Gödel's Incompleteness Theorem

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Gödel's Incompleteness Theorem

or

Is there any guaranty that there would always be a job for physicists & mathematicians ?

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End of Physics ?

- "When I began my physical studies [in Munich in 1874] and sought advice from my venerable teacher Philipp von Jolly... he portrayed to me physics as a highly developed, almost fully matured science... Possibly in one or another nook there would perhaps be a dust particle or a small bubble to be examined and classified, but the system as a whole stood there fairly secured, and theoretical physics approached visibly that degree of perfection which, for example, geometry has had already for centuries." - from a 1924 lecture by Max Planck (Sci. Am, Feb 1996 p.10)

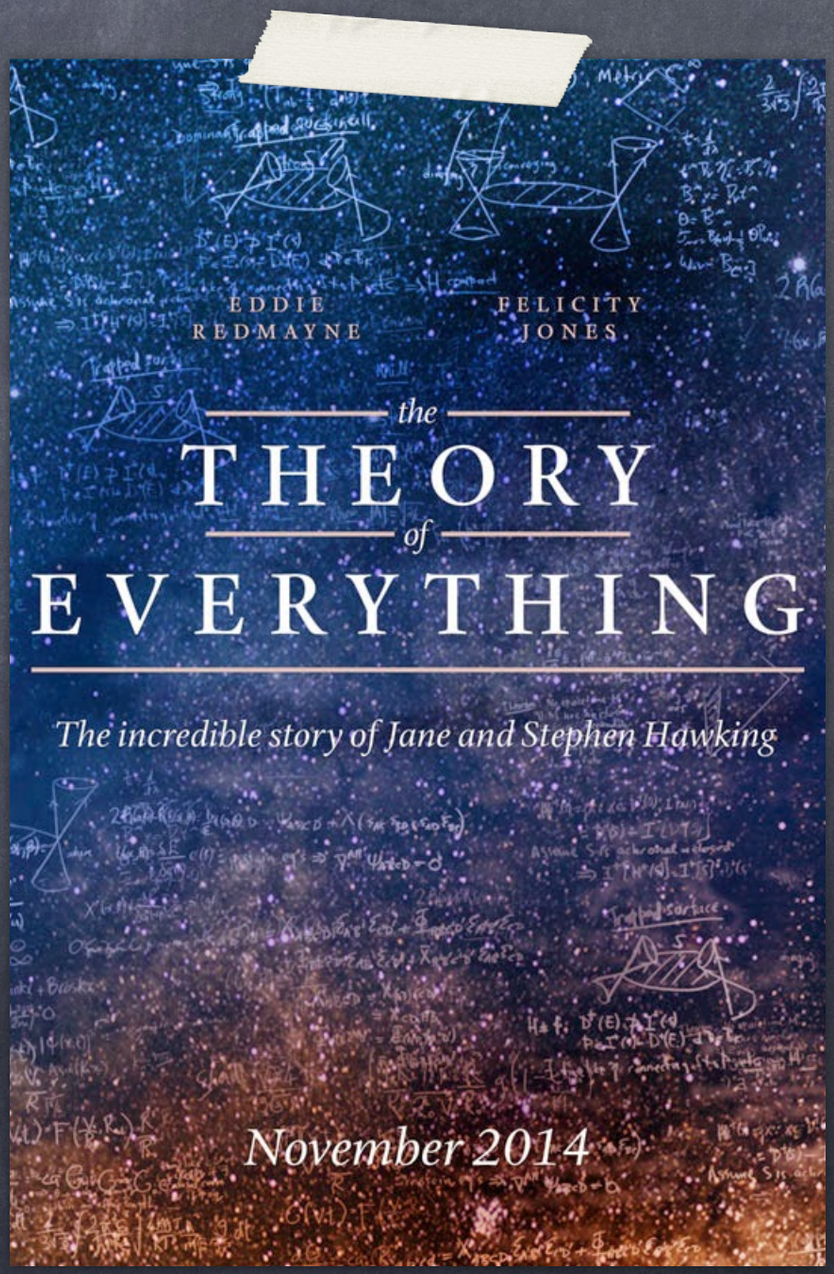
- "The more important fundamental laws and facts of physical science have all been discovered, and these are now so firmly established that the possibility of their ever being supplanted in consequence of new discoveries is exceedingly remote.... Our future discoveries must be looked for in the sixth place of decimals." - Albert. A. Michelson, speech at the dedication of Ryerson Physics Lab, U. of Chicago 1894

- "There is nothing new to be discovered in physics now. All that remains is more and more precise measurement" - Lord Kelvin (1900)

Theory of everything ?

