rate falls by 6% and the EU rate increases by 10% on impact and then quickly falls back to about the same level as before the shock. The unemployment rate for these workers increases by about 5%. For the oldest group of workers, the UE rate falls by 5% and the EU rate falls by 2%. The unemployment rate for these workers increases by about 2%.

The disaggregated analysis highlights two additional implications of  $r$ -shocks. First, while the  $r$ -shock generates a negative comovement between the UE and EU rates of younger workers, it still generates a positive comovement between the UE and EU rates of older workers. Second, the  $r$ -shock generates a much larger response in the UE, EU and unemployment rates of younger workers than of older workers.



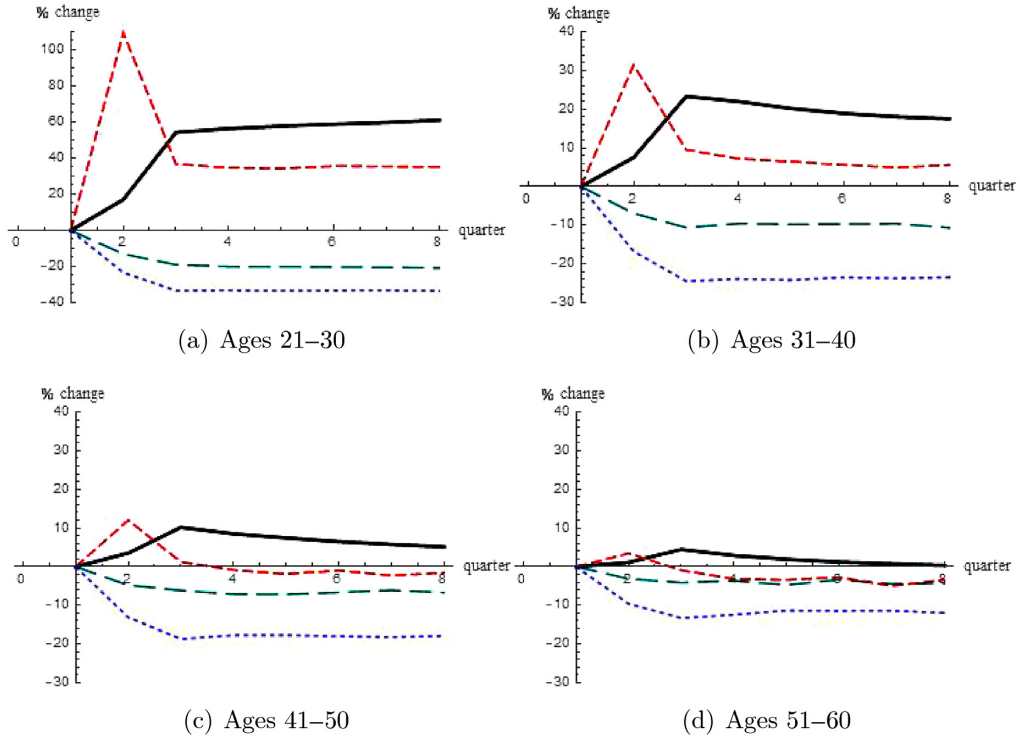


Fig. 7. High Discounts by Worker's Age.

Notes: Percentage change relative to steady state for  $u$  (black, solid), UE rate (green, long dash), EU rate (red, medium dash), EE rate (blue, short dash) for different age groups.

The findings in Figures 4 and 5 are easy to understand. First, consider the UE rate. The higher is the rate at which workers accumulate human capital on the job, the more backloaded is the income generated by a worker when he is employed relative to the income generated by the same worker when he is unemployed. For this reason, the higher is the rate of human capital accumulation, the more sensitive to an increase in the discount rate become the worker's value of moving from unemployment to employment and, in turn, the UE rate. Next, consider the EU rate. When workers do not accumulate human capital on the job, the income generated by a worker who is employed at the reservation quality  $R$  is frontloaded relative to the income generated by the same worker in unemployment. In this case, dissolving an employment relationship of quality  $R$  is an investment and, for this reason,  $R$  tends to fall in response to an increase in the discount rate. When workers accumulate human capital on the job at a sufficiently high rate, the income generated by a worker who is employed at  $R$  becomes backloaded relative to the income generated by an unemployed worker. In this case, keeping an employment relationship of quality  $R$  is an investment and, for this reason,  $R$  tends to rise in response to an increase in the discount rate.

At the aggregate level, the rate at which workers accumulate human capital is high enough for the  $r$ -shock to produce a large decrease in the UE rate and a substantial rise in the EU rate. The rate at which workers accumulate human capital on the job, though, falls sharply over the

life cycle and is low for older workers.<sup>17</sup> For this reason, the  $r$ -shock produces a much larger response in the UE rate of younger than older workers, a positive response in the EU rate of younger workers, and a negative response in the EU rate of older workers. To paint a picture, the response of labour market outcomes to an  $r$ -shock is similar to Kehoe *et al.* (2019) for younger workers, as these workers face a high return of experience on human capital. The response of labour market outcomes to an  $r$ -shock for older worker is essentially the same as in our baseline model, as these workers face a low return of experience.

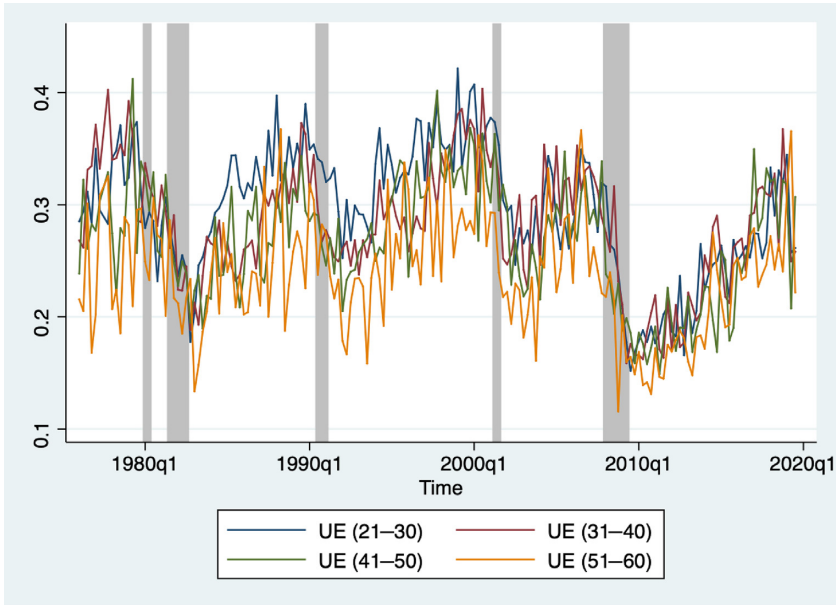
The findings in Figure 6 appear to vindicate the hypothesis of labour market fluctuations caused by  $r$ -shocks. Indeed, as it is observed in a typical recession, the response of the labour market to a positive  $r$ -shock features a sizeable decline in the aggregate UE rate, an equally large increase in the aggregate EU rate, and an even larger increase in unemployment. The findings in Figure 7 show, however, that the same forces that lead to a counterfactual *aggregate* implication of  $r$ -shocks in the baseline model, also lead to a counterfactual *disaggregate* implication of  $r$ -shocks in the model with human capital accumulation.<sup>18</sup> In fact, in a typical recession, the UE and the EU rates move against each other for all age groups (see Figure 8). In contrast, a positive  $r$ -shock lowers both the UE and EU rates of older workers. Moreover, in a typical recession, the UE and the EU rates move by approximately the same percentage for all age groups (see again Figure 8). In contrast, a positive  $r$ -shock generates much larger responses in the UE and EU rates of younger than older workers.

For the sake of comparison, we now compute the response of the labour market to a negative shock to the aggregate component of productivity. As in Section 2, we assume that the economy is at the steady state associated with a discount rate  $r$  of 4% and an aggregate component of productivity  $y$  equal to 1. We then hit the economy with an unanticipated and permanent negative shock to the aggregate component of productivity of 5%. Figure 9 shows the response of the aggregate UE, EU and EE rates to the  $y$ -shock (left panel) and the response of the aggregate unemployment and vacancy rates (right panel). The aggregate UE rate falls by about 7.5%; the aggregate EU rate increases by 35% on impact and then settles to a level that is 20% higher than before the shock; the aggregate EE rate falls by 10%. The aggregate unemployment rate increases by about 20%, while the aggregate vacancy rate falls by about 10%. Figure 10 shows the response of the UE, EU, EE and unemployment rates by age group. The response of UE, EU, EE and unemployment rates is very similar across workers of different ages. The decline in the UE rate is around 7.5% for workers aged 21–30, 31–40, 41–50 and 51–60. The EU rate increases for workers in all age groups, although the increase is slightly larger for younger workers than for older ones (as young workers are more likely to be in marginal matches than older ones).