- A theory of everything (ToE) or final theory refers to the hypothetical presence of a single, all-encompassing, coherent theoretical framework of physics that fully explains and links together all physical aspects of the universe.

- In parallel to the intense search for a ToE (like M-theory), various scholars have seriously debated the possibility of its discovery. Is it really possible to have a ToE and actually put an end to theoretical physics or it is just an illusion and the situation is quite like the end of the 19th century ?



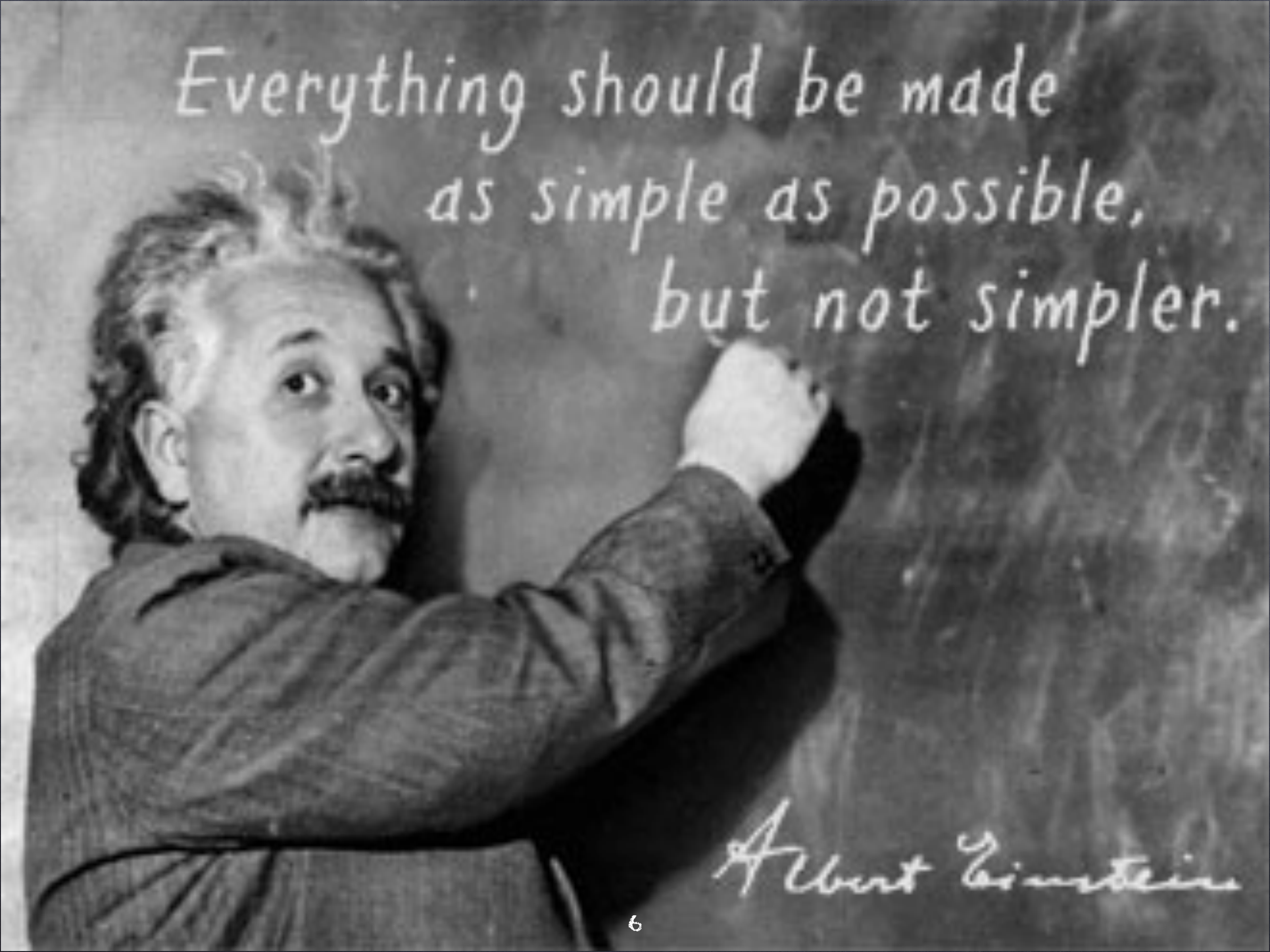
Gödel's Incompleteness Theorems

- First theorem: Any effectively generated theory capable of expressing elementary arithmetic cannot be both consistent and complete. In particular, for any consistent, effectively generated formal theory that proves certain basic arithmetic truths, there is an arithmetical statement that is true, but not provable in the theory (Kleene 1967, p. 250).

- Second theorem : For any formal effectively generated theory T including basic arithmetical truths and also certain truths about formal provability, if T includes a statement of its own consistency then T is inconsistent.

"Gödel proved that the world of pure mathematics is inexhaustible; no finite set of axioms and rules of inference can ever encompass the whole of mathematics; given any set of axioms, we can find meaningful mathematical questions which the axioms leave unanswered. I hope that an analogous situation exists in the physical world. If my view of the future is correct, it means that the world of physics and astronomy is also inexhaustible; no matter how far we go into the future, there will always be new things happening, new information coming in, new worlds to explore, a constantly expanding domain of life, consciousness, and memory."

-Freeman Dyson


Everything should be made
as simple as possible,
but not simpler.
Albert Einstein

Gödel's Incompleteness Theorems

- With these worries in mind, let us look a little more closely at what Gödel's result might have to say about physics. The situation is not so clear-cut as some commentators would often have us believe. It is useful to layout the precise assumptions that underlie Gödel's deduction of incompleteness. Gödel's first theorem says that if a formal system is :