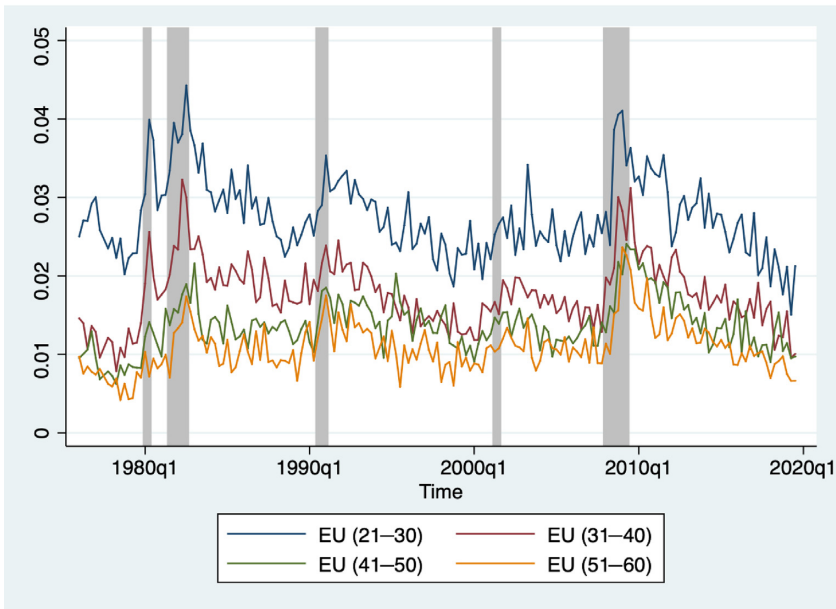
At the aggregate level, the response of the labour market to a negative  $y$ -shock is very similar to the response to a positive  $r$ -shock, both with respect to the magnitude of the response of different variables and with respect to the comovement of the response of different variables. Moreover, the aggregate response of the labour market to both  $y$  and  $r$  shocks is broadly consistent with the

<sup>17</sup> There is a consensus that older workers hardly accumulate any human capital on the job. The consensus is based on the properties of models where workers allocate their time on the job between learning and productive activities (see, e.g., Ben Porath, 1967), as well as from models in which workers mechanically accumulate human capital on the job estimated to the empirical life-cycle profile of wages, which flattens around age 45 (see, e.g., Bagger *et al.*, 2014). The consensus is so strong that the lack of human capital accumulation for older workers is used as a source of identification (see, e.g., Heckman *et al.*, 1998; Huggett *et al.*, 2011; Lagakos *et al.*, 2018; Gregory, 2020).

<sup>18</sup> We do not claim that discount rate shocks would generate the same counterfactual predictions in the context of any life-cycle model. We only claim that, if the return to experience falls with age, there is a natural economic force that makes the response of the UE and EU rates to a discount rate shock different for younger and older workers. Our life-cycle model illustrates this force.



(a) u, UE, EU, EE rates



(b) u, v and tightness

Fig. 8. UE and EU Rates by Worker's Age.

Notes: The empirical time series for the UE and EU rates by age are constructed using the CPS microdata from IPUMS (2020): (1) Compute the sample-weighted average of the fraction of unemployed workers in month  $t$  that report to be employed in month  $t + 1$  (raw UE rate) and, similarly, the fraction of employed workers in month  $t$  that report to be unemployed in month  $t + 1$  (raw EU rate). (2) Seasonally adjust the raw transition rates using a ratio-to-MA filter as in Shimer (2012). (3) Correct the resulting EU rate for time-aggregation bias.

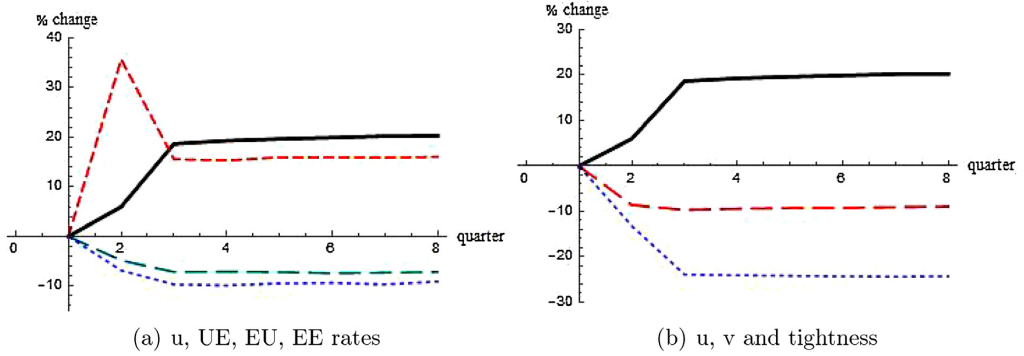


Fig. 9. *Low Productivity.*

Notes: Percentage change relative to steady state for  $u$  (black, solid), UE rate (green, long dash), EU rate (red, medium dash), EE rate (blue, long dash),  $v$  (red, long dash),  $\theta$  (blue, short dash).