(i) finitely specified

(ii) large enough to include arithmetic

(iii) consistent

then it is incomplete.

- Condition 2 means that the formal system includes all the symbols and axioms used in arithmetic. The symbols are 0, 'zero', S, 'successor of', +, x, and =.

"Gödel's theorem is proved using statements that refer to themselves. Such statements can lead to paradoxes. An example is, this statement is false. If the statement is true, it is false. And if the statement is false, it is true. Another example is, the barber of Corfu shaves every man who does not shave himself. Who shaves the barber? If he shaves himself, then he doesn't, and if he doesn't, then he does. Gödel went to great lengths to avoid such paradoxes by carefully distinguishing between mathematics, like 2+2 =4, and meta mathematics, or statements about mathematics, such as mathematics is cool, or mathematics is consistent. That is why his paper is so difficult to read. But the idea is quite simple. First Gödel showed that each mathematical formula, like 2+2=4, can be given a unique number, the Gödel number. The Gödel number of 2+2=4, is *. Second, the meta mathematical statement, the sequence of formulas A, is a proof of the formula B, can be expressed as an arithmetical relation between the Gödel numbers for A and B. Thus meta mathematics can be mapped into arithmetic, though I'm not sure how you translate the meta mathematical statement, 'mathematics is cool'. Third and last, consider the self referring Gödel statement, G. This is, the statement G can not be demonstrated from the axioms of mathematics. Suppose that G could be demonstrated. Then the axioms must be inconsistent because one could both demonstrate G and show that it can not be demonstrated. On the other hand, if G can't be demonstrated, then G is true. By the mapping into numbers, it corresponds to a true relation between numbers, but one which can not be deduced from the axioms. Thus mathematics is either inconsistent or incomplete."

-Stephen Hawking; Gödel and the End of the Universe.

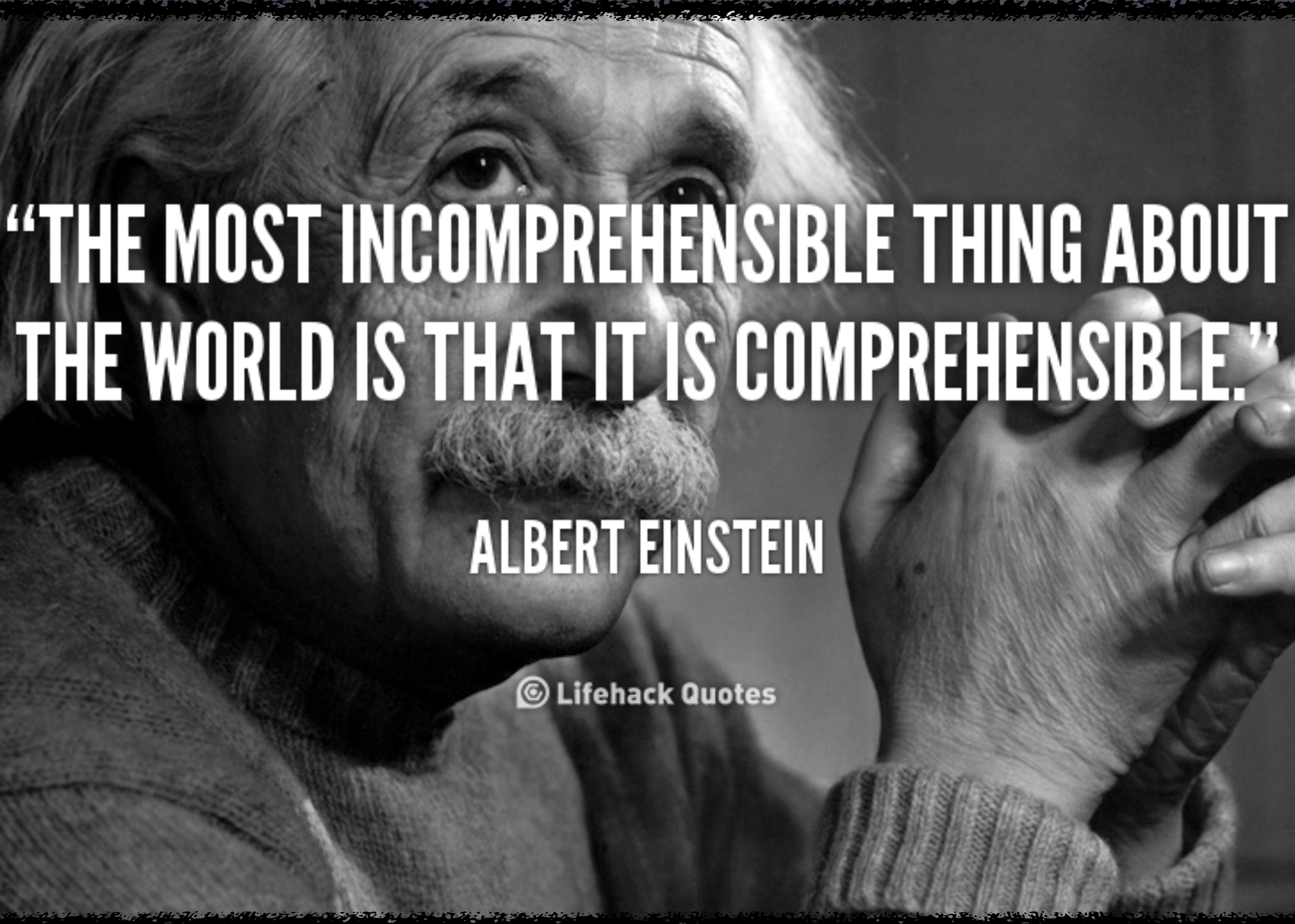
http://www.hawking.org.uk/godel-and-the-end-of-physics.html

Gödel's Incompleteness Theorems

- The structure of arithmetic plays a central role in the proof of Gödel's theorem. Special properties of numbers, like their factorisations and the fact that any number can be factored in only one way as the product of prime divisors (eg. 130 = 2×5×13), were used by Gödel to establish a crucial correspondence between statements of mathematics and statements about mathematics. Thereby, linguistic paradoxes like that of the 'liar' could be embedded, like Trojan horses, within the structure of mathematics itself. Only logical systems which are rich enough to include arithmetic allow this incestuous encoding of statements about themselves to be made within their own language.

"It's indeed the case that if the laws of physics are formulated in a formal system S which includes the concepts and axioms of arithmetic as well as physical notions such as time, space, mass, charge, velocity, etc., and if S is consistent then there are propositions of higher arithmetic which are undecidable by S. But this tells us nothing about the specifically physical laws encapsulated in S, which could conceivably be complete as such."

-Solomon Feferman


“THE MOST INCOMPREHENSIBLE THING ABOUT THE WORLD IS THAT IT IS COMPREHENSIBLE.”
ALBERT EINSTEIN
Lifehack Quotes

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Thank You