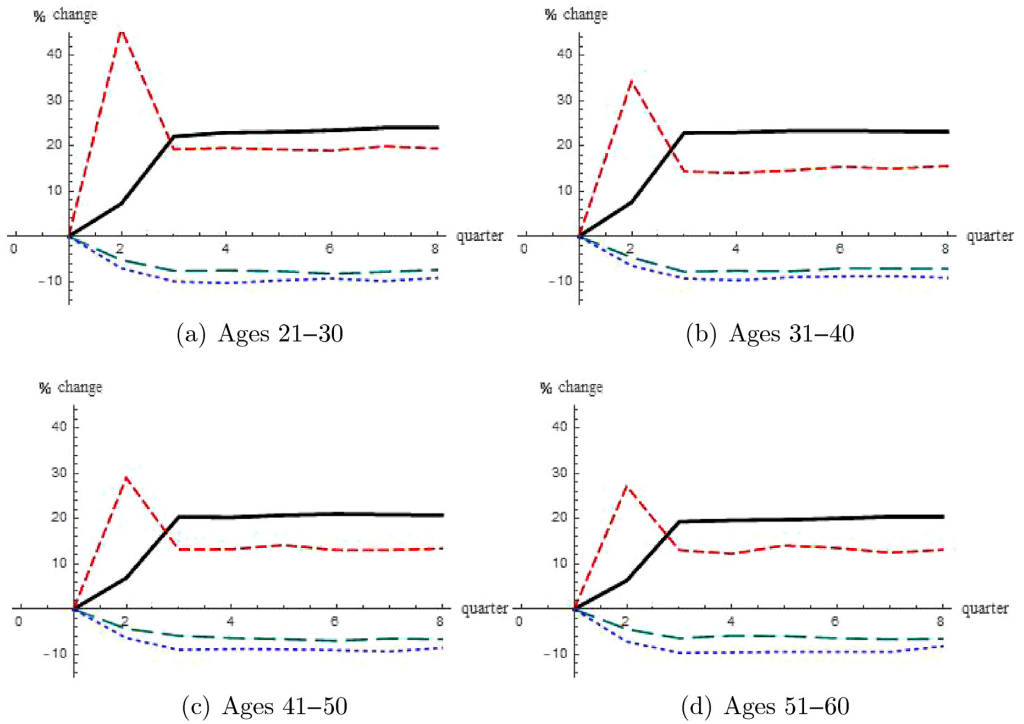


Fig. 10. *Low Productivity by Worker's Age.*

Notes: Percentage change relative to steady state for  $u$  (black, solid), UE rate (green, long dash), EU rate (red, medium dash), EE rate (blue, short dash) for different age groups.

behaviour of the US labour market in a typical recession. At a more disaggregate level, though, the two shocks have different implications. A positive  $r$ -shock generates labour market responses that are very heterogeneous across different age groups, while a negative  $y$ -shock generates responses that are fairly homogeneous across different age groups.

#### 4. Conclusions

Using a simple but rich search-theoretic model of the labour market in which the UE, EU and EE rates are all endogenous, we revisited Hall's hypothesis that cyclical fluctuations in unemployment are caused by shocks to the rate at which agents discount future income. Analytically, we showed that an increase in the discount rate not only lowers the UE rate—as pointed out by Hall (2017)—but it also lowers the EU rate—under some rather natural conditions. Quantitatively, we showed that—when the model is calibrated to match the average UE, EU and EE rates as well as the relationship between EU and EE rates and job tenure—an increase in the discount rate from 4 to 10% generates a 3.5% decline in the UE rate and a 6% decline in the EU rate. The response of the unemployment rate is extremely small. These findings are at odds with the actual behaviour of the US labour market over the business cycle, which features a negative comovement between the UE and EU rates and large fluctuations in the unemployment rate.

We showed that our findings are robust to several alternative specifications of our baseline model. Among the alternative specifications that we considered, the most interesting is one in which workers accumulate human capital on the job (as in Kehoe *et al.*, 2019). We found that, at the aggregate level, a discount rate shock does generate a negative comovement between the UE and the EU rate and a large increase in the unemployment rate. The shock, though, still generates a positive comovement between the UE and EU rate and a very small response in the unemployment rate for older workers, because their return to experience is close to zero. In contrast, in the US labour market, the UE and EU rates move against each other for all age groups, and they move by approximately the same percentage for all age groups.

We do not view our findings as definitive evidence against the high discount hypothesis. Rather, we view our findings as an illustration of a natural economic force which tends to produce—in the context of our simple model—a counterfactually positive comovement between the UE and the EU rate in response to discount rate shocks. We hope that our findings will prove useful in directing further research on the topic.

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Additional Supporting Information may be found in the online version of this article:

#### Replication Package

